for the QCD vacuum. This point could be demonstrated in the same way that it has been done before[5,6] with the group SU(2). This issue will be discussed in detail in a forthcoming communication.

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# Do Total Cross Sections for Scattering of Off-Shell Particles Grow Like a Power of the Energy?

C. López and F. J. Yndurâin
*Departamento de Física Teórica, Universidad Autónoma de Madrid, Canto Blanco, Madrid 34, Spain*
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Let $\sigma_P(Q^2, s)$ be the total cross section for scattering of meson $P$ on some target, with $-\boldsymbol{Q}^2$ the off-shell mass of $P$. It is shown that, when $-Q^2 \neq m_P^2$ (the physical $\boldsymbol{P}$ mass), quantum chromodynamics suggests that, as the c.m. energy squared $s$ approaches $\infty$,

$$\sigma_P(Q^2, s) \sim C_1 f(Q^2) s^\lambda + \ldots, \quad \lambda > 0,$$

where $f(Q^2)$ is calculable for large $Q^2$. This is compatible with the Froissart bound, but only for on-shell particles, provided $f(-m_P^2)=0$. It is shown that there is experimental evidence supporting such behavior.

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Let $\sigma_P(Q^2,s)$ be the total cross section for scattering on some target of the off-shell projectile $P$ (say, a meson) with (unphysical) mass $-Q^2$; $s$ is the square of the c.m. energy. Usually, the Froissart bound is assumed for $\sigma$, and hence its high-energy behavior is taken to be controlled by the Pomeron. However, all proofs of the Froissart bound require unitarity[1] and therefore there is no reason why it would hold away from the mass shell. In this Letter we will argue that quantum chromodynamics (QCD) strongly suggest a behavior of the type

$$\sigma_P(Q^2,s) \underset{s\to\infty}{\sim} C_1 f(Q^2) s^\lambda + C_{\mathrm{Pom}}(Q^2), \tag{1a}$$

where $\lambda$ is strictly positive and where, for sufficiently *large* $Q^2$ that perturbation theory be applicable,

$$f(Q^2) \simeq [\alpha_c(Q^2)]^{-d_+(\lambda)}; \tag{1b}$$

$\alpha_c(Q^2)=12\pi/(33-2n_f)\ln(Q^2/\Lambda^2)$ is the usual QCD running coupling constant and $d_+$ is calculable in terms of $\lambda$ (see below). Actually, from QCD one can only prove that $\sigma_P(Q^2,s)$ grows faster than any $\ln^N s$, for large enough $Q^2$; and, indeed, behavior of the type

$$\sigma_P(Q^2,s)\simeq g(Q^2)(\ln s)^a \exp[b(Q^2)(\ln s)^{1/2}] \qquad (2)$$

has been considered in the literature.[2] Equations (1) follow if we assume a Regge-type character for the leading singularity; the arguments in favor of it will be discussed later on, and we now turn to the proof of (1), considering a typical example. Let us take a $\rho$ meson whose width we neglect, and use as its interpolating field the current $J^\mu=f_\rho\bar{u}\gamma^\mu d$. The off-shell cross section for $\rho p$ scattering may be defined by the formula[3]

$$\sigma_\rho(Q^2,s)\equiv\frac{4\pi^2}{\lambda^{1/2}(s,-Q^2,m_\rho^2)}\sum_{\text{spin}}\epsilon_\mu{}^*(q)\epsilon_\nu(q)\int d^4x\, e^{iq\cdot x}(q^2-m_\rho^2)^2\langle p|[J^\mu(x)^+,J^\nu(0)]|p\rangle, \qquad (3)$$

where $q$ is the four-momentum of the $\rho$ and $Q^2\equiv -q^2$. We will consider the limit $s,Q^2$ large but $x=Q^2/s$ small. In this situation we have

$$\sigma_\rho(Q^2,s)\simeq KF_2(x,Q^2),$$

where $K$ is a known coefficient and $F_2$ is the usual deep-inelastic structure function.[4] Let us assume Regge behavior for $\sigma_\rho$, or, equivalently,[5] that as $x\to 0$ one has the behavior

$$F_2(x,Q^2)\underset{x\to 0}{\simeq} g(Q^2)x^{-\lambda(Q^2)}, \qquad (4)$$

which corresponds to $\sigma_\rho(Q^2,s)\sim\varphi(Q^2)(Q^2)^{-\alpha+1}s^{\alpha-1}$, where $\alpha$ is the intercept of the leading Regge trajectory and $\alpha=1+\lambda$. One could also introduce Regge cuts in the form of (finite) powers of $\ln s$; since they alter nothing essential, we will stick to (4) as it stands.

First of all, it is not difficult to see that only the singlet piece of $F_2$ is relevant as $x\to 0$. We then consider the standard quark singlet ($i=S$) and glue ($G$) moments,

$$\mu_i(n,Q^2)=\int_0^1 dx\, F_{2i}(x,Q^2)x^{n-2}. \qquad (5)$$

It has been proved from QCD that one has[6]

$$\mu_i(n,Q^2)=\sum_{i'}[\exp\rho(Q_0^2,Q^2)D(n)]_{ii'}\mu_{i'}(n,Q_0^2), \qquad (6)$$

where $\rho(Q_0^2,Q^2)=\ln[\alpha_c(Q_0^2)/\alpha_c(Q^2)]$ and

$$D(n)=\frac{16}{33-2n_f}\begin{pmatrix}\dfrac{1}{2n(n+1)}+\dfrac{3}{4}-S_1(n) & \dfrac{3n_f}{8}\,\dfrac{n^2+n+2}{n(n+1)(n+2)}\\[2ex] \dfrac{n^2+n+2}{2n(n^2-1)} & \dfrac{9}{4n(n-1)}+\dfrac{9}{4(n+1)(n+2)}+\dfrac{33-2n_f}{16}-\dfrac{9S_1(n)}{4}\end{pmatrix}, \qquad (7)$$

$$S_1(n)\equiv\sum_{j=1}^{n}\frac{1}{j}=n\sum_{k=1}^{\infty}\frac{1}{k(k+n)}.$$

The last expression defines $S_1$ for arbitrary $n$; with it, Carlson's theorem assures us that Eqs. (6) and (7) remain valid for noninteger (even complex) $n$. Moreover, $D(n)$ becomes an analytic function of $n$ to the right of $\mathrm{Re}\,n=0$ except for an obvious pole at $n=+1$.

According to Eqs. (5), the $\mu_i(n,Q^2)$ will diverge as const/$\epsilon$ when $n\sim 1+\lambda_i(Q^2)+\epsilon$. But since all the dependence of $\exp\rho D$ in $Q^2$ is contained in $\rho$, Eq. (6) implies directly that $\lambda_S(Q^2)=\lambda_G(Q^2)=\lambda$ independent of $Q^2$. Moreover, since $D(n)$ is singular for $n=1$, but not to the right of this point, it follows that $1+\lambda+\epsilon$ cannot be smaller than 1 for any $\epsilon>0$: hence, $\lambda\geqslant 0$. To exclude the case $\lambda=0$ is easy. If we had (8) with $\lambda_i = \lambda = 0$, substituting into (5) and (6) with $n = 1+\epsilon$, we would get

$$\frac{1}{\epsilon}\begin{pmatrix} g_S(Q^2) \\ g_G(Q^2) \end{pmatrix} = \frac{1}{\epsilon}\{\exp[\rho D(1+\epsilon)]\}\begin{pmatrix} g_S(Q_0^2) \\ g_G(Q_0^2) \end{pmatrix},$$

which cannot be satisfied when $\epsilon \to 0$ because $D(1+\epsilon)$ diverges as $1/\epsilon$. This finishes the proof that $\lambda_G(Q^2) = \lambda_S(Q^2) \equiv \lambda > 0$. To complete the proof of Eq. (4), we let $d_+(1+\lambda)$ be the largest, $d_-(1+\lambda)$ the smallest eigenvalue of $D(1+\lambda)$, and $U_\lambda$ the (numerical) matrix that diagonalizes $D(1+\lambda)$. As $Q^2 \to \infty$, and with $n = 1+\lambda+\epsilon$, $\epsilon \to 0$, Eqs. (6) and (7) give

$$\begin{pmatrix} g_S(Q^2) \\ g_G(Q^2) \end{pmatrix} = U_\lambda \begin{pmatrix} [\alpha_c(Q_0^2)/\alpha_c(Q^2)]^{d_+(1+\lambda)} & 0 \\ 0 & [Q_0(Q_0^2)/\alpha_c(Q^2)]^{d_-(1+\lambda)} \end{pmatrix} U_\lambda^{-1} \begin{pmatrix} g_S(Q_0^2) \\ g_G(Q_0^2) \end{pmatrix},$$

from which the behavior

$$F_2(x,Q^2) \simeq B[\alpha_c(Q^2)]^{-d_+(1+\lambda)} x^{-\lambda}, \quad \lambda > 0, \tag{8}$$

and hence Eqs. (1) follow directly to leading order in QCD. This result, Eq. (8), is interesting on its own for analysis of deep-inelastic structure functions. The corresponding implications will be presented elsewhere,[7] and we turn back to the problem at hand.

Clearly, the key assumption of our analysis is the Regge-type behavior, Eq. (4). To trace it, we consider that the moments of a structure function are given,[4,6] in shorthand notation, by

$$\mu(n,Q^2) = C_n(Q^2/\mu^2, \alpha_c)\langle p|O^n|p\rangle,$$

where the $C_n$ are calculable in perturbation theory and the matrix elements of the local operators $\langle O^n\rangle$ embody the unknown, nonperturbative hadronic structure. The right-most singularity of $C_n$ is at $n=1$, for singlet functions ($n=0$ for nonsinglet ones). If the right-most singularity of $\langle p|O^n|p\rangle$ is to the right of this, we will have a Regge behavior; otherwise, we will obtain something like Eq. (2), as in Ref. 2. For the nonsinglet case there is little doubt that we have the singularity of $\langle p|O_{ns}{}^n|p\rangle$ to the right of that of $C_n{}^{ns}$, and hence Regge behavior (the $\rho$ trajectory); we assume the same to be true for the singlet one. Actually, there are some cases in which the confinement problem does not arise and then one can *compute* $\langle p|O^n|p\rangle$. This occurs for scattering on photon targets where Witten[8] has *proved* a behavior precisely like (4) with $\lambda \simeq 0.6$ independent of $Q^2$ except for a slight variation with the number of flavors. Although Witten's result only refers to the pointlike piece of $\sigma$, it certainly lends support to our hypothesis.

Another point is that our results have been obtained working only to leading order in QCD. That the exponent $\lambda$ is independent of $Q^2$ and strictly positive may be verified to hold also to second order in QCD, as it only depends on the fact that $D(n)$ has a pole at $n=1$, and none other to the right of $\mathrm{Re}n = 0$. For this, one uses the recently found analytical expressions of $D(n)$ to second order[9]: Only the expression for $f(Q^2)$, Eq. (2b), will be altered by $O(\alpha_c)$ terms. It is likely that this be valid to all orders in perturbation theory, but we are aware that the extension to small $Q^2$ is a nonperturbative problem. All we can say in this respect is that in the nonsinglet case, nonperturbative effects do not appear to spoil the constancy of the exponent, and so perhaps this is also true in our case.

To finish with the theoretical discussion, we want to comment on the relevance of our results for the Froissart bound. Equations (1) only hold for $Q^2$ large enough that perturbation theory be valid, and hence they pose no threat to the Froissart bound provided $f(-m_P^2) = 0$. It is clear that

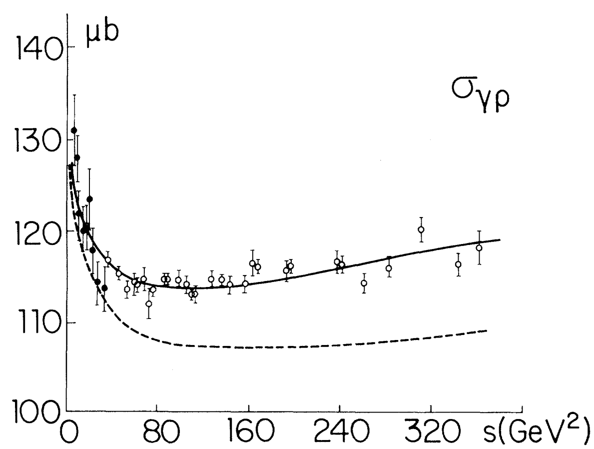


FIG. 1. Fit with Eq. (9), $\overline{C}_{1\gamma} = 1.40$ $\mu$b, $s$ in GeV$^2$ (solid line). Dashed line: $\sigma_{\mathrm{Pom}}$. Data from Armstrong *et al.* (Ref. 11), solid black dots and Caldwell *et al.* (Ref. 11), open dots.

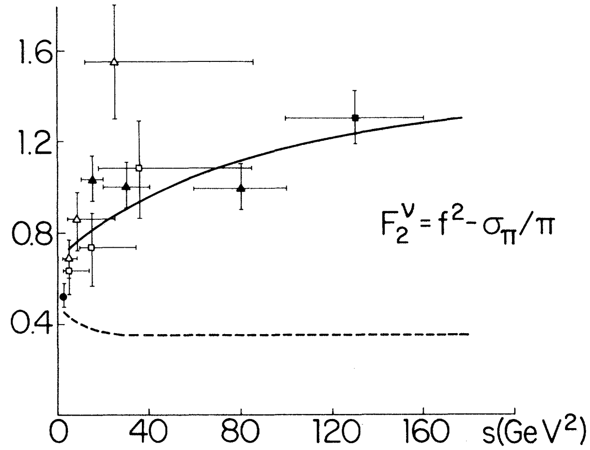


FIG. 2. Fit with Eq. (10), $\overline{C}_{1\gamma} = 0.17\pi/f_\pi^2$, $s$ in GeV$^2$ (solid line). Dashed line, physical pion cross section. Open data points, BEBC data; solid data points, CDHS data (Ref. 13). Squares, $Q^2 = 1.75$ GeV$^2$; triangles, $Q^2 = 1.25$ GeV$^2$. The solid dot is a point at $Q^2 = 0.15$ GeV$^2$, included to show the compatibility between $\nu$ data and physical $\pi$ data (Adler's theorem).

one cannot use expression (1b) below, say 2 GeV$^2$; but it is certainly encouraging that, as calculated here, $f$ diminishes as $Q^2$ decreases, suggesting the existence of a zero for $|Q^2| \sim \Lambda^2$. The mechanism that generates this zero is of course unknown in detail, being of a nonperturbative nature, but one may guess that it is related to the unitary iteration of the starting Regge behavior. In fact, as is known, in Gribov's Reggeon calculus one starts from an amplitude with a leading trajectory with $\alpha_L(0) > 1$; unitarization then brings the actual behavior to one dominated by the usual Pomeron.

To conclude, we want to present a preliminary comparison of Eqs. (1) with experiment in two typical cases. First, we have Compton scattering: Since we are working to lowest order in electromagnetic interactions, we could well have

$$\sigma_\gamma(Q^2, s) \underset{s\to\infty}{\simeq} C_{1\gamma} f(Q^2) s^\lambda + C_{\rm Pom}(Q^2),$$

where $f(Q^2)$ vanishes at $Q^2 = -m_\rho^2$, but not necessarily at $Q^2 = 0$. The value of $C_{\rm Pom}(0)$ is approximately given by[10] $(\sigma_{\pi^+p} + \sigma_{\pi^-p})/440$; $\lambda$ may be obtained (since it is independent of $Q^2$) from small-$x$ fits to deep-inelastic structure functions, where one finds[7] $\lambda \simeq 0.3$ to 0.5. The quality of the fit to all data[11] with only one parameter, $\overline{C}_{1\gamma} = C_{1\gamma} f(0)$, is excellent (Fig. 1).

The second example is the cross section

$$\sigma_\pi(Q^2, s) = (\pi/f_\pi^2) F_2^{\,\nu}(Q^2/s, \tfrac{1}{2}s), \quad f_\pi = 0.95 m_\pi,$$

where $F_2^{\,\nu}$ is the usual neutrino structure function. The reason for the above identification is Adler's theorem[12] that states that, if we neglect the pion mass, $\sigma_\pi(0,s)$ coincides with the total physical pion cross section. Hence, we expect that

$$\sigma_\pi(Q^2, s) - \sigma_\pi(-m_\pi^2, s) \simeq \overline{C}_\pi s^\lambda,$$

and we have neglected the variation with $Q^2$ of the second term in Eq. (1a). The agreement with experiment (at $Q^2 \sim 1.5 \pm 0.25$ GeV$^2$; see Fig. 2) is as good as the quality of the data[13] permits. Both fits were made with $\lambda = 0.4$, but results practically as good are obtained provided $0.3 \leq \lambda \leq 0.6$. Certainly, one cannot argue that the quality of the fits proves the behavior of $\sigma$ given by Eq. (1); but these results, together with those gathered in Ref. 7 for electroproduction at large $Q^2 > 4$ GeV$^2$ using the behavior found, Eq. (8), constitute evidence that, at least, the conclusions of this note are supported by experiment.

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[3]It is clear that the extrapolation away from the mass shell is not unique. Different extrapolations will merely give different $C_1$, and $f$ in (1); the main result, however, remains: $\lambda$ in Eqs. (1) and (8) is positive.

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# Excitation Exchange and Collisional Relaxation of Rydberg Levels of Helium

J. Boulmer, G. Baran, F. Devos, and J.-F. Delpech

*Groupe d'Electronique dans les Gaz, Institut d'Electronique Fondamentale, Université Paris XI, F-91405 Orsay, France*

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Isotope-selective laser excitation has been used to study excitation exchange in collisions of helium Rydberg atoms with ground-state helium; this process is dominated by the interaction of the ionic core with the neutral atom and proceeds at the charge-exchange rate, the excited electron remaining a spectator.

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Recent experimental results (Devos, Boulmer, and Delpech,[1] designated as I in what follows) suggest that neutral-induced collisional transfers between Rydberg levels of an atom involve the three-body interaction between the ionic core, the Rydberg electron, and the neutral perturber. We report here a more detailed study of this interaction; isotope-selective laser excitation has been used to study excitation exchange in collisions of helium Rydberg atoms with ground-state $^3$He and $^4$He and to show that this process is dominated by the interaction of the ionic core with the neutral atom and proceeds essentially as a charge-transfer reaction, the excited electron remaining a spectator.

The present study was carried out in a room-temperature helium afterglow at a pressure of 2.6 Torr. Its basic features are carefully diagnosed (see I and Delpech, Boulmer, and Stevefelt[2]): The role of the discharge is simply to create a large enough population ($\simeq 10^{11}$ cm$^{-3}$) of metastable helium atoms. At the time of the laser pulse the electrons are swept out by a microwave heating pulse[2]; their density is below $10^9$ cm$^{-3}$ and thus collisions involving electrons remain always negligible here compared to those involving ground-state atoms (their respective rate coefficients have been reported in I).

Rydberg He($9^3P$) atoms are produced by laser excitation; after frequency doubling, the dye laser delivers 3-ns, 1- to 6-$\mu$J pulses at a repetition rate of 25 Hz. The wavelength is pressure tuned around 2696 Å with a resolution of 0.3 Å; either $^3$He($2^3S$-$9^3P$) or $^4$He($2^3S$-$9^3P$) is selectively populated with excellent rejection, as they are separated[3] by 12 Å.

As noted in I, $l$ sublevels reach statistical equilibrium in a time short compared to the duration of the laser pulse; in what follows, states will thus be designated simply by their principal quantum number $p$. The population of either $^3$He($p$) or $^4$He($p$) is monitored by transient fluorescence by means of a coupled holographic grating monochromator and Fabry-Perot etalon of 1.35-cm$^{-1}$ free spectral range with a finesse of 5; the multiplet structures of the two isotopes are well separated with this combination.

At low energies, below about 2 $\mu$J per laser pulse, the population density [He(9)] of the $p=9$ level increases linearly with laser energy and collisional transfers play a dominant role in the population of all other levels, as described in I. However, above this energy threshold, [He(9)] begins to saturate while [He(8)] increases faster than would be warranted by purely collisional transfer: Cooperative radiative phenomena[4-6] begin to play a substantial role. When the energy of the laser reaches about 6 $\mu$J under our experimental conditions, superradiant transfer becomes so fast that both [He(8)] and [He(9)] reach their maximum during the laser pulse (Fig. 1); collisional transfers retain, however, a dominant role in the population of all other levels, including [He(7)], and their population densities (as deduced from their fluorescence intensities) remain small, at early times, compared to [He(8)] and [He(9)].

Conceivably, one could populate simply a given

 