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[1]P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. 43, 718 (1979).

[2]B. Abeles, *Applied Solid State Science*, edited by R. Wolfe (Academic, New York, 1976), Vol. 6, p. 1.

[3]G. Deutscher, H. Fenichel, M. Gershenson, E. Grünbaum, and Z. Ovadyahu, J. Low Temp. Phys. 10, 231 (1973).

[4]T. Worthington, P. Lindenfeld, and G. Deutscher, Phys. Rev. Lett. 41, 316 (1978).

[5]G. Deutscher, Y. Imry, and L. Gunther, Phys. Rev. B 10, 4598 (1974).

[6]G. Deutscher, O. Entin-Wohlman, S. Fishman, and Y. Shapira, to be published; G. Deutscher and M. L. Rappaport, J. Phys. (Paris), Colloq. 39, C6-581 (1978), and J. Phys. (Paris) 40, L-219 (1979). This work is reviewed by G. Deutscher, O. Entin-Wohlman, M. Rappaport, and Y. Shapira, in Proceedings of the International Conference on Inhomogeneous Superconductors, Berkeley Springs, West Virginia, November, 1979 (to be published).

[7]M. Gershenson and W. L. McLean, to be published.

[8]B. Abeles, Phys. Rev. B 15, 2828 (1977); W. L. McLean and M. J. Stephen, Phys. Rev. B 19, 5925 (1979).

[9]W. L. McLean, P. Lindenfeld, and T. Worthington, in *Electrical Transport and Optical Properties of Inhomogeneous Media—1977*, edited by J. C. Garland and D. B. Tanner, AIP Conference Proceedings No. 40 (American Institute of Physics, New York, 1978), p. 403.

[10]W. L. McLean, T. Chui, B. Bandyopadhyay, and P. Lindenfeld, in Proceedings of the International Conference on Inhomogeneous Supercondcutors, Berkeley Springs, West Virginia, November, 1979 (to be published).

[11]P. Sheng, B. Abeles, and Y. Arie, Phys. Rev. Lett. 31, 44 (1973).

[12]D. Stauffer, Phys. Rep. 54, 1 (1979).

[13]B. B. Mandelbrot, *Les Objets Fractals: Forme, Hasard, et Dimension* (Flammarion, Paris and Montreal, 1975) [translation: B. B. Mandelbrot, *Fractals: Form, Chance, and Dimension* (W. H. Freeman, San Francisco, 1977)].

[14]G. Dolan and D. D. Osheroff, Phys. Rev. Lett. 43, 721 (1979).

[15]N. Giordano, W. Gilson, and D. E. Prober, Phys. Rev. Lett. 43, 725 (1979).

[16]C. C. Tsuei, Solid State Commun. 27, 691 (1978).

[17]R. C. Dynes, J. P. Garno, and J. M. Rowell, Phys. Rev. Lett. 40, 479 (1978).

[18]W. L. McLean, to be published.

Nonmetallic Conduction in Electron Inversion Layers at Low Temperatures

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We have measured the resistance of electron inversion layers in Si metal-oxide-semiconductor field-effect transistors at low temperatures (~50 mK) and low electric fields (~0.1 V/m). At low values of $R_\square$ we observe logarithmic dependences of the resistance on both temperature and applied electric field which scale only on $R_\square$. We observe a gradual transition to an exponential dependence at $R_\square \gtrsim 10$ kΩ. The logarithmic dependences agree qualitatively but not quantitatively with current theories of localization.

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A number of recent experimental and theoretical papers have addressed the subject of electron transport and localization in a quasi two-dimensional (2D) system.[1-5] We report measurements of the electrical conductivity of electron inversion layers in silicon metal-oxide-semiconductor field-effect transistors (MOSFET's) at low temperatures and compare them with recent theoretical ideas and experimental results on metal films.[5] Our measurements show that at sheet resistances $R_\square \sim 10$ kΩ/□ there is a smooth and gradual crossover from an exponential to a logarithmic dependence on temperature and applied electric field. Similar to the previous observations of Dolan and Osheroff,[5] we see no evidence for true metallic behavior below 10 kΩ/□. In addition, our results show that these effects scale only with $R_\square$ of the inversion layer.

Our measurements were performed on n-channel MOSFET's fabricated on (100) and 111) surfaces of p-type silicon with peak mobilities of ~2000 cm^2/V·sec at 4.2 K. The Si electron inversion layer is a 2D electron gas whose density is determined by the applied gate voltage. Also the mobility can be varied by applying a substrate bias which moves the electron wave function closer to, or further away from, the Si-oxide interface. This system has been extensively studied by a

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number of investigators.[6-11] However, the present investigation at low temperatures (~50 mK) examines in detail the source-drain field (~0.1 V/m) dependence of the conductivity and we have observed temperature and electric field dependences not previously seen in these systems.

The measurements were performed in a He^3-He^4 dilution refrigerator at temperatures from 50 mK to 4 K. The devices allowed direct four-terminal measurements of the channel resistance as a function of temperature, source-drain electric field, gate voltage, substrate bias, and magnetic field.[12] The conducting channel is 0.25 mm wide and 1.0 mm long, with potential probes arranged in the usual Hall bridge geometry. The two potential probes were separated by 0.25 mm and so the $R_\square$ quoted here is the resistance of a 0.25×0.25-mm^2 area.

The *I*-*V* characteristics were measured using a dc bridge such that a constant resistance R_{sub} could be subtracted off and small deviations from linearity more clearly observed. In Fig. 1 we show representative I-δV curves at different temperatures obtained in this fashion with $R_{sub}=4.9$ $k\Omega$ from a sample on (111) Si($\delta V=V-IR_{sub}$). Note that the curves become non-Ohmic at extremely low values of the electric field. The relevant experimental quantities are the zero-bias resistances as a function of temperature and the low-temperature resistance as a function of applied electric field. The zero-bias resistance is shown as a straight line for the lowest temperature (0.057 K) curve.

Measurements of the zero-bias resistance as a function of temperature demonstrate two different types of behavior which depend on the $R_\square$ of the channel. In the high-resistance regime ($R_\square\gtrsim10$ k$\Omega/\square$) down to the lowest temperatures studied (~50 mK) we find an exponential dependence of zero-bias resistance on temperature best fitted by the form $R\sim R_0\exp(A/T^{1/3})$, the dependence expected for two-dimensional variable-range hopping.[10]

In the low-resistance regime ($R_\square\lesssim10$ k$\Omega/\square$, we find that the zero-bias resistance decreases logarithmically with increasing temperature for *all* values of the electron density and $R_\square$. We therefore see no evidence for true metallic conduction at *any* value of $R_\square$ studied (to date this result holds down to 1000 $\Omega/\square$ and in metal films to 160 $\Omega/\square$).[5] Our results therefore are consistent with the fundamental conclusion of Abrahams, Anderson, Licciardello, and Ramakrishnan (AALR)[2] that in two dimensions there should exist no true metallic behavior.

Representative curves for this logarithmic region are shown in more detail in Fig. 2. At intermediate temperatures we see the logarithmic dependence of the zero-bias resistance on temperature. The logarithmic dependence saturates at high temperatures and eventually turns around as seen by other workers.[13] Also at low temperatures this logarithmic dependence either saturates or changes its slope (the data are not adequate to distinguish). As can be seen in Fig. 2 this "saturation" temperature increases with increasing electron density from $T\lesssim60$ mK to $T\sim200$ mK. The first possible explanation is a simple noise heating effect. This seems unlikely

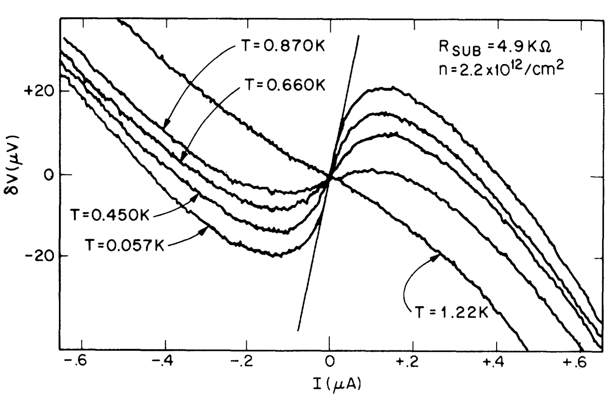

FIG. 1. I-δV characteristics for an electron inversion layer at various temperatures ($\delta V=V-IR_{sub}$). The curves shown are therefore the deviations from linearity after the constant resistance R_{sub} is subtracted off.

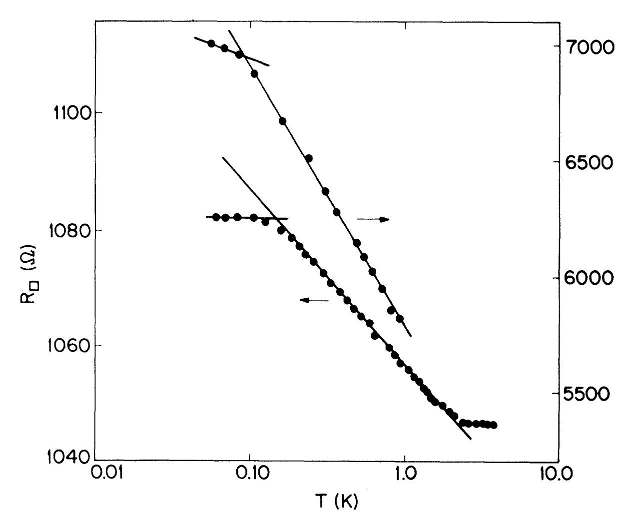

FIG. 2. The zero-bias resistance on an expanded scale as a function of temperature for inversion layers with densities of 2.03×10^{12} electrons/cm^2 (right) and 5.64×10^{12} electrons/cm^2 (left).

because the noise in our system is low (~1 μV) and the "saturation" moves to higher temperatures with decreasing resistance, opposite to conventional noise heating. A possible second explanation is that this is due to the finite size of the sample as at these low temperatures the relevant lengths can be quite long.

From data of the type shown in Fig. 1 we can also extract the electric field dependence of resistance. For $R_\square \lesssim 10$ k$\Omega/\square$ this non-Ohmic behavior is logarithmic with source-drain voltage. In Fig. 3 the logarithmic slopes of resistance vs voltage and temperature plotted as a function of $R_\square$. Both slopes show a linear dependence on $R_\square$ at low enough values of $R_\square$.

At $R_\square \gtrsim 5$ k$\Omega/\square$ the logarithmic slopes begin to deviate from this linear dependence. At $R_\square \gtrsim 10$ k$\Omega/\square$ the curves of resistance versus temperature and voltage are no longer logarithmic and at $R_\square \gtrsim 15$ k$\Omega/\square$ they are exponential. There is no evidence of any abrupt behavior and our data qualitatively support the intuitive arguments of AALR[2] who predict that there should exist no sharp mobility edge in two dimensions and that a smooth and continuous crossover from exponential to logarithmic behavior should occur.

Using a substrate bias we can test the validity of conductance as single scaling parameter. The observed experimental behavior should only be determined by $R_\square$ of the two-dimensional system and not by any other single parameter such as

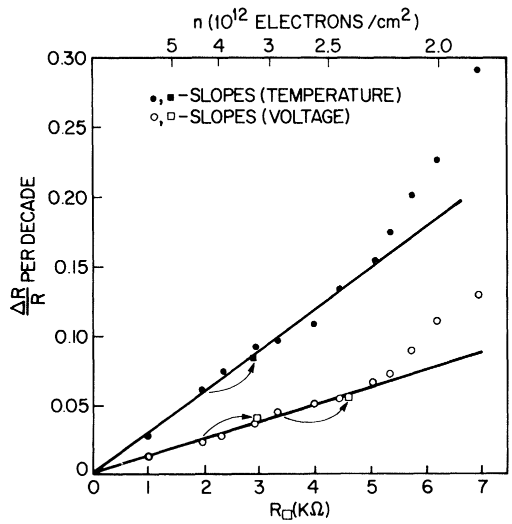

FIG. 3. The logarithmic slopes vs temperature and voltage (circles) as a function of $R_\square$ ($T = 1$ K) (lower scale) and electron density (upper scale) for zero substrate bias. The squares show the change slopes with $V_{\text{sub bias}} = -10$ V.

electron density, mobility, or effective mass. The rough quantitative agreement between the logarithmic slopes observed in metal films[5,14] and Si MOSFETS already support this contention, but a more rigorous test can be made. By applying a substrate bias to the inversion layer we can change the electron mobility without affecting the density. In these samples it was possible to change the resistance by almost a factor of 2. Logarithmic slopes have been measured for various values of electron mobility. These data are shown as the squares in Fig. 3 and show that the slopes fit on the universal curves when plotted as a function of $R_\square$, not mobilities or electron densities. In addition to (111) devices we also measured samples fabricated on (100) Si and observed this same universal behavior, independent of electron effective mass. Therefore these measurements provide strong support for the concept of a single scaling parameter in the current theories of conduction in two dimensions. Experimentally this single parameter manifests itself as $R_\square$.

Extending the ideas of Thouless, AALR show that the $T = 0$ K conductance of a disordered electronic system depends in a universal manner on its length scale L. They argue that in two dimensions there should exist *no* true metallic behavior and that for the resistance ($R \ll \pi^2\hbar/e^2$) one finds

$$R_\square(L) = R_\square(L_0)\left[1 + \frac{\alpha e^2}{\pi^2\hbar} R_\square(L_0) \ln\left(\frac{L}{L_0}\right)\right]. \qquad (1)$$

At finite T, using a diffusion model, Thouless has suggested the geometric mean of the elastic and inelastic mean free paths as the relevant length scale L. It follows then that $L^2 \sim T^{-P}$ with P the power of T for the appropirate inelastic scattering power, i.e., $P = 2$ if the appropriate inelastic length scale is determined by electron-electron scattering and $P = 2$, 3, or 4 for electron-phonon scattering, depending upon phonon dimensionality and dirty or clean limits. Therefore, from (1) the logarithmic slopes versus temperature should be given by $(\alpha e^2/\pi^2\hbar)(P/2)R_\square$. From the slope of the solid line in the upper curve in Fig. 3, $\alpha P/2$ has the value 0.52 ± 0.05

Utilizing an electron heating model we can also analyze the voltage slopes shown in Fig. 3. Anderson, Abrahams, and Ramakrishnan[4] show that the ratio of the coefficients of $\ln V$ and $\ln T$ for any given $R_\square$ should be a constant given by $2/(2 + P')$, where P' is the temperature exponent of the electron-phonon scattering rate. This ratio is independent of the physical mechanism which determines the coefficient of the $\ln T$ dependence.

Our data shown in Fig. 3 provide a value for P' of 2.7 ± 0.5, which suggests an exponent of $P' = 3$. At these temperatures and elastic scattering rates we are in the dirty limit, $q l_{el} << 1$ ($q = k_B T/\hbar s$; s = sound velocity). In this limit $P' = 3$ is expected, as the selection rules for phonon emission should reflect the two-dimensional nature of the Fermi surface.

Using the electron heating model we can now estimate the Thouless length $L = (\frac{1}{2} l_{el} l_{inel})^{1/2}$, where l_{inel} is determined by the electron-phonon interaction. For our sample $l_{el} \sim 100$ Å. Our estimates of L_{ep} indicate that it is approximately the size of the sample at the temperatures where we observe saturations in $R_\square$ vs $\ln T$ (see Fig. 2). This implies that the saturation might be due to finite-size effects and from the slope measurements we determine $P = 3$, and so $\alpha \sim \frac{1}{3}$. However, our estimates for the electron-electron scattering length are that it is one to two orders of magnitude shorter and so it should define L. For electron-electron scattering $P = 2$ and hence $\alpha \sim \frac{1}{2}$. Reference 4 predicts $\alpha = 1$ for two noncommunicating gases of electrons with spins up and down and $\alpha = \frac{1}{2}$ if the elastic spin-flip scattering length is short compared to the other lengths in the problem. A third possibility is that the relevant length is set by some other inelastic process as suggested by the experiments of Dolan and Osheroff on thin metal films, which indicate that neither L_{ee} or L_{ep} can be responsible for determining the dimensionality of their system. A final possibility has been suggested by Altshuler, Aronov, and Lee.[5] In two dimensions Coulomb interactions may give logarithmic terms in the conductance which are comparable in size to the effects which have been observed. Unfortunately the present experiments are not sufficient to determine which of the relevant ideas is correct.

Finally we note the low value of $R_\square$ at which exponential localization effects are first observed. For all our devices above a value of $R_\square \sim 10$ kΩ/□ we see deviations of the logarithmic slopes from a linear dependence on $R_\square$. By $R_\square \sim 15$ kΩ/□ we see a well-defined exponential dependence. This behavior is in agreement with earlier work on metal film[5,14] results but is lower than the expected value of 30 kΩ/□.

In conclusion we have measured the conductance of Si MOSFET's at low temperatures and have observed logarithmic dependences on termperature and applied electric field, indicating the lack of true metallic conduction in this 2D electronic system. Our data show that the crossover from logarithmic to activated behavior is smooth and continuous and that the logarithmic effects scale only with $R_\square$ and not with any other parameter of the system. Within current localization ideas we obtain $\alpha P \sim 1$ while one would expect $\alpha P = 3$ for electron-phonon scattering or $\alpha P = 2$ for electron-electron scattering.

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[1]D. J. Thouless, Phys. Rev. Lett. 39, 1167 (1977).

[2]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979).

[3]P. A. Lee, Phys. Rev. Lett. 42, 1492 (1979).

[4]P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. 43, 718 (1979).

[5]G. J. Dolan and D. D. Osheroff, Phys. Rev. Lett. 43, 721 (1979), and private communications.

[6]D. C. Tsui and S. J. Allen, Jr., Phys. Rev. Lett. 32, 1200 (1974), and 34, 1293 (1975); S. J. Allen, D. C. Tsui, and F. DeRosa, Phys. Rev. Lett. 35, 1359 (1975).

[7]M. Pepper, S. Pollitt, C. J. Adkins, and R. E. Oakley, Phys. Lett. 47A, 71 (1974), and 48A, 113 (1974); for a review, see C. J. Adkins, J. Phys. C 11, 851 (1978).

[8]E. Arnold, Appl. Phys. Lett. 25, 705 (1974).

[9]A. Fowler, Phys. Rev. Lett. 34, 15 (1975).

[10]N. F. Mott, M. Pepper, S. Pollitt, R. H. Wallis, and C. J. Adkins, Proc. Roy. Soc. London, Ser. A 345, 169 (1975).

[11]M. E. Sjöstrand and P. J. Stiles, Solid State Commun. 16, 903 (1975).

[12]The data reported here were taken in low magnetic fields (≲10 mOe, double Mumetal shields).

[13]K. M. Cham and R. G. Wheeler, Bull. Am. Phys. Soc. 25, 389 (1980).

[14]R. C. Dynes, J. P. Garno, and J. M. Rowell, Phys. Rev. Lett. 40, 479 (1978).

[15]B. L. Altshuler, A. G. Aronov, and P. A. Lee, to be published.