Interaction Effects in Disordered Fermi Systems in Two Dimensions

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(Received 11 February 1980)

Interaction effects in disordered Fermi systems are considered in the metallic regime. In two dimensions, logarithmic corrections are obtained for conductivity, density of states, specific heat, and Hall constant. These results are compared with a recent theory of localization as well as some experiments.

PACS numbers: 71.55.Jv, 72.15.Cz, 73.60.Dt

Recently there are reports of logarithmic rise of resistivity at low temperatures in thin films[1] and in the silicon inversion layer.[2] Many features of these experiments are in agreement with a prediction of Abrahams, Anderson, Licciardello, and Ramakrishnan[3,4] that electronic states are always localized in two dimensions (2D). On the other hand a numerical scaling study[5] appears to be inconsistent with this conclusion. We note that the localization theory deals only with noninteracting electrons. Before these experiments are accepted in support of the localization theory it is necessary to consider the effects of interactions in disordered systems. This problem has been treated by Altshuler and Aronov[6] for three-dimensional systems. In this paper we extend their treatment to two dimensions and consider additional physically observable quantities such as Hall effect and specific heat.

Before dealing with the electron gas we first consider a model problem of a fermion system subject to a random external potential and interacting with a weak short-range potential $v(\vec{r}-\vec{r}')$. We shall be in the "metallic" regime, meaning that $k_F l \gg 1$, where $l = v_F \tau$ is the electronic mean free path. In this regime the conventional diagrammatic technique[7] is applicable[8] and a systematic calculation to lowest order in $v(\vec{r})$ is possible. Let us consider the vertex correction in Fig. 1(a). A straight forward calculation shows that for $|\omega_m|\tau \ll 1$ and $Dq^2\tau \ll 1$, where $D = \frac{1}{2} v_F{}^2 \tau$ in 2D,

$$\Gamma(q, \omega_m, \epsilon_n) = (|\omega_m| + Dq^2)^{-1}\tau^{-1} \tag{1}$$

if ϵ_n and $\epsilon_n - \omega_m$ have opposite signs and is unity otherwise. The singularity for small ω and q is associated with diffusion as is clear from the fact that Eq. (1) produces the proper diffusive behavior for the density-density correlation function. As an illustration we consider the exchange self-energy correction shown in Fig. 1(c) which leads to the following correction to the impurity-averaged (denoted by a bar) single-particle density of states:

$$\delta N(\epsilon) = -\pi^{-1}\,\mathrm{Im}\sum_{\vec{p}} \bar{G}(\vec{p}, \vec{p}, i\epsilon_n \to \epsilon + i\eta)$$

$$= \mathrm{Im}\Big\{-i\pi^{-1}\sum_{\vec{p},\vec{q}} v(q)\int_{\epsilon}^{1/\tau}(d\omega/2\pi)\tau^{-2}(-i\omega + Dq^2)^{-2} G_{0+}{}^2(\vec{p}, \epsilon) G_{0-}(\vec{p}-\vec{q}, \epsilon-\omega)\Big\}. \tag{2}$$

Here we have specialized to $T = 0$ for simplicity of presentation and $G_{0\pm}(p, \omega) = [\omega - \epsilon(p) \pm i2\tau]^{-1}$. The important term in Eq. (2) is the square of the diffusion pole which comes from the two factors of vertex correction in Fig. 1(c). Clearly q, ω can be set to be zero in the rest of the expression of the $\vec{p}$ integration performed. Upon q and ω integration the diffusion poles then lead to the following logarithmic correction to the density states due to the exchange diagram

$$\frac{\delta N_{ex}(\epsilon)}{N_1} = \frac{1}{\pi}\frac{v(q)N_1}{\epsilon_F \tau}\ln(|\epsilon|\tau), \tag{3}$$

where N_1 is the single-spin unperturbed density of states. In 3D similar considerations[6] have led to a correction of order $(\epsilon\tau)^{1/2}(\epsilon_F\tau)^{-2}$. Such corrections should be observable in tunneling experi-

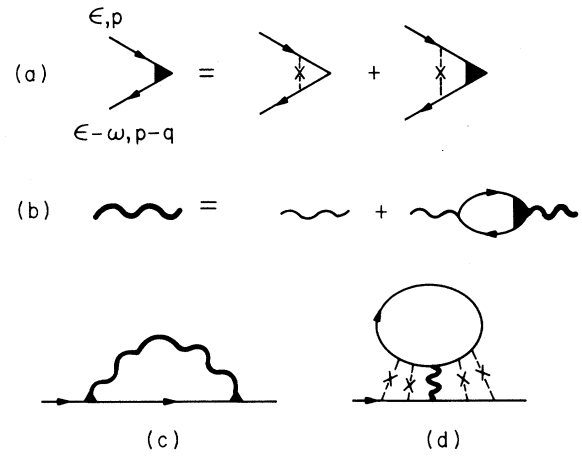

FIG. 1. (a) Vertex correction where dashed lines with a cross denote impurity scattering, (b) thick wavy line denotes dynamically screened Coulomb interaction, (c) and (d) exchange and Hartree self-energy correction.

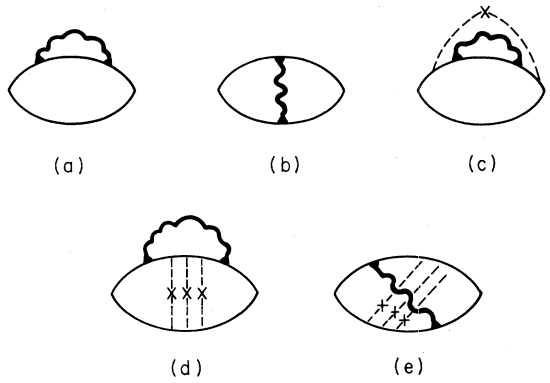

FIG. 2. Conductivity diagrams.

ments. Furthermore for this model problem the density-of-states corrections comes from corrections in the eigenvalues and nor from the quasiparticle spectral weight. Therefore the corrections should appear in the specific heat, as can be confirmed by direct calculations. Physically the eigenvalue of an added particle with energy ϵ is shifted by exchange interactions with the occupied states. This shift depends on ϵ because the overlap matrix element with a state of energy ϵ' depends on $\epsilon-\epsilon'$ in a way given by Eq. (1). This energy-dependent shift is sufficient to give a logarithmic correction to the density of states. While it is convenient to calculate in momentum representation using averaged Green's functions, the physics is much clarified in a representation using the exact impurity eigenstates. This latter point of view will be present elsewhere.[9]

The conductivity can be calculated using standard diagrammatic techniques. Figure 2 represents all the diagrams generated in a conserving approximation from the self-energy correction Fig. 1(c). Again it is a systematic approximation to first order in the interaction and in the $k_F l \gg 1$ limit. Instead of weakly-interacting-fermion problem we now treat the electron gas. In the small-q,ω limit, the dynamically screened Coulomb interaction is given [(see Fig. 1(b)] by

$$v_s(q,\omega)=v_B(q)[1+v_B(q)\Pi(q,\omega)]^{-1}. \quad (4)$$

Here the bare Coulomb interaction $v_B(q)=2\pi e^2/|\vec{q}|$ in 2D and $\Pi(q,i\omega_n)=sN_1Dq^2/(|\omega_n|+Dq^2)$, where s is the spin and other degeneracies. Note that in the limit $q,\omega\to 0$, we may ignore the unity term in Eq. (4) and $v_s(q,\omega)\to\Pi^{-1}(q,\omega)$ which is independent of the bare coupling constant e^2. Diagrams 2(a), 2(b), and 2(c) represent self-energy correction, vertex correction, and correction to the scattering rate, respectively, and are found to exactly cancel each other. Figures 2(d) and (e) arise from the fact that impurity averaging of the particle-hole propagator is not the product of the averaged Green's functions. The real and imaginary part of v_s is found to contribute equally giving a conductivity correction due to the exchange diagrams

$$\delta\sigma_{ex}(\Omega,T)=\frac{e^2}{\hbar}\frac{1}{2\pi^2}\times\begin{cases}\ln(\Omega\tau), & T\ll\Omega\ll\tau^{-1}\\ \ln(T\tau), & \Omega\ll T\ll\tau^{-1}.\end{cases} \quad (5)$$

Note that the coefficient of the logarithm is independent of the interaction constant for reasons already mentioned. Thus the ratio $\delta\sigma/\sigma$ is proportional to the resistance, consistent with the experimental observation.[1,2] Indeed the constant $(2\pi^2)^{-1}$ is in excellent agreement with experiment.[2] The nonlinear conductivity is explained by the heating model[4] once a temperature-dependent conductivity is obtained.

The fact that we obtain a logarithmic correction even for a short-range instantaneous interaction imply that corresponding corrections must exist also for the Hartree term. This is because if the interaction is a δ function in space, the parallel-spin Hartree and exchange terms must cancel. The corresponding diagram is shown in Fig. 1(d). Usually the Hartree term is canceled by the uniform background. However, in the presence of impurities the electron density $n(r)$ is not uniform so that when a particle with wave functions ψ_α is added, a residual interaction of the form $\psi_\alpha^*(r)\psi_\alpha(r)n(r)$ remains. The impurity average

of this term cannot be factorized and the wave-function overlap with the occupied states again produce an energy-dependent Hartree shift. However unlike the exchange case, we see from Fig. 1(d) that the momentum transfer in the interaction line is not small. The static screened interaction has the form $V(q)=2\pi e^2/(|q|+\kappa)$, where $\kappa=2\pi e^2 s N_1$ is the inverse screening length in 2D. Compared with the exchange term, the Hartree term is opposite in sign and reduced by the factor

$$F=(2\pi\tau N_1)^{-2}V^{-1}(q=0)\sum_{\vec{p},\vec{p}'}V(\vec{p}-\vec{p}')G_{0+}(p)G_{0-}(p)G_{0+}(p')G_{0-}(p')\approx\int(d\theta/2\pi)[1+(2k_F/\kappa)\sin\tfrac{1}{2}\theta]^{-1}, \quad (6)$$

where θ is the angle between $\vec{p}$ and $\vec{p}'$ and we have used the fact that as $\tau^{-1}\to 0$, $|p|\approx k_F$. The factor F approaches unity for short-range interaction ($2k_F/\kappa\to 0$) as expected and vanishes for long screening length. The total correction to conductivity is given by

$$\delta\sigma=(e^2/4\pi^2\hbar)(2-2F)\ln(T\tau), \quad (7)$$

where the factor $2F$ comes from spin degeneracy and the fact that only the real part of v_s contribute to the Hartree term. Unlike the exchange term, the Hartree contribution depends on the coupling constant via $2k_F/\kappa$. As noted earlier the experiments appear to be consistent with the exchange term being dominant, i.e., $2k_F/\kappa\gg 1$. It will be interesting to decrease the electron density in the inversion layer to look for a systematic deviation.

We emphasize that the present theory does not involve localization.[8] Apart from the density dependence just discussed comparing our results with the localization theory is complicated by the fact that the localization prediction depends on the inelastic scattering rate $\tau_{in}^{-1}\alpha T^p$ and p is not very well known at the moment.[4] In the localization theory $\delta\sigma$ is proportional to s. For $s=2$ the frequency-dependent conductivity[4,10] in fact identical to Eq. (5) except that the condition is $\Omega\gg\tau_{in}^{-1}$. The dc conductivity in the localization theory is proportional to $p\ln(T\tau)$ and is identical to Eq. (5) for $p=1$. It is worth noting here that the inelastic rate due to electron-electron scattering is modified from the usual T^2 dependence in the presence of impurities so that $p=\frac{3}{2}$ in 3D[6,11] and $p=1$ in 2D.[9] Still another point of distinction between the two theories is the crossover from higher to lower dimensions. In the present theory the important region of integrations is of order kT, so that crossover occurs when one dimension is of order d given by

$$Dd^{-2}=kT. \quad (8)$$

In the localization picture it is the diffusion length in the time τ_{in} that sets the scale,[12] thus kT is again replaced by τ_{in}^{-1} in Eq. (8). Present experiments[1] are consistent with Eq. (8) and require a surprisingly large τ_{in}^{-1} if interpreted in terms of localization. Again, our lack of an independent measurement of τ_{in}^{-1} precludes a detailed comparison at present.

To gain more insight into the conductivity process we have also calculated the Hall current. The Hall current involves a second-order response function linear in the electric and magnetic field. This is obtained by adding a magnetic vertex that carries momentum but not frequency in all possible ways into Fig. 2. The computation is rather complicated and the details will be reported elsewhere.[13] The result is simply that corrections to the Hall current vanishes. Since the Hall constant R_H goes like the resistance squared, this implies

$$\delta R_H/R_H=2\delta R/R. \quad (9)$$

This result shows that the conductivity correction cannot be interpreted in the conventional sense of a density-of-states correction or a scattering-rate correction.

While we have not carried out the calculation to higher orders in $v(q)$, our estimates indicate that it is a series in powers of $(vN_1/\epsilon_F\tau)\ln(T\tau)$ which indicates the limit of validity of the present theory for small T. It has previously been stated[6] that higher-order terms are much more singular, but we now find that those terms are in fact canceled.

Finally we would like to summarize the results for the density of states and specific heat. The exchange contribution has been given in Eq. (3) for a static potential. For a retarded interaction such as Eq. (4), the density-of-states and specific-heat corrections are not the same because the density-of-states correction is due partly to a redistribution in the quasiparticle spectral weight and partly to shifts in the eigenvalues. Only the latter contribute to the specific heat. Indeed direct calculation of the free energy in the standard way using a coupling-constant integration[7] shows that only the real part of $v_s(q,\omega)$ contribute a log-

arithmic term to the specific heat. Thus instead of the usual specific heat C_0 that is linear in T we we find a correction $\delta C/C_0 = (\pi\epsilon_F\tau)^{-1}(1-2F)\ln(T\tau)$. On the other hand, both the real and imaginary parts of $v_s(q,\omega)$ contribute to the single-particle density of states. Peculiar features of screening in 2D result in the following

$$\frac{\delta N(\epsilon)}{N_1} = -\frac{1}{2\pi\epsilon_F\tau}\ln(|\epsilon|\tau)\ln\left|\frac{\epsilon}{D\kappa^2}\right|. \quad (10)$$

We recall from the definition that this is a density of states for bare particles and tunneling is probably the only way to observe it. No correction is expected for the magnetic susceptibility because the corrections are tied to the up- and down-spin Fermi energy. (A similar effect is well known for phonon enhancement of the density of states.) The specific-heat correction will be difficult to measure experimentally in 3D since it is smaller than the usual linear term by a factor $(T/\epsilon_F)^{1/2}(\epsilon_F\tau)^{-3/2}$ which is small even for highly disordered metals. The logarithmic correction in 2D may have a better chance of being observable. Possible systems are doped semiconductors of helium three adsorbed on Grafoil.

This work is the result of our participation in the Joint Research Group in Condensed Matter Physics of the Joint US-USSR Commission on Scientific and Technological Cooperation that met in Lake Sevan, Armenia, USSR. We thank other participants, especially D. Khmel'nitzkii, A. I. Larkin, L. P. Pitaevskii, and W. F. Brinkman for discussions. One of us (P.A.L.) is thankful to P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan for many stimulating discussions.

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Temperature Dependence of the Angle-Resolved Photoemission in Itinerant-Electron Ferromagnets

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(Received 25 January 1980)

The prediction of the local band theory of itinerant-electron ferromagnetism for the temperature dependence of the single-hole Green's function is obtained. Qualitative agreement with observation is found, but the interpretation is novel. The underlying spectral function is more complex than the simple two-peaked structure heretofore assumed. Although the width of this complex shrinks with temperature, the shrinkage does not correspond to a diminution of the exchange splitting as that quantity is most conveniently defined.

PACS numbers: 75.10.Lp, 79.60.Cn

The exchange splitting, and particularly its temperature dependence, in band ferromagnets such as iron and nickel, is fundamental to the understanding of these materials. It is therefore surprising that no direct measure of it exists. Perhaps even more surprising is that there is no

