general conclusion: The large-N behavior of the quantum theory is directly determined by the classical equations of motion supplemented with some special constraints on the generators of the symmetry group. This statement is valid irrespective of whether the theory under consideration involves only bubble diagrams or the more complex planar diagrams.

The foregoing conclusion may seem surprising. Indeed there are statements in the literature that the classical solutions do not seem to be relevant in the large-N limit. Obviously, not only do we sharply disagree with such statements but moreover in the present framework we show that appropriate classical solutions in fact directly determine the large-N quantum theory.

The generalization of the above phenomenon to Yang-Mills gauge theory would be as follows. The quantum theory is given by the Hamiltonian

$$\hat{H} = \int \mathrm{Tr}[\tfrac{1}{2}\hat{E}_i^{\,2} + \tfrac{1}{4}\hat{F}_{ij}^{\,2}]\, d\vec{x} \tag{42}$$

and the requirement of gauge invariance,

$$\hat{G}(\vec{x})\,|\,\rangle = 0; \quad \hat{G}(\vec{x}) = \hat{D}\cdot\hat{E}(\vec{x}). \tag{43}$$

We then expect that the large-N behavior and the sum of all planar diagrams is given by the classical solutions of Yang-Mills equations with a special constraint $G=\rho$ representing an effective source. We have checked that this is true for the simple one-plaquette Yang-Mills theory and believe that it holds for the full three-dimensional theory.

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Measurements of Cross Sections for Pion Absorption by Nuclei

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Cross sections for true absorption by nuclei (Al, Ti, Cu, Sn, Au) of pions ($\pi^{\pm}$) in the energy range from 20 to 280 MeV were measured by a new method of detecting nuclear γ rays following the reaction. The incident-energy dependence of the cross sections in the low-energy region ($T_\pi < 50$ MeV) was well reproduced by an optical-model calculation, while the higher-energy part seems to indicate complex mechanisms of pion absorption.

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The pion absorption process is essentially an "exclusive" reaction in which no pion is left in the final state, so that its study provides unique information on pion-nucleus interactions. Despite extensive studies at meson factories, very few data have been available on the pion (true) absorption cross section.[1,2] We have measured the cross sections with a new method by detecting γ rays from residual nuclei following the pion absorption. In this Letter we shall first describe the principle and practical problems of the method, and then present experimental data.

A nucleus is excited after pion absorption and emits several nucleons until the nuclear excitation energy becomes less than the binding energy of nucleons. The nucleon emission is followed by low-energy γ-ray emissions from the excited residual nucleus with essentially the same mechanism as that well studied in the low-energy (particle, xn) reactions.[3] In the present method, we measured these low-energy γ rays to determine both the cross section and the multiplicity of γ rays.

The γ rays are emitted also in other processes of pion interactions. The total cross section (σ_{tot}) of pions on a nucleus consists of cross sections for elastic (σ_{el}) and inelastic (σ_{inel}) scattering, absorption (σ_{abs}), and single (σ_{scx}) and double (σ_{dcx}) charge-exchange processes:

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel} + \sigma_{abs} + \sigma_{scx} + \sigma_{dcx}. \tag{1}$$

Observed γ rays can be from any one of the last four processes. We can, however, subtract those "background" γ rays from the processes other than absorption by measuring γ rays in coincidence with the secondary pions.

The experiments were carried out at the National Laboratory for High Energy Physics, KEK. The pions from a low-momentum meson channel (π-μ channel) at the KEK 12-GeV proton synchrotron were used on three targets; Al (2.7 g/cm^2), Ti (1.8 g/cm^2), Cu (2.7 g/cm^2), Sn (2.2 g/cm^2), and Au (2 g/cm^2). The incident pions were identified by the time of flight between two counters R_1 and R_2 placed in the beam line. A beam-defining counter R_3 with a size slightly smaller than the target (2.5 cm $\times$ 2.5 cm) was set in front of the target to determine the incident pion intensity. An anticoincidence counter (R_4; 9 cm $\times$ 9 cm) was set 13 cm behind the target to identify reaction events. The nuclear γ rays were detected with eight NaI(Tl) detectors (7.5 cm diam $\times$ 7.5 cm long) placed at 19.5 cm from the target and at angles of ($\theta = 55$, 145, -125, $-35°$ with $\varphi = 45°$, and $\theta = 35$, 125, -145, $-55°$ with $\varphi = 135°$). With absorbers (Cu 3 mm) placed in front of the detectors, the energy dependence of the γ-ray detection efficiency was made to be nearly flat in the energy range from 0.1 to 2 MeV.

Single and multiple-coincidence spectra of γ-ray detectors ($G_1, G_2, \ldots, G_5$) and pulse-height signals of all counters were stored on a magnetic tape by a trigger signal of ($R_1R_2R_3\bar{R}_4G_i$). Then the γ-ray single (Y_1) and coincidence (Y_2) counts were obtained in an off-line analysis with a gate on the R_1-R_2 time-of-flight spectrum to identify the pion-induced events.

A correction was made to the data for the fact that when any proton following the pion absorption triggered R_4, the event was not assigned as an absorption event. For this correction, data were taken also with the trigger $R_1R_2R_3G_i$, and the ratio of secondary protons to the beam (and secondary) pions in the R_4 counts was determined with use of the ΔE information from R_4 and R_5 to identify particles.

In order to subtract γ rays following the inelastic scattering, we prepared seven liquid scintillation counters ($N_1, N_2, \ldots, N_7$; 12.5 cm diam $\times$ 5 cm long) with ΔE counters placed at different angles from $\theta = 30°$ to $135°$, and measured inelastically scattered pions in coincidence with γ rays ($R_1R_2R_3G_iN_j$). The scattered pions were identified by the ΔE-E information. From this measurement, the $\pi\gamma$ and $\pi\gamma\gamma$ coincidence counts were obtained as a function of the scattered pion angle. The angular distribution was fitted with Legendre polynomials, and then integrated over the pion angles except for the solid angle subtended by the counter R_4. The single (Y_1') and coincidence (Y_2') γ-ray counts associated with the scattered pion event were thus obtained.

The γ-ray counts associated with the pion absorption are now obtained as $(Y_1)_{abs} = Y_1 - Y_1'$ and $(Y_2)_{abs} = Y_2 - Y_2'$, and the pion absorption cross section and the multiplicity of γ rays can be deduced by use of the following relations:

$$(Y_1)_{abs} = I_B N_T n\Omega \langle m\rangle \sigma_{abs}, \tag{2}$$

$$(Y_2)_{abs} = I_B N_T [n(n-1)/2]\Omega^2 \langle m(m-1)\rangle \sigma_{abs}, \tag{3}$$

where I_B, N_T, n, and Ω are beam intensity, number of target nuclei, number of NaI(Tl) detectors, and γ-detection efficiency, respectively. The average multiplicity of γ rays M_γ is given as $M_\gamma = \langle m\rangle$. Effects of angular distributions and correlations of γ rays were negligible. We checked the effect by comparing Y_1's and Y_2's deduced from data taken with various partial combinations out of the eight NaI detectors, and observed no significant difference.

From Eqs. (2) and (3), we get $\langle m\rangle\sigma_{abs} = A$ and $\langle m^2\rangle\sigma_{abs} = B$. Since $\langle m^2\rangle/\langle m\rangle^2 = 1 + (s/M_\gamma)^2$, where the variation of multiplicity distribution s is given as $s^2 = \langle m^2\rangle - \langle m\rangle^2$, the cross section σ_{abs} is obtained as

$$\sigma_{abs} = [1 + (s/M_\gamma)^2]A^2/B = fA^2/B. \tag{4}$$

Although the γ-ray multiplicity distribution is not well known yet, we made a conservative estimate of s/M_γ to be 0.5 ± 0.5; $f = 1.25 \pm 0.25$ from much experimental knowledge about deexcitation of highly excited states following the (particle, xn) reaction.[3] This ambiguity can be reduced by further studies of the pion-absorption process.[4]

The cross sections obtained by Eq. (4) were corrected for the contributions of the charge-exchange processes. The γ rays following the

TABLE I. Summary of measured cross sections. Errors shown are only statistical ones. Additional uncertainties of 20% due to the assumption of the multiplicity distribution should be taken into account for the absolute values of cross sections.

T_π (MeV)	Al (mb)	Ti (mb)	Cu (mb)	Sn (mb)	Au (mb)
			Positive pion		
23	102 ± 15	200 ± 35	332 ± 35	388 ± 45	557 ± 75
37	148 ± 15	292 ± 25	379 ± 29	565 ± 35	763 ± 59
52	220 ± 15	395 ± 25	490 ± 25	664 ± 25	945 ± 60
68	260 ± 20	435 ± 20	521 ± 25	667 ± 25	970 ± 30
83	282 ± 20	452 ± 20	554 ± 20	751 ± 30	1031 ± 30
100	275 ± 29	453 ± 20	552 ± 20	753 ± 25	1047 ± 30
125	256 ± 20	495 ± 20	571 ± 30	800 ± 35	1045 ± 40
151	259 ± 29	507 ± 25	557 ± 25	800 ± 25	1072 ± 50
187	256 ± 20	442 ± 55	540 ± 30	780 ± 50	1064 ± 70
233	211 ± 20	396 ± 20	463 ± 45	674 ± 40	885 ± 60
280	187 ± 25	404 ± 55	412 ± 30	652 ± 60	798 ± 60
			Negative pion		
37	253 ± 20	616 ± 50	736 ± 45	1220 ± 100	2082 ± 100
52	307 ± 20	637 ± 20	758 ± 20	1130 ± 30	1781 ± 40
68	304 ± 20	637 ± 20	712 ± 20	995 ± 30	1746 ± 60
83	303 ± 20	602 ± 25	695 ± 15	991 ± 30	1600 ± 30
100	306 ± 25	600 ± 25	683 ± 20	952 ± 30	1531 ± 30
125	292 ± 15	570 ± 20	655 ± 20	927 ± 30	1375 ± 40
151	267 ± 20	525 ± 25	584 ± 25	885 ± 45	1316 ± 40
187	217 ± 35	417 ± 30	495 ± 30	757 ± 45	1125 ± 80
233	225 ± 25	415 ± 30	449 ± 45	753 ± 50	1001 ± 80
280	170 ± 15	386 ± 25	400 ± 45	625 ± 40	862 ± 100

double-charge-exchange process had been included in Y_1' and Y_2', and subtracted properly because the method was insensitive to the sign of pion charge. On the other hand, it is difficult to subtract the contribution of the single-charge-exchange process experimentally. However, since the contribution was considered to be small, we made a crude estimate of σ_{scx}. For this purpose, we calculated cross sections for the inelastic scattering followed by γ-ray emission, σ_{IESG}, from Y_1' and Y_2' using equations similar to (2)–(4). The estimate of σ_{scx} was then obtained by multiplying σ_{IESG} by a ratio R which is deduced from free pion-nucleus cross sections under the assumption of a quasifree nature of the processes;

$$R(\pi^-) = Z\sigma(\pi^- p \to \pi^0 n)/[Z\sigma_{tot}(\pi^- p) + N\sigma_{tot}(\pi^+ p)],$$

$$R(\pi^+) = N\sigma(\pi^- p \to \pi^0 n)/[Z\sigma_{tot}(\pi^+ p) + N\sigma_{tot}(\pi^- p)].$$

The pion-absorption cross sections obtained are listed in Table I and plotted in Figs. 1(a)–1(c). Shown in Figs. 1(d)–1(f) are average multiplicities of γ rays. For comparison, the pion-absorption cross sections of Al, Fe, and Bi for 125-MeV pions reported by Navon *et al.*[2] are plotted in Fig. 1. These were in agreement with the present data though the methods were different.

The low-energy part of the present data shows drastic effects of the Coulomb interaction, which should be reproduced theoretically without any assumption or parametrization. Indeed, the behavior of cross sections at low energies is well reproduced by an optical-model calculation. The theoretical cross sections by Stricker, McManus, and Carr[5] are plotted in Fig. 1. In the figure, the theoretical cross sections for Ni and Pb are compared with the present data for Cu and Au, respectively. The theoretical values for Al were obtained by an interpolation between C and Ni. Good agreement with present data is seen in the low-energy part.

We have made similar optical-model calculations for Al, Cu, and Au and obtained essentially the same results as the calculations of Ref. 5. While good agreement was obtained for low-energy part ($T_\pi < 50$ MeV), the theoretical calculations failed to reproduce the observed behavior

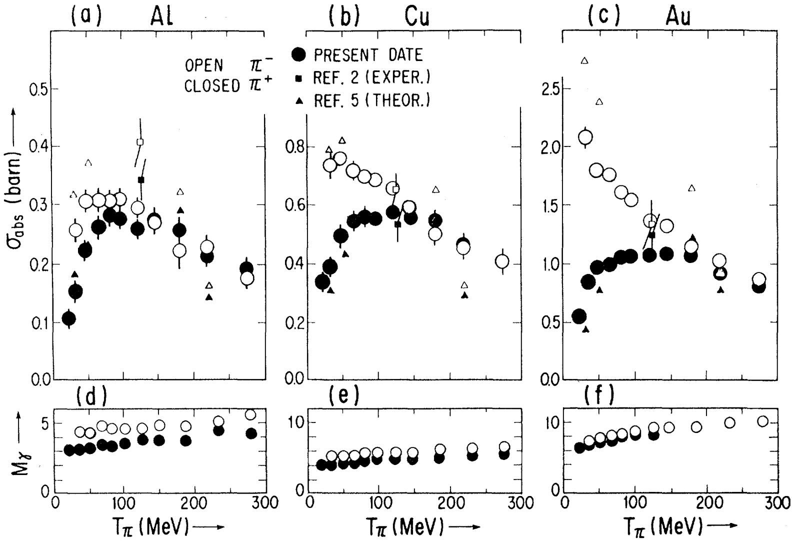

FIG. 1. Cross sections for pion absorption by (a) Al, (b) Cu, and (c) Au; (d)–(f) multiplicities of γ rays. Closed and open circles are for positive and negative pions, respectively. Squares are experimental data for Al, Fe, and Bi by Navon *et al.* (Ref. 2) and triangles are theoretical values by Stricker, McManus, and Carr (Ref. 5).

of the cross sections around the (3, 3)-resonance region. The calculated cross sections decrease with increasing incident energy faster than the experimental data. In the energy region $T_\pi > 50$ MeV, calculations become unreliable unless the absorption following a quasielastic scattering process is properly taken into account. Another interesting approach was reported by Hüfner and Thies,[6] who developed a transport theory to calculate the cross sections for light nuclei. We note that the relatively slow decrease of the absorption cross section above the (3, 3)-resonance region seems to be well reproduced. The present data provide a critical test for those theoretical models of the pion-nucleus interactions.

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[4]Combining the present results with those of separate experiments to measure the M_γ by observing coincidences between a Ge(Li) and the eight NaI detectors, we obtained the following preliminary results: f(100-MeV π^- on Au) = 1.32 ± 0.18, f(100-MeV π^- on Co) = 1.10 ± 0.13, f(150-MeV π^- on Co) = 1.32 ± 0.19, and f(100-MeV π^- on Al) = 1.23 ± 0.18.

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