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Polarization of the Primeval Radiation in an Anisotropic Universe

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The results are given of accurate computations of the polarization of the cosmic microwave background radiation in homogenous anisotropic universes with flat spacelike hypersurfaces. The degree of polarization never exceeds twice the maximum temperature quadrupole anisotropy, and its value is found to be very sensitive both to the mass fraction in the form of hydrogen and to its ionization history. There is no spectral distortion to first order.

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Meausrement of the polarization of the cosmic microwave background radiation, generally considered to be the relict radiation from the big bang, can set important constraints on the possible anisotropy of the universe. Other properties of the radiation, such as its large-scale isotropy[1] and blackbody spectrum,[2] have generally tended to confirm the standard big-bang interpretation. The observed dipole temperature is generally assumed to be due to the motion of the earth relative to the cosmological frame of reference. However, anisotropic cosmological models could in principle result in a similar effect.[3] Polarization measurements of the cosmic background radiation could provide a unique signature of cosmological anisotropy.[4,5] Moreover, the degree of polarization in an anisotropic universe is very sensitive to its ionization history and to its fractional hydrogen content. We describe below new calculations that correct and extend previous estimates of polarization in anisotropic universes with flat spacelike hypersurfaces (Bianchi type I).

The photon distribution function is assumed to be that of an isotropically radiating blackbody at a sufficiently early epoch. Its subsequent evolution is determined by the collisional Boltzmann equation, the interaction between matter and radiation being determined by Thomson scattering at red shifts $z \lesssim 10^7$. For simplicity, we restrict ourselves initially to axisymmetric homogeneous model universes of Bianchi type I (although our results can readily be generalized to nonaxisymmetric Bianchi I universes). Such a universe is described by scale factors $a(t)$, $a(t)$, and $w(t)$, normalized to unity at the present epoch t_0. Then we define[6] the shear $\Delta H = \dot{w}/w - \dot{a}/a$ and the average "Hubble constant" (anisotropic expansion rate) $H = (2\dot{a}/a + \dot{w}/w)/3$, where $\Delta H \propto (a^2 w)^{-1}$ and, when the universe is matter dominated, $a \propto t^{2/3}$ and $w = a + Ka^{-1/2}$ $(-\infty < K < \infty)$.

The anisotropy in the radiation is assumed to be small. If θ is the angle between the photon momentum $\vec{k}$ and the symmetry axis (x_3 say), then the distribution function for photons in the polarization mode α can be written

$$f_\alpha(\nu_0,\theta,t) = \bar{f}(\nu_0,t)[1 + \delta_\alpha(\nu_0,t) - \tfrac{2}{3}\epsilon_\alpha(\nu_0,t)P_2(\theta)],$$

where $\bar{f}$ is the unperturbed distribution function (if the metric were isotropic), ν_0 is the comoving frequency, and $P_2 = \frac{1}{2}(3\cos^2\theta - 1)$.[4,5] To specify α, the polarization basis vectors are defined as $\vec{P}_a$ in the plane containing $\vec{k}$ and x_3 and $\vec{P}_w$ perpendicular to this plane. If we insist that the radiation be unpolarized along x_3, we can then express the polarization and anisotropy in the radiation as[7]

$$\mathrm{Pol}(\nu_0,\theta,t) \equiv (f_w - f_a)/\bar{f} = (\epsilon_w - \epsilon_a)\sin^2\theta,$$

and

$$\mathrm{Anis}(\nu_0,\theta,t) \equiv \tfrac{1}{2}(f_w + f_a)/\bar{f} - 1 = \tfrac{1}{2}(\epsilon_w + \epsilon_a)(\sin^2\theta - \tfrac{1}{3}).$$

Their maximum values are $P \equiv \epsilon_w - \epsilon_a$ and $\epsilon \equiv \frac{1}{2}(\epsilon_w + \epsilon_a)$, respectively.

The polarization is produced by electron scattering of the anisotropic radiation field. The evolution of the quadrupole anisotropy coefficients ϵ_a and ϵ_w can be inferred from the Boltzmann equation for the photon distribution function, the right-hand side of which includes the Thomson scattering collisional term. One finds that ϵ_a and ϵ_w satisfy[4,8]

$$\frac{d}{dt}\begin{pmatrix}\epsilon_a\\ \epsilon_w\end{pmatrix} = \left(-\frac{\partial \ln\bar{f}}{\partial \ln\nu_0}\right)\Delta H\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{\tau}\begin{pmatrix}-1 & 0\\ -1/2 & -3/10\end{pmatrix}\begin{pmatrix}\epsilon_a\\ \epsilon_w\end{pmatrix}, \tag{1}$$

where the mean time between Thomson scatterings (cross section σ_T) is $\tau = [n_e(t)\sigma_T c]^{-1}$. Since $\bar{f}$ is a blackbody, $\partial \ln\bar{f}/\partial \ln\nu_0 (=-1$ in the Rayleigh-Jeans region) is time independent and we absorb it into ΔH henceforth for convenience. The principal complication in solving this equation is associated with the time variation of the hydrogen ionization fraction $I(t)$, defined by $n_e(t) = I(t)n_H(t)$, where $n_H(t)$ is the hydrogen atom number density.

Prior to recombination at epoch t_r, the ionization fraction is $I(t) = 1$, and $\tau \propto (\Delta H)^{-1}$. Both the temperature and polarization anisotropies are constant, and the corresponding solutions to (1) are $\epsilon_a = \Delta H\tau$, $\epsilon_w = 5\Delta H\tau/3$, whence

$$\epsilon = 4\Delta H\tau/3 \text{ and } P = 2\Delta H\tau/3. \tag{2}$$

To illustrate the effects of recombination, we first consider a simple step-function model: $I(t) = 1$ $(t < t_r)$; $I(t) = I_0$ $(t > t_r)$. Solution (2) applies prior to recombination at red shift $z_r \approx 1500$; subsequently, the asymptotic polarization achieved depends on the number of scatterings N_0 between epochs t_r and t_0, where

$$N_0 = \int_{t_r}^{t_0} dt/\tau = 4.61\times10^{-2}(t_0/t_r)I_0\Omega_H h. \tag{3}$$

Here Ω_H is the hydrogen mass fraction, h is the Hubble constant (H_0) in units of 100 km s^{-1} Mpc^{-1}, and subscript zero deletes the present epoch. We distinguish two regimes.

When $N_0 \ll 1$, the temperature anisotropy increases to the maximum value induced by the shear,

$$\epsilon(t) = \Delta H_0\tau_0[\tfrac{4}{3} + (1/I_0 - 1)(1 - t_r/t)N_0],$$

there being insufficient scattering to isotropize the radiation in either polarization mode; the polarization anisotropy does not change substantially from its prerecombination value. On the other hand, if $N_0 \gtrsim 1$, the anisotropy of the radiation in the two polarization modes grows at different rates and once there is a nonnegligible number of scatterings, this difference manifests itself as an increase in the polarization anisotropy over its prerecombination value. However, as the number of scatterings is increased, the temperature anisotropy attains a smaller asymptotic value and the polarization anisotropy is correspondingly reduced. Specifically, if $N_0 \gtrsim 3$ the asymptotic values are[5]

$$\epsilon(t_0) = \tfrac{4}{3}\Delta H_0\tau_0/I_0; \quad P(t_0) = \tfrac{2}{3}\Delta H_0\tau_0/I_0. \tag{4}$$

Thus increasing N_0 (or I_0) reduces the temperature and polarization anisotropies.

The preceding discussion demonstrates how the polarization is affected by the number of scatter-

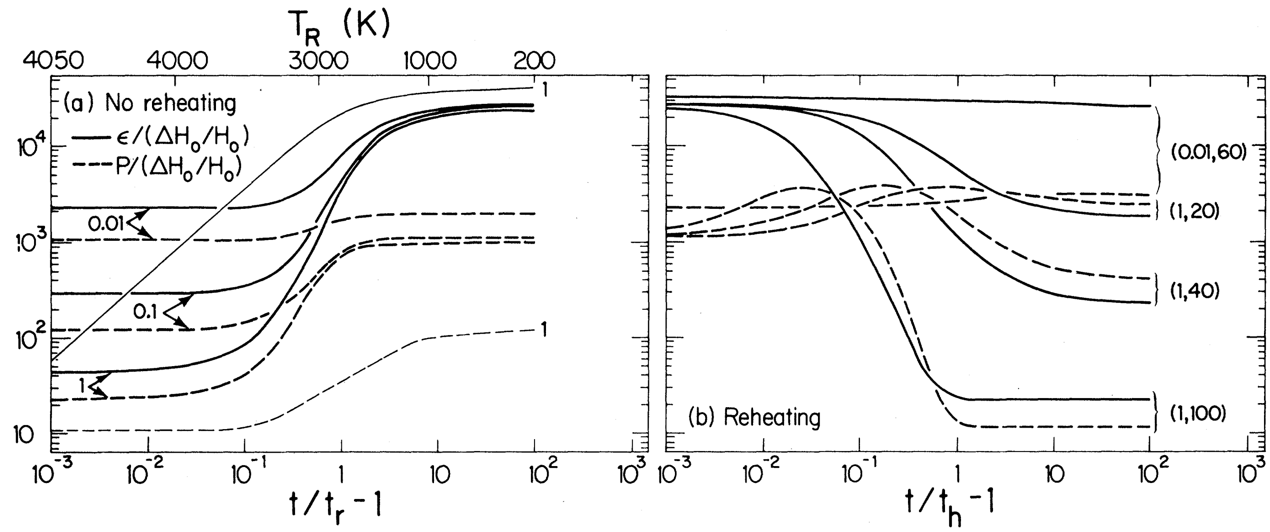

FIG. 1. Polarization anisotropy (dashed lines) and temperature anisotropy (solid lines) in units of present shear to Hubble-constant ratio $\Delta H_0/H_0$ for $T_0 = 2.7$ K, $\Omega = h = 1$, as a function of epoch. (a) Results are shown which utilize (i) a step-function approximation of the ionization history $[I(t) = 1,\ t \leq t_r;\ I(t) = 10^{-5},\ t > t_r]$ (thin lines) where epoch t_r corresponds to a redshift of 1500, and (ii) an exact solution[8] to the ionization fraction (thick lines). The mass fraction in the form of hydrogen, Ω_H, is indicated. The upper abscissa gives the radiation temperature. (b) At epoch t_h, the ionization fraction increases to unity from the value given by the numerical model. The appropriate values of Ω_H and the redshift z_h are indicated for each model by the label (Ω_H, z_h).

ings after recombination. In fact, the analytic step-function model for the ionization fraction during recombination results in a considerable underestimate of the polarization anisotropy. The time variations of $\epsilon(t)$ and $P(t)$ have been computed, utilizing an accurate model[9] of the ionization fraction during the recombination epoch. Results of a numerical integration of Eq. (1) are shown in Fig. 1(a). All models have $h = 1$; Ω_H is allowed to vary between 0.01 and 1. Prior to recombination, the polarization and anisotropy are given by (4) with $I_0 = 1$. A considerable number ($\sim 10^2 - 10^3$) of scatterings occur in the numerical models $\Omega_H = 0.01-1$). Nevertheless, the polarization increases after recombination, especially in the higher Ω_H models, rising to a value $P \sim 1000(\Delta H_0/H_0)$. If I_0 is chosen to be the asymptotic ionization fraction attained,[9] the analytical model [also shown in Fig. 1(a) for $N_0 \approx 0.03$] provides a reasonable estimate of the asymptotic temperature anisotropy; however, it underestimates the polarization by up to an order of magnitude. The reason models with differing Ω_H result in approximately the same asymptotic polarization and anisotropy is that the asymptotic ionization fraction is approximately inversely proportional to Ω_H.[9]

Finally, we consider the question of matter reionization subsequent to the recombination epoch. There is little doubt that reionization has occurred,[10] but whether it occurred sufficiently early ($z \geq 10$) to result in additional scatterings of the background radiation is not known. Any additional scatterings do tend to enhance the polarization anisotropy. To demonstrate this, we approximate the ionization fraction by $I(t) = 1$ ($t < t_r$); 0 ($t_r < t < t_h$); I_1 ($t > t_h$). At $t_h(\gg t_r)$, the temperature and polarization anisotropies amount to $\epsilon(t_h) = \frac{2}{3}(\Delta H_0/H_0)(t_0/t_h)$; $P(t_h) = \frac{2}{3}(\Delta H_0 \tau_0)$. The subsequent behavior of $\epsilon(t)$ and $P(t)$ depends on the number of scatterings between t_h and t, which we write as $N(t_h, t) = N_h(1 - t_h/t)$, where

$$N_h = (t_0/\tau_0)(t_0/t_h)I_1$$
$$= (\tfrac{2}{3})(1 + z_h)^{3/2} I_1 H_0^{-1}\tau_0^{-1}.$$

If $N_h \ll 1$, reionization has no effect on the polarization and temperature anisotropies. If $N_h \gtrsim 1$, scattering decreases ϵ_a from its value at t_h more efficiently (by a factor $\sim 10/3$) than ϵ_w. Explicitly, we find that the maximum polarization anisotropy attainable is $P_{\max} \equiv P(t_c) \approx 0.12\epsilon_a(t_h)$, occurring at epoch t_c, when the number of scatterings is $N_c \equiv N(t_h, t_c) \approx 1.72$.

If $N_h \sim N_c$, this epoch of maximal polarization anisotropy occurs late, with $t_c \sim t_0$, yielding the case of greatest interest to observers. As N_h is increased, more scatterings occur that tend to damp the reionization anisotropy of both the polarization modes. Consequently, in the limit of $N_h \gtrsim 30$, the polarization anisotropy drops to the asymptotic value (4) (with I_0 replaced by I_1) as if there had been no free expansion between t_r and t_h. For $I_1 = 1$, this implies that (4) is valid only for reheat epochs $z_h \gtrsim 75$, in contrast to $z_h \gtrsim 7$ obtained previously[5] (since there is $\lesssim 1$ scattering over $z \lesssim 7$). One can easily show from the preceding analysis that the ratio of polarization anisotropy to temperature anisotropy, P/ϵ, attains a maximum value of about 2 (an earlier derivation[4] was in error) when $N(t_h, t) \sim 9$, corresponding to a later epoch than t_c. A somewhat more accurate model which includes reionization is one in which the ionization fraction is computed exactly throughout recombination, rather than being approximated by a step function from unity to zero. The results are shown in Fig. 1(b).

The mass content in luminous material (visible stars, gas, etc.) amounts to $\Omega_{\text{lum}} \approx 0.01$, whereas dynamical estimates[11,12] yield $0.05 \lesssim \Omega \lesssim 1$; we conservatively assume that a lower limit on Ω_{H} at recombination is given by Ω_{lum}. It is not known whether the dark matter that contributes most of the clustered mass density in the universe was in gaseous form at recombination. Certainly, it cannot be gaseous now, and some hypothesized forms for the dark mass, such as hypothetical heavy neutral leptons[13] or primordial black holes,[14] would not have contributed to the gas density at recombination. Other possibilities for dark matter are that it consists of very low-mass stars or black holes formed *after* recombination, but preceding galaxy formation.[15] The dependence of polarization anisotropy on Ω_{H} could therefore provide an interesting constraint on the dark matter, should the temperature anisotropy be confirmed.

We note that generalization of these calculations to the case of unequal expansion along three axes (the most general Bianchi type I) is straightforward since Eq. (1) holds for the parameters describing the azimuthal as well as the polar asymmetry. Thus, all the results derived above for P and ϵ are valid for their azimuthal and polar generalizations, the only difference being the angular dependence of the temperature, which becomes quadrupolar in azimuth as well as in polar angle.[16]

Lubin and Smoot[17,18] have looked for polarization of the cosmic background radiation at 33 GHz at ten declinations between 37 °S and 63 °N. A χ^2 fit of their data with the axisymmetric model yields $P < 9 \times 10^{-5}$, with 95% confidence. If there were no reheating subsequent to recombination, this upper limit implies that the present shear to Hubble-constant ratio is $\Delta H_0/H_0 = 1.07 \times 10^{-3} P < 9.5 \times 10^{-8}$ for $\Omega_{\text{H}} = h = 1$. However, if reionization of a fraction Ω_{H} of the hydrogen occurred at a redshift $z \gtrsim 75$, this constraint relaxes to $\Delta H_0/H_0 = 1.5 P H_0^{-1} \tau_0^{-1} < 9.2 \times 10^{-6} \Omega_{\text{H}} h$. A recent 95% confidence limit for the quadrupole temperature anisotropy is[19] $\Delta T/T \lesssim 1$ mK, whence $\epsilon \lesssim 3 \times 10^{-4}$. The corresponding constraints on the shear are $\Delta H_0/H_0 = 4.6 \times 10^{-5} \epsilon < 1.7 \times 10^{-8}$ and $0.75\, \epsilon H_0^{-1} \tau_0^{-1} < 1.9 \times 10^{-5} \Omega_{\text{H}} h$ for the two cases of no reheating and $z_h \gtrsim 75$, respectively. We conclude that the polarization provides a stronger constraint than the temperature anisotropy only if reheating occurred at $z_h \gtrsim 75$. However, a fit of the temperature anisotropy data with the axisymmetric model should enable the upper limit on ϵ to be reduced.

In summary, polarization measurements provide a constraint on the shear of marginal interest compared with that inferred from measurement of temperature anisotropy. However, confirmation of an actual quadrupole anisotropy would enable polarization to be an important probe of the ionization history and hydrogen mass fraction of the universe at recombination. The frequency dependence of the anisotropy and polarization is entirely determined by the factor $(\partial \ln \bar{f}/\partial \ln \nu_0)$ which multiplies ϵ and hence the effect is minimal except for measurements near the peak of the spectrum. It is interesting to note that there is no first-order distortion of the blackbody spectrum, contrary to an earlier suggestion.[4]

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Six lines above Fig. 1, the value given for the excess radiation density should read 0.14 eV/cm^3 rather than 1.14 eV/cm^3.