Nonlinear Saturation of the Buneman Instability

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An analytical model is developed for the nonlinear evolution of electron-ion two-stream instability. The instability saturates when the electric field energy reaches $\sim 2(m/M)^{1/3} W_0$ (W_0 = initial electron drift energy density), and is then followed by an algebraic growth. Complete stabilization is caused by electron trapping in deformed potential wells due to the rise of higher harmonics. The analytical model results are in close agreement with nonlinear kinetic computer simulations.

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The Buneman instability[1] is expected to occur when relative streaming velocity between electrons and ions much exceeds electron thermal velocity, as in early stages of various pulsed heating experiments (θ pinch, turbulent heating). The knowledge of the saturation level of the instability is of crucial importance for estimation of various transport coefficients such as anomalous resistivity due to the instability. Since the instability is "strong" with the growth rate comparable with the frequency, and the phase velocity is remotely separated from the electron drift velocity, it has conventionally been presumed that the instability can saturate only when the field energy becomes comparable with the initial electron-drift energy density, W_0.[2] Recently, Hirose[3] has pointed out that the linear growth of the instability should break down when the field energy becomes of the order of $(m/M)^{1/3}W_0$, and concluded that the anomalous resistivity associated with the instability scales as $(m/M)^{2/3}$, rather than $(m/M)^{1/3}$, with m/M the electron/ion mass ratio. However, it is not obvious whether a slower growth stage (probably with algebraic, rather than exponential, growth) can follow the apparent saturation and if this is the case, it is of importance to determine the ultimate saturation level.

In this report, it is shown that (a) the existence of the algebraic growth stage critically depends on the mass ratio; (b) the final saturation is caused by electron trapping in a nonsinusoidal potential well. The harmonics play an essential role in electron trapping, which takes place at a relatively low ($\simeq 0.1W_0$) level of field energy.

The theoretical method presented here is to solve a nonlinear dispersion relation for the Buneman instability, which takes into account the renormalization of the electron velocity distribution function, namely, the deceleration of the initial drift velocity and the "heating" of the electrons. Harmonics play a key role in the initial saturation[4] and in the eventual electron trapping found in our computer simulations. Therefore, nonlinear interaction between the fastest growing fundamental mode and its higher harmonics, due to forced oscillations, is also taken into account. Furthermore, we find it essential to allow a shift in the frequency as well as the change in the growth rate. This frequency shift is important since, in the Buneman instability, the frequency ($\mathrm{Re}\Omega$) and the growth rate ($\mathrm{Im}\Omega$) are quite comparable. In fact, we have found that the frequency shift occurs first, and the reduction in the growth rate is induced by the frequency shift.

We assume a one-dimensional plasma composed of a cold electron beam drifting relative to ions with an initial drift velocity V_0. If the initial thermal fluctuation level is sufficiently small, the electron-ion two-stream instability should be dominated by the most unstable mode,[1]

$$\Omega_k(0) = \omega_k(0) + i\gamma_k(0) = \left\{\left[1+\frac{1}{2}\left(\frac{m}{2M}\right)^{1/3}\right] + i\sqrt{3}\left[1-\frac{1}{2}\left(\frac{m}{2M}\right)^{1/3}\right]\right\}\frac{1}{2}\left(\frac{m}{2M}\right)^{1/3}\omega_{pe}, \tag{1}$$

for $kV_0 = \omega_{pe}$ is the electron plasma frequency. The dispersion relation for the electron-ion two-stream instability may be expressed as[5]

$$1+\chi_{e,QL}+\chi_{e,MC}+\chi_i=0, \tag{2}$$

where

$$\chi_i=\omega_{pi}^2\exp\left[i\int_0^t\Omega_k\,dt'\right]\int_{-\infty}^t dt'(t-t')\exp\left(-i\int_0^{t'}\Omega_k\,dt''\right) \tag{3}$$

and the quasilinear electron susceptibility is defined by

$$\chi_{e,QL}=\frac{-\omega_{pe}^2}{[kV_{QL}(t)-\Omega_k(t)]^2-3k^2T_{QL}(t)/m}\,. \tag{4}$$

The mode-coupling electron susceptibility can be approximated in terms of electric field as

$$\chi_{e,MC}=-\frac{3}{2}\left(\frac{e}{m}\right)^2\frac{k^2|E_k|^2}{\omega_{pe}^4}\left[1+\frac{179}{48}\left(\frac{e}{m}\right)^2\frac{k^2|E_k|^2}{\omega_{pe}^4}\right]. \tag{5}$$

To obtain this expression we have solved the Vlasov equation together with the Poisson equation for an electric field

$$E=\tfrac{1}{2}\textstyle\sum_k\{E_k(0)\exp[i(kx-\int_0^t\Omega_k\,dt')]+\text{c.c.}\}$$

in the presence of higher harmonics ($2k$ and $3k$ modes). Contributions to the mode coupling term of the perturbed distribution function f_k come from the interactions k with $2k$ and $2k$ with $3k$. The drift velocity $V_{QL}(t)$ and the electron temperature $T_{QL}(t)$ are given by the conventional quasilinear equations

$$\frac{dV_{QL}}{dt}=-\left(\frac{e}{m}\right)^2\sum_k\frac{\gamma_k k(kV_0-\omega_k)|E_k|^2}{[(kV_0-\omega_k)^2+\gamma_k^2]^2}\,, \tag{6}$$

$$\frac{dT_{QL}}{dt}=\frac{e^2}{m}\sum_k\frac{\gamma_k|E_k|^2}{(kV_0-\omega_k)^2+\gamma_k^2}\,. \tag{7}$$

The nonlinear dispersion relation given above is valid until resonance or particle trapping becomes important. As we will show later, particle trapping takes place very abruptly, and the equation can well describe the development of the instability up to the time when trapping occurs.

Since we are interested in the temporal evolution of the complex frequency $\Omega_k(t)$, it is convenient to differentiate Eq. (2) with respect to time. Substituting Eqs. (6) and (7) into Eq. (2) yields two closed, simultaneous differential equations for $\omega_k(t)$ and $\gamma_k(t)$ which have been numerically solved for various mass ratios. Initial amplitude of the electric field E_k does not affect the nonlinear behavior of the instability, but determines the linear growth period. The time is normalized by the inverse of the initial growth rate, $\tau=\gamma_k(0)t$.

Figure 1 shows the time evolution of the electric field energy in the case of an argon plasma ($M/m=1836\times40$). The linear growth of the instability breaks down when the field energy becomes of the order of $(m/M)^{1/3}W_0$, after which the growth is no longer exponential but is well characterized by an algebraic growth. The oscillating behavior after the first saturation is due to the strong modulation in the growth rate, which is caused by the frequency shift as explained earlier and not by particle trapping. The instability finally saturates

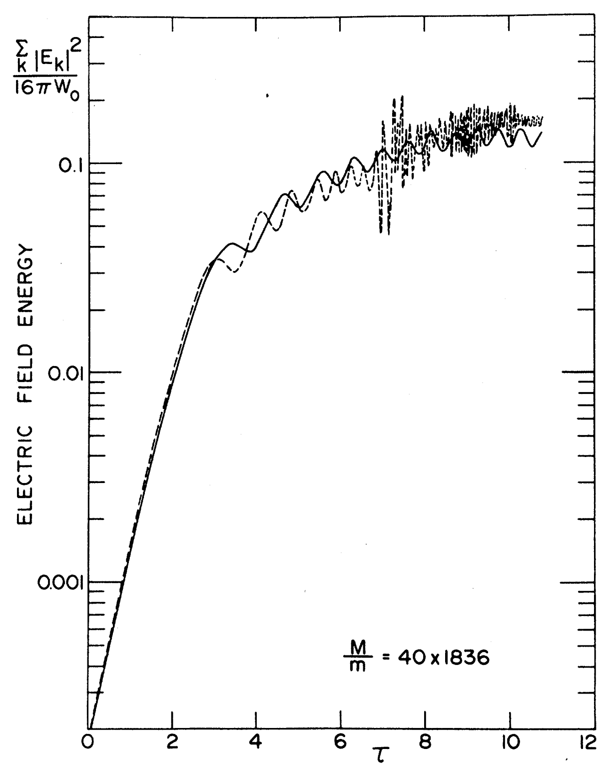

FIG. 1. Time variation of electric field energy density. Solid line is a solution of Eq. (2). $|E_k(0)|^2/16\pi W_0 = 1.6\times10^{-4}$. W_0 = initial electron drift energy density. Broken line is a computer simulation. The initial time of computer simulation is shifted to $\tau=-2.9$ for the convenience. $\tau=\gamma_k(0)t$.

at the level of $0.13W_0$. In the Buneman instability, the harmonics must appear as forced oscillations because of a strong dispersive nature. Thus the harmonics return their energy back to the fundamental to destabilize further. At the first saturation, the field energies differ by a factor of 2.5 with and without the harmonics, as predicted by Bartlett.[4] At a later time, the difference becomes smaller, and the dispersion relation obtained by Bartlett becomes inaccurate.

Qualitatively, the substantial reduction in the growth rate is caused by the "heating" of the electrons and, what amounts to the same thing, electron deceleration. However, the algebraic growth stage can be revealed only by solving the nonlinear dispersion relation as an initial-value problem.

We have solved the nonlinear dispersion relation for a wide range of the mass ratio, $M/m = 1836\times 1 \sim 1836\times 200$, to see how the first and the final saturations depend on the mass ratio. The first saturation (breakdown of the linear growth) occurs at

$$\sum_k |E_k|^2/16\pi \simeq 1.5(m/M)^{1/3}W_0$$

in agreement with the previous estimate.[3] The final saturation level weakly depends on the mass ratio,

$$\sum_k |E_k|^2/16\pi \simeq 0.2W_0(A=1) \sim 0.1W_0(A=200)$$

with A the atomic mass. For a hydrogen plasma, both saturation levels almost coincide, and there hardly exists an algebraic growth stage.

Whether the final saturation level found above is real or not can be checked by electron trajectory calculation. If electron trapping takes place at the field energy, the electron drift energy is completely thermalized and no further growth is expected. In calculating electron trajectories, 200 electrons per fundamental wavelength are employed. The motion of each electron is followed in the electric field found from the nonlinear dispersion relation. Two relevant quantities, electron drift velocity $V(t)$ and "thermal" energy T_e, are then computed from

$$V(t) = \frac{1}{N}\sum_{i=1}^{N} v_i(x,t) \quad (N=200),$$

$$\tfrac{1}{2}T_e(t) = \frac{m}{N}\sum_{i=1}^{N} \tfrac{1}{2}[v_i(x,t) - V(t)]^2.$$

In Fig. 2, the time evolution of the thermal energy $T_e(t)/2$ thus found is shown. The sudden jump in the thermal energy is due to electron trapping.

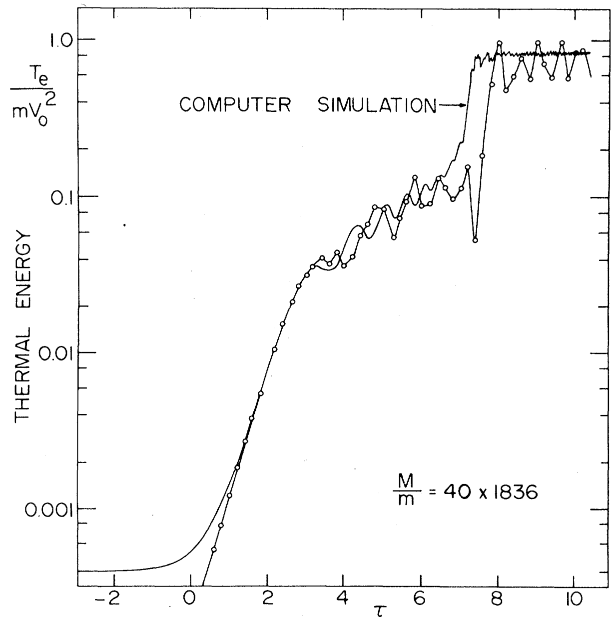

FIG. 2. Time variation of thermal energy of electrons. Line with circles, trajectory calculation based on the analytical model; solid line, computer simulation.

It should be noticed that trapping occurs at the field energy level very close to the final saturation level predicted from the analytical model. The trapping occurs very abruptly, and our analytical model seems to hold very well up to the trapping. The field energy after the trapping is expected not to vary appreciably, and we may conclude that the final saturation predicted from the model is real. A similar comparison has been made for other mass ratios. The field energy level at which electron trapping takes place is rather independent of the mass ratio, and is given by $(0.13\pm 0.02)W_0$. This is about four times larger than that expected from the conventional trapping condition, $2e\varphi \geq mV_0^2/2$, where V_0 is the initial electron drift velocity and we have neglected the phase velocity ω/k [$\simeq (m/M)^{1/3}V_0$] compared with V_0. This discrepancy may be explained as follows. As the instability grows, both the drift and thermal velocities of the electrons change. The drift velocity decreases by the amount $\Delta V = (e^2/2m^2)k^3\varphi^2/\omega_{pe}^3$. The thermal or oscillatory electron velocity defined by $v_\sim = [T(t)/m]^{1/2} \simeq ek\varphi/\sqrt{2}m\omega_{pe}$ is a quantity of the first order in φ, and makes a large contribution to the kinetic energy of an individual electron. At the bottom of the potential well (minimum of $-e\varphi$), the instantaneous electron velocity is then given by V_0

$-\Delta V+\sqrt{2}v_{\sim}$, and the trapping condition becomes $2e\varphi \geq mV_0^2[1-(2e\varphi/mV_0^2)^2/8+e\varphi/mV_0^2]^2/2$. Solving this inequality for φ, we find $e\varphi \geq 0.92mV_0^2/2$ or, in terms of the field energy, we obtain $|E|^2/8\pi n_0 mV_0^2 \geq 0.11$ which is close to that found in our numerical analysis.

One-dimensional computer simulation for the electron-ion two-stream instability in an argon plasma has been carried out. A fully nonlinear and kinetic particle model[6] is used, moving 4096 electrons and 4096 ions on a periodic system of length 2π with 256 cells. Initially ions are cold and stationary while electrons are drifting with small spread $\langle[v(0)-\bar{V}(0)]^2\rangle = 4\times 10^{-4}$, where $\bar{V}(0)=V(0)/V_0=1$. The simulation and analytic results agree throughout, confirming our nonlinear model of the Buneman instability. With argon mass ratio, the fluctuation energy shows the first saturation at the level of $0.035W_0$ which is followed by algebraic growth in an oscillating manner, and saturates completely because of electron trapping at the level of $\sim 0.15W_0$ (Fig. 1). The initial time is shifted to $\tau=-2.9$ so that we could easily compare the simulation result with that of the analytical model. The thermal energy of electrons is shown in Fig. 2. The thermal energy increases abruptly when electron trapping sets in.

In conclusion, we have shown that a simple nonlinear dispersion relation can well describe the time evolution of the Buneman instability provided mode-coupling effects are properly taken into account. For a large ion-to-electron mass ratio, the algebraic growth stage exists between first and final saturation. The final saturation takes place at the field energy $\sim 0.1\times$ initial electron drift energy, consistent with the trapping condition based on quasilinear moment quantities. The results are in close agreement with computer simulation of the Buneman instability for the argon plasma.

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Time-Resolved Observations of the Three-Halves Harmonic Spectrum from Laser-Produced Plasmas

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Time-resolved $\frac{3}{2}\omega_0$ spectra from glass microballoons irradiated by 100-ps, 2×10^{16}-$\mathrm{W\cdot cm^{-2}}$ laser pulses have been obtained with a temporal resolution of 20 ps and a spectral resolution of 15 Å. Pulsed emission is observed, with a pulse duration of less than the instrument limit. Both red and blue peaks appear simultaneously, with their separation varying in time.

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In recent years much theoretical and experimental work has been performed to study the various parametric processes which occur in laser-produced plasmas at high irradiances. At one quarter of the critical density both stimulated Raman scattering[1] and two-plasmon decay[2] can

