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Order-R Vacuum Action Functional in Scalar-Free Unified Theories with Spontaneous Scale Breaking

Stephen L. Adler

The Institute for Advanced Study, Princeton, New Jersey 08540

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It is shown that in unified theories containing only fermions and gauge fields and in which scale invariance is spontaneously broken, radiative corrections induce an order-R term in the vacuum action functional which is uniquely determined by the flat–space-time theory.

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In recent papers Minkowski,[1] Zee,[1] and Smolin[2] have suggested that spontaneous scale-invariance breaking may play an important role in a fundamental theory of gravitation. The basic mechanism considered by both involves an action[3]

$$S=\int d^4x(-g)^{1/2}[\tfrac{1}{2}\epsilon\varphi^2R-V(\varphi^2)+\text{kinetic terms}+\text{other fields}], \tag{1}$$

with φ a scalar field. The potential $V(\varphi^2)$ is assumed to have a minimum away from $\varphi=0$,

$$V'(\kappa^2)=0,\quad V''(\kappa^2)>0, \tag{2}$$

so that spontaneous symmetry breaking induces an effective gravitational action

$$S_{\text{grav}}=\int d^4x\,(-g)^{1/2}\tfrac{1}{2}\epsilon\kappa^2R, \tag{3}$$

with ϵ a free parameter. In this note I examine the analog of the above mechanism in unified theories which contain no fundamental scalar fields and in which scale invariance is spontaneously broken.[4] I show that in such theories the vacuum action functional contains an order-R term which is *explicitly calculable* in terms of the flat–space-time parameters of the theory. This result is basically an extension, to curved space-times, of the known fact that in such theories all mass ratios are explicitly calculable.

Consider a theory based on a scale-invariant classical Lagrangian, constructed from spin-$\frac{1}{2}$ fermion and spin-1 gauge fields, with the generally covariant renormalized matter action

$$\tilde{S}=\int d^4x\,(-g)^{1/2}(\mathcal{L}_{\text{matter}}+\text{counter terms}). \tag{4}$$

Because quantum effects induce nonlocal interactions with the space-time curvature, the vacuum action functional $\langle\tilde{S}\rangle_0$ *cannot* be related to its flat–space-time value by the equivalence principle. Instead, we have a formal decomposition

$$\langle\tilde{S}\rangle_0=\int d^4x\,(-g)^{1/2}\langle\tilde{\mathcal{L}}\rangle_0,$$

$$\langle\tilde{\mathcal{L}}\rangle_0=\sum_{n=0}^{\infty}\kappa^{4-2n}\langle\tilde{\mathcal{L}}\rangle_0^{(2n)}=\kappa^4\langle\tilde{\mathcal{L}}\rangle_0^{(0)}+\kappa^2\langle\tilde{\mathcal{L}}\rangle_0^{(2)}+\langle\tilde{\mathcal{L}}\rangle_0^{(4)}+\dots, \tag{5}$$

with κ the unification mass of the flat–space-time theory and with $\langle\tilde{\mathcal{L}}\rangle_0^{(2n)}$ homogeneous of degree $2n$ in derivatives ∂_x acting on the metric. Because the curvature scalar R is the only Lorentz scalar of order $(\partial_x)^2$, the second term in Eq. (5) has the form[5]

$$\langle \tilde{\mathcal{L}} \rangle_0^{(2)} = \beta R, \tag{6}$$

with β in general nonzero. According to a criterion of Weinberg,[6] the coefficient β will be calculable in the flat-space-time field theory, provided that there are *no possible* Lagrangian counter terms which contribute to this term. The only relevant counter terms of the general form

$$\Delta\mathcal{L} = \mathcal{O}_2 R, \tag{7}$$

with $\mathcal{O}_2$ a gauge-invariant operator with canonical dimension 2. However, in a theory with no fundamental scalars, and with spontaneous breaking of scale invariance (and hence no bare-mass parameters), the only dimension-2 operators are of the form $b_\mu{}^a b^{\mu a}$, with $b_\mu{}^a$ a gauge potential. But such operators are not gauge invariant, and hence no counter terms of the form of Eq. (7) are possible. Therefore, in the theories under consideration, β is finite and calculable (as opposed to merely renormalizable).

Following[7] Sakharov[8] and Klein,[9] it is tempting to regard the $\kappa^2\beta R$ term in Eq. (5) as the entire gravitational action, rather than as just an additional finite contribution to the gravitational action. This interpretation is clearly justified if the unified matter theory predicts the correct sign and magnitude of the Einstein action *and* if the virtual integrations contributing to β are dynamically cut off at energies well below the Planck mass. If the virtual integrations extend beyond the Planck mass, then use of the semiclassical, background metric analysis given above requires further justification or corrections, involving an analysis of possible quantum gravity effects.[10]

Added notes.—Calculations by Hasslacher and Mottola,[11] Mottola,[11] and Zee[11] in models obeying the premises of this note all give a nonvanishing induced order-R term, and show that the sign can correspond to attractive gravity.

Guo has brought to my attention a number of further references on R^2-type gravity Lagrangians.[12] In particular, it is known that a $C_{\mu\nu\lambda\sigma} \times C^{\mu\nu\lambda\sigma}$ gravity theory is renormalizable, but has a dipole ghost. Hence the extended matter-gravity theories discussed in Ref. 10 of this note are renormalizable; they could also be unitary (by the Lee-Wick[13] mechanism) if scale-symmetry breakdown causes the dipole ghost to split into a single positive-residue graviton pole at $k^2 = 0$, and a pair of complex-conjugate unstable ghost poles at $k^2 = M \pm i\Gamma$ (with M and Γ of order the unification mass). Detailed dynamical studies of the extended matter-gravity theories will be needed to settle the unitarity issue. Tomboulis[14] has given an interesting model with a dynamical Lee-Wick mechanism, and I wish to thank him for a discussion about this point.

Because dimension-4 and dimension-0 operators are always available (e.g., $\mathcal{L}_{\text{matter}}$ and the gauge-field bare coupling, respectively), the arguments of this note do not apply to the order-(0) or -(4) terms in Eq. (5). Hence, even in scalar-free theories with spontaneous scale-invariance breaking, the cosmological constant contains renormalizable infinities. An additional symmetry, very likely relating the boson and fermion sectors of the theory, will be needed to give a calculable cosmological constant. L. S. Brown and J. C. Collins point out that because dimension-0 operators are available, the induced R^2 term in the vacuum action can become divergent in high loop order; if this happens, a quadratic gravitational Lagrangian must include an R^2 term in addition to the term $C_{\mu\nu\lambda\sigma}C^{\mu\nu\lambda\sigma}$ discussed above.

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[1]P. Minkowski, Phys. Lett. 71B, 419 (1977); A. Zee, Phys. Rev. Lett. 42, 417 (1979).

[2]L. Smolin, Nucl. Phys. B160, 253 (1979). Smolin discusses a conformal-invariant scalar-vector theory, where the equation governing the classical minimum is $(\partial/\partial\varphi - 4/\varphi)V = 0$, rather then simply $V' = 0$ as in Minkowski's or Zee's model.

[3]I follow the conventions of C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[4]See, e.g., L. Susskind, Phys. Rev. D 20, 2619 (1979); S. Weinberg, Phys. Rev. D 13, 974 (1976), and D 19, 1277 (1979); or the $m_0 = 0$ version of S. L. Adler, Phys. Lett. 86B, 203 (1979), and "Quaternionic Chromodynamics as a Theory of Composite Quarks and Leptons," Phys. Rev. D (to be published).

[5]The term $\langle \mathcal{L} \rangle_0^{(4)}$, in addition to local contributions proportional to R^2, ... has nonlocal contributions arising from the effects of massless fields. See, for example, L. S. Brown and J. P. Cassidy, Phys. Rev. D 16, 1712 (1977). I am assuming that such nonlocal

terms do not appear in $\langle \mathfrak{L} \rangle_0^{(2)}$, but the presence of such terms would not alter the argument for the calculability of β. In noncompact manifolds there can be Lorentz scalars other than R which contribute (I wish to thank S. M. Christensen for this remark), but again these do not alter the argument given for the βR term.

[6]S. Weinberg, Phys. Rev. Lett. 29, 388 (1972). For a recent pedagogical review of the renormalization algorithm implicit in the calculability criterion, see L. S. Brown, "Dimensional Renormalization of Composite Operators in Scalar Field Theory" (unpublished).

[7]See C. W. Misner, K. S. Thorne, and J. A. Wheeler, Ref. 3, pp. 426–428.

[8]A. D. Sakharov, Dokl. Akad. Nauk. SSSR 177, 70 (1967) [Sov. Phys. Dokl. 12, 1040 (1968)].

[9]O. Klein, Phys. Scr. 9, 69 (1974).

[10]One possibility is that the metric is not a quantum variable, but is a classical dynamical variable governed by the Euler-Lagrange equations, in which case the background metric analysis given in the text is exact. Another possibility consistent with the viewpoint of the text is that the metric is a quantum variable, with dynamics governed by a scale-invariant fundamental Lagrangian (see L. Smolin, Ref. 2). The only generalization of Eq. (4) to include a scale-invariant gravitational action is $\tilde{S} = \int d^4x\,(-g)^{1/2}\,(\mathfrak{L}_{\text{matter}} + \delta C_{\mu\nu\lambda\sigma}C^{\mu\nu\lambda\sigma} + \text{counterterms})$, with $C_{\mu\nu\lambda\sigma}$ the Weyl tensor, and with δ a dimensionless coupling constant. Recent work of Stelle [K. S. Stelle, Phys. Rev. D 16, 953 (1977)] on quadratic gravitational actions suggests that this extended theory should still be renormalizable. The $\kappa^2\beta R$ term in Eq. (5) would still be calculable, even with quantum gravitational effects taken into account, but the β value calculated from the flat-space-time matter theory would be subject to a finite, δ-dependent renormalization. This renormalization could be important if the virtual integrations contributing to β extend to energies beyond the Planck mass.

[11]B. Hasslacher and E. Mottola, unpublished; E. Mottola, unpublished; A. Zee, unpublished.

[12]D. E. Neville, Phys. Rev. D 18, 3535 (1978), and unpublished; E. Sezgin and P. van Nieuwenhuizen, unpublished; H.-Y. Guo *et al.*, unpublished.

[13]T. D. Lee and G. C. Wick, Nucl. Phys. B9, 209 (1969); B10, 1 (1964); Phys. Rev. D 2, 1033 (1970).

[14]T. Tomboulis, Phys. Lett. 70B, 361 (1977).

Choosing an Expansion Parameter for Quantum Chromodynamics

L. F. Abbott
Department of Physics, Brandeis University, Waltham, Massachusetts 02254
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When expanded in powers of $\bar{\alpha}$ where $\bar{\alpha}$ is chosen to satisfy $\bar{\alpha}(Q_0^2) = 4\pi/\beta_0 \ln(Q_0^2/\Lambda^2) = \alpha_{\overline{\text{MS}}}(Q_0^2)$ with $Q_0^2 \approx 10$ GeV^2, the quantum chromodynamics (QCD) perturbation series seems to be extremely well behaved. In particular, the large third-order corrections discussed recently by Moshe do not appear when this $\bar{\alpha}$ is used as the QCD expansion parameter.

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The behavior of the perturbation expansion in quantum chromodynamics (QCD) depends critically on the choice of the expansion parameter used to define the perturbation series. It has been noted previously that nonleading corrections to QCD predictions for deep-inelastic scattering[1,2] and for e^+e^- annihilation,[3] calculated in the minimal-subtraction (MS) scheme,[4] are significantly reduced if one expands in powers of an α defined by momentum-space subtraction[5] (α_{MOM}) or by the $\overline{\text{MS}}$ scheme[2] ($\alpha_{\overline{\text{MS}}}$) in which factors of $\gamma_E - \ln 4\pi$ are removed along with the divergences. This suggests that in QCD, as in quantum electrodynamics (QED), a "physically" defined coupling constant like α_{MOM} or the closely related $\alpha_{\overline{\text{MS}}}$ makes a good expansion parameter for the perturbation series. However, recently Moshe[6] has presented an interesting estimate of the third-order corrections to the QCD predictions for moments of deep-inelastic structure functions. He finds that even if the $\overline{\text{MS}}$ definition of α is used, third-order corrections seem to be enormous for $Q^2 \lesssim 10$ GeV^2. Although this result is speculative, a close examination shows that large contributions to the third-order corrections come from terms like $C_{2,2}^{(N)}(\alpha/4\pi)^2 \ln^2(\alpha/4\pi)$, where the constant $C_{2,2}^{(N)}$ is completely *known*, and it seems unlikely that the unknown constant terms would conspire to cancel this large known term. Moshe's work seems to indicate a disaster in QCD perturbation theory. What has gone wrong?

Consider a perturbative QCD calculation for the moments of the nonsinglet structure function[7] xF_3, performed with the MS scheme.[4] The β function