Local $B-L$ Symmetry of Electroweak Interactions, Majorana Neutrinos, and Neutron Oscillations

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Interpretation of the U(1) generator of the left-right–symmetric electroweak model in terms of $B-L$ enables us to study the spontaneous breaking of local $B-L$ symmetry. The same Higgs mechanism at the "partial unification" level of $SU(2)_L \otimes SU(2)_R \otimes SU(4')$ that produces $\Delta_L = 2$ processes (e.g., Majorana neutrinos) also yields $\Delta B = 2$ processes (e.g., "neutron oscillations"). The observation of "neutrinoless" double β decay and $\Delta B = 2$ nucleon transitions without proton decay would favor this model and an intermediate mass scale.

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Recent successes[1] in interpreting the results of neutral-current weak-interaction experiments involving neutrinos and electrons in terms of the standard gauge model[2] of weak and electromagnetic interactions has led to wide acceptance of $SU(2)_L \otimes U(1)$ as the local electroweak symmetry at low energy. This symmetry is supposed to manifest itself above the mass scale of the order of 100 GeV, the mass of W and Z bosons. It is necessary for further understanding of the electroweak interaction to probe beyond the above energy scale and look for new effects which could signal the existence of any possible higher local symmetries. One such local symmetry,[3] suggested earlier to restore parity to the status of a short-distance symmetry of weak interactions, is $SU(2)_L \otimes SU(2)_R \otimes U(1)_{L+R}$. This model has been studied extensively and it is known[4] that if subsequent to spontaneous breakdown, $m_{W_R} \gg m_{W_L}$, the structure of the low-energy neutral-current weak interaction is indistinguishable from the standard model for $(m_{W_L}/m_{W_R})^2$ less than 10%.

Two recent developments have inspired the investigations reported in this paper. First, it was noted[5] that unlike the case of the $SU(2)_L \otimes U(1)$ model, the vector U(1) generator in the left-right-symmetric model can be identified with $B-L$ symmetry. One implication of this observation is that the mass scale associated with spontaneous breakdown of parity could be associated with the breakdown of local $B-L$ electroweak symmetry. To see this explicity, we note that in the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ model, electric charge is given by $Q = I_{3L} + I_{3R} + \frac{1}{2}(B-L)$, where $\vec{I}_{L,R}$ are the generators of the $SU(2)_{L,R}$ groups. The above relation implies that $\Delta I_{3R} = -\frac{1}{2}\Delta(B-L)$ in the energy region where $SU(2)_L \otimes U(1)_{Y_W}$ is a good symmetry (Y_W is the weak hypercharge of the standard model). Furthermore, the left-right–symmetric model enables us to study the phenomenological implications of *local* $B-L$ breaking[6] and the possible existence of intermediate mass scales without reference to grand unification models. The second development is the observation by Senjanović and one of the authors (R.N.M.)[7] that relating the breaking of $B-L$ symmetry to that of the discrete parity symmetry implies a Majorana neutrino and, furthermore, the smallness of neutrino mass is then related to the dominant $V-A$ nature of the weak interaction at low energy. Observed upper limits on the masses of all three neutrinos are consistent with a value of $m_{W_R} \gtrsim 3m_{W_L}$.

In the present paper, we extend the above considerations by generalizing the model to include the full quark-lepton correspondence. The most elegant formulation appears to be in terms of the "partial unification" group $SU(2)_L \otimes SU(2)_R \otimes SU(4')$,[8] where $SU(4')$ unifies color and $B-L$ symmetry. We find that within this framework, an immediate implication of $(B-L)$-symmetry breaking is the existence not only of Majorana neutrinos[7] ($\Delta L = 2$ in the lepton sector) but also the new phenomenon of $n \leftrightarrow \bar{n}$ transitions ($\Delta B = 2$ in the hadron sector), which we call "neutron oscillations." The experimental feasibility of measuring $\Delta B = 2$ nucleon transitions is also noted.

As stated, we work within the "partial unification" group $SU(2)_L \otimes SU(2)_R \otimes SU(4')$ for electroweak and strong interactions[8]; we consider one generation of quarks and leptons for simplicity. The fermions are assigned to the following representations:

$$\Psi_{L,i} \equiv (\tfrac{1}{2},0,4); \quad \Psi_{R,i} \equiv (0,\tfrac{1}{2},4),$$

where the color index $i = 1, 2, 3, 4$, or, explicitly

$$\Psi = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e^- \end{pmatrix}.$$

The U(1) generator identified with $B-L$ is given by

$$B-L = (\tfrac{2}{3})^{1/2} F_{15}{}' = \begin{bmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \qquad (1)$$

There are two gauge couplings: $g_L = g_R = g$; and f, the SU(4′) coupling. We envision a mass scale m_X such that the "partial unification" group breaks down to the level of $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes SU(3)_c$, which breaks down in stages to $SU(2)_L \otimes U(1) \otimes SU(3)_c$ and subsequently to $U(1) \otimes SU(3)_c$ with m_{W_R} and m_{W_L} being the corresponding mass scales, respectively. Since we expect $m_X \gtrsim 10^4$ GeV because of the absence of $K_L \to \mu^+ e^-$ decay,[3] the mass hierarchy would be $m_X > m_{W_R} >> m_{W_L} >> 1$ GeV. In Refs. 3 and 7, we have argued from considerations based on the neutrino mass as well as the neutral current interaction[1] that $m_{W_R} \gtrsim 300$ GeV. We note that the breakdown of $SU(4') \to U(1)_{B-L} \otimes SU(3)_c$ could be achieved either dynamically or via a Higgs multiplet Σ that belongs to the representation $(1,1,\underline{15})$ under the gauge group with $\langle\Sigma\rangle_{vac} = \mathrm{diag}\,(1,1,1,-3)m_X/f$.

It is in the remaining Higgs multiplets that the full implications of $(B-L)$-symmetry breakdown surfaces. We therefore display them below. We consider the "minimal" model, namely only those kinds of Higgs multiplets that can arise as bound states of existing fermion multiplets. Our results therefore do *not* depend on the existence or nonexistence of physical Higgs mesons but simply on the pattern of symmetry breakdown. The relevant Higgs multiplets are $\Phi \equiv (\frac{1}{2},\frac{1}{2},1)$, $\Delta_L \equiv (1,0,\underline{10})$, and $\Delta_R \equiv (0,1,\underline{10})$. Making the color and flavor indices explict, the Δ's can be written as $\Delta_{L,ij}{}^a$ and $\Delta_{R,ij}{}^a$, where i, j are SU(4′) indices; a is the flavor index and the Δ's are symmetric in i,j. The breakdown of parity and $B-L$ local symmetry are achieved by

$$\langle\Delta_{R,44}^{1+i2}\rangle \equiv v \neq 0; \quad \langle\Delta_{L,ij}\rangle = 0. \qquad (2)$$

Equation (2) reduces the local flavor symmetry to $SU(2)_L \otimes U(1)$ for $m_{W_R} \approx gv$, which is further broken[3] by choosing $\langle\Phi\rangle = \begin{pmatrix}\kappa & 0 \\ 0 & \kappa'\end{pmatrix}$, where $g\kappa \approx 100$ GeV, the mass scale of W_L. As is well known,[3] the masses of charged fermions arise due to $\langle\Phi\rangle \neq 0$; this constrains the Yukawa-Higgs couplings to be of order $h \sim g(m_q/m_{W_L})$.

To study the implications of $(B-L)$ breakdown in detail, we write down the gauge-invariant Yukawa couplings

$$\mathcal{L}_Y = ih(\psi_{L,i}{}^T \tau_2 \tau_a C^{-1} \psi_{L,j} \Delta_{L,ij}{}^a + \Psi_{R,i}{}^T \tau_2 \tau_a C^{-1} \Psi_{R,j} \Delta_{R,ij}{}^a) + h_1 \bar{\Psi}_L \varphi \Psi_R + h_1{}' \bar{\Psi}_L \tilde{\varphi} \Psi_R + \mathrm{H.c.} \qquad (3)$$

As was noted in Ref. 7, $\langle\Delta_{R,44}^{1+i2}\rangle \neq 0$ along with the contribution of $\langle\Phi\rangle$ leads to the following mass matrix for the neutrinos (let $\nu \equiv \nu_L$ and $N \equiv \nu_R$):

$$\begin{matrix} & \nu & N \\ \nu & 0 & m_e \\ N & m_e & m_N \end{matrix}, \qquad (4)$$

where $m_e \approx h_1 \kappa$ and $m_N \approx hv \approx g m_{W_R}$. From this it follows that the usual neutrinos must be Majorana particles with[7,9] $m_\nu \simeq m_e{}^2/g m_{W_R}$. We thus see that the smallness of the neutrino mass is related to the dominant $V-A$ nature of the charged weak current at low energy. On the other hand, since $g \sim e$, $m_N \gtrsim 100$ GeV, reflecting the breaking[10] of I_{3R}. This implies the breaking of $B-L$ which is consistent with a Majorana neutrino (for which $\Delta L = 2$).

The Majorana character of ν_e predicts the existence of double β decay coming from the exchange of two W_R bosons with the heavy Majorana neutrino counterpart as intermediate state. Thus far, the existing lower limit on the lifetime for double β decay is consistent with the predictions of our model (see Ref. 7). Another consequence of the Majorana neutrino is the production of "wrong"-type charged leptons in inverse β decay; here, the model prediction is much lower than the experimental limit[11] because of the (m_ν/E_ν) suppression factor (E_ν is the neutrino energy).

We now discuss the implications of $B-L$ nonconservation for the hadronic sector. First we note that there exists a self-coupling of the scalar multiplet $\Delta_{L,R}$ as follows:

$$\mathcal{L}_S = \lambda[\epsilon^{ikmp}\epsilon^{ilnq}\Delta_{L,ij}{}^a\Delta_{L,kl}{}^a\Delta_{L,mn}{}^b\Delta_{L,pq}{}^b + (L \leftrightarrow R) + \mathrm{H.c.}]. \quad (5)$$

From Eqs. (2), (3), and (5), it follows that there exists a six-fermion vertex of type (see Fig. 1)

$$\mathcal{L}_{6f} = h_{\mathrm{eff}} d_{R,\alpha}{}^T C^{-1} d_{R,\beta} d_{R,\gamma}{}^T C^{-1} d_{R,\lambda} u_{R,\rho}{}^T C^{-1} u_{R,\sigma} \epsilon^{\alpha\gamma\rho}\epsilon^{\beta\lambda\sigma} + \text{similar terms}, \quad (6)$$

where the Greek letters α, β, ... denote color. This Lagrangian causes the transition $n \leftrightarrow \bar{n}$ which we call neutron oscillation.[12] The important point, which becomes obvious looking at Fig. 1, is that the $\Delta B = 2$ n-$\bar{n}$ transition is caused by the same spontaneous breaking mechanism (i.e., $\langle\Delta_{R,44}\rangle \neq 0$) that causes $\Delta L \neq 0$. We now estimate the strength of the n-$\bar{n}$ transition $h_{\mathrm{eff}} \approx \lambda h^3\langle\Delta_{R,44}\rangle / m_{\Delta_R}{}^6$. We see that as $\langle\Delta_{R,44}\rangle \to 0$ (i.e., restoration of parity as well as $B-L$ symmetry), the n-$\bar{n}$ oscillation disappear. We may choose the coupling $h \sim 10^{-2}$ (since it is related to the mass of the heavy Majorana neutrino[7]) and it becomes of interest to relate the characteristic time $t_{n\text{-}\bar{n}}$ for the neutron oscillation to m_{Δ_R}. If we use the limiting lifetime resulting from the observed nuclear stability,[13] of 10^{30} yr, this corresponds to $m_{\Delta_R} \simeq 10^4$ GeV and $t_{n\text{-}\bar{n}} \simeq 10^5$ sec.[14] We stress that in our "minimal" model (without any additional Higgs beyond those already introduced), the proton is stable.[15] This is just the reverse of the situation with the "minimal" SU(5) model where $\Delta B = 2$ transitions are forbidden.[16]

Thus, baryon number nonconservation—with or without $B-L$ conservation—becomes a very interesting test of unification models. It would seem that essentially the same experimental setup as the one which will be used to search for proton decay could yield information about $\Delta B = 2$ nucleon transitions.[17] The observation of such transitions without proton decay would be strong evidence for the existence of a "partial unification" model of the type that we are considering.

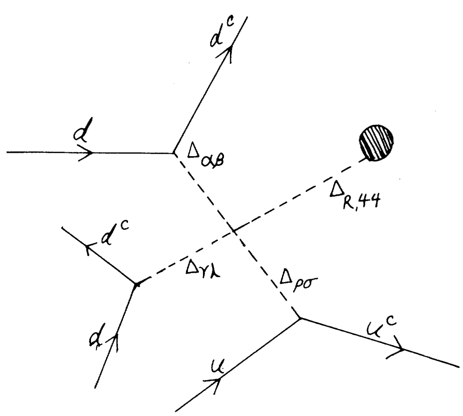

FIG. 1. The tree graph that induces the six-fermion $\Delta B = 2$ vertex that leads to $n \leftrightarrow \bar{n}$ oscillation.

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[8]The SU(4′) "color" group was introduced by J. C. Pati and A. Salam (Ref. 3) with L as the fourth color; in our scheme $B-L$ is the fourth "color".

[9]Other recent works that use Majorana neutrinos are M. Gell-Mann, P. Ramond and R. Slansky, unpublished;

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[10]The large value of m_n for each generation could reestablish the same order-of-mass spectra for the "up" and "down" components of the quark and lepton isodoublets (see Ref. 5).

[11]R. Davis, in *Proceedings of the International Conference on Radio-Isotopes in Scientific Research, Paris, 1957* (Pergamon, London, England, 1958).

[12]The same Lagrangian plus a "spectator" quark line will also lead to a $\Delta B=2$ nucleon transition of the type $n+p\to\pi$'s.

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[14]The value $t_{n\text{-}\bar{n}}\gtrsim 10^5$ sec for a free neutron follows from the relation $\Delta m\approx(\Gamma M)^{1/2}$ derived by S. L. Glashow [Harvard University Report No. HUTP-79/A059 (to be published)] with $\Gamma^{-1}\gtrsim 10^{30}$ yr; in this relation, $\Delta m\approx 1/t_{n\text{-}\bar{n}}$ is the mass difference between the $n+\bar{n}$ and $n-\bar{n}$ mass eigenstates, Γ is the decay width for $\Delta B=2$ nucleon transitions (see Ref. 12) and M is of the order of a hadronic mass (say 10 GeV). The effect of the overlap of the wave functions of the two nucleons in the nucleus has been estimated in connection with the possible $p+p\to e^++e^+$ process by G. Feinberg, M. Goldhaber, and B. Steigman, Phys. Rev. D 18, 1602 (1979).

[15]The reason for proton stability is the existence of a hidden discrete symmetry in the model $q_\alpha\to e^{i\pi/3}q_\alpha$ and $\Delta_{\alpha\beta}\to e^{-2i\pi/3}\Delta_{\alpha\beta}$, $\Delta_{\alpha4}\to e^{-i\pi/3}\Delta_{\alpha4}$ (everything else unaffected), under which the Lagrangian is invariant even after spontaneous symmetry breakdown. This hidden discrete symmetry is broken by adding extra Higgs bosons which are antisymmetric in the interchange of SU(4′) indices thereby allowing for proton decay (with $B-L$ conservation). It should be emphasized that the "symmetric" Higgs bosons are essential in our model to explain the parity nonconservation of the weak interaction at low energy whereas the "antisymmetric" Higgs bosons play no such role.

[16]Even the "nonminimal" SU(5) model can not induce $\Delta B=2$ nucleon transitions that are competitive with proton decay (private communication from G. Senjanović).

[17]See L. Sulak, in Proceedings of the Weak Interaction Workshop, Virginia Polytechnic Institute, December 1979 (unpublished); $\Delta B=2$ nucleon transitions in this experiment would be expected to yield multipion final states with an average multiplicity of about 4. The average energy per pion (~500 MeV) would therefore be approximately the same as in the experiment designed to detect proton decay. In principle, the $\Delta B=2$ $n\to\bar{n}$ transition could be observed directly with reactor neutrons (private communication from R. Wilson).

Relativistic Wave Equation and Mass Spectrum of Gluonium

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A relativistic wave equation is derived for gauge-invariant $J=0$ gluonium amplitudes, and show that its reduced eigenvalue equation is identical with that for a quark-antiquark system in $^3P_1$ states. Analyzing relations between potentials in the two respective systems, I obtain an upper limit of 2 GeV to the ground-state mass if I take light quarkonium as a reference system, whereas I estimate the ground state to lie between 2.5 and 3 GeV if I use charmonium potential parameters. The scalar and the pseudoscalar gluonium states are degenerate in the present approach.

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Gluonium spectra have been studied by several authors[1] in various models including the Massachusetts Institute of Technology bag model and lattice-gauge theory. Recently, Ishikawa[2] studied a potential model of gluoniums using a variational calculation and obtained a relatively high ground-state mass compared with previous works. In the present paper, I derive and solve in the framework of quantum chromodynamics a relativistic wave equation with a confinement potential, satisfied by a gauge-invariant two-gluon amplitude, which represents a string of electric flux connecting the gluons. A gluonium in this model is very much like a system of a massless quark and an antiquark, except for a difference in potentials. The potential for the former has a group-theoretical weight of $\frac{9}{4}$ relative to the one for the latter, as will be shown. The Coulomb potential has the same weight, but it appears to be less important for systems of massless particles as

