PHYSICAL REVIEW LETTERS

VOLUME 44 | 21 JANUARY 1980 | NUMBER 3

Condition for Nonexistence of Aharonov-Bohm Effect

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(Received 19 September 1979)

It is shown that electromagnetic effects of a vector potential on a charged particle confined to a region R (multiply or simply connected) are completely determined by field strengths in R, if the potential obeys certain conditions. Such potentials are adequate in experiments purporting to show Aharonov-Bohm effect. Therefore these experiments have not established effects of inaccessible fields.

The classic proposal[1] of Ehrenberg and Siday and of Aharonov and and Bohm is that in quantum mechanics, unlike in classical mechanics, charged particles can be affected by inaccessible fields. The effect has deeply stirred both theorists[2,3] and experimentalists.[4] Particularly inspiring is the exposition by Wu and Yang,[5] who conclude that field strengths underdescribe electromagnetism and also propose a *Gedanken* generalized Aharonov-Bohm experiment for zero-mass non-Abelian gauge fields. Exactly the opposite conclusion has been reached recently[6-8] on the basis of equivalence of the Schrödinger equation with a system of hydrodynamical-type nonlinear equations in which only field strengths (and not potentials) appear. Strocchi and Wightman[6] and Casati and Guarneri[8] emphasize the importance of boundary conditions in formulating the equivalence; Bocchieri and Loinger[7] discuss ambiguities in defining the canonical momentum operator and propose modification of continuity conditions on the wave function to eliminate the Aharonov-Bohm effect. I show under standard continuity conditions that no effect of inaccesible fields can exist if the vector potential satisfies a condition proposed here. The condition is satisfied in actual experiments on Aharonov-Bohm effect.

The crux of the Aharonov-Bohm argument is that the canonical quantum formalism involves potentials rather than fields. When an electron is confined to the field-free region outside an impenetrable, infinitely long and narrow cylinder carrying flux F, a Stokesian potential outside is (in cylindrical coordinates ρ, z, φ with the z axis along the cylinder)

$$A_0=0, \quad A_\rho=A_z=0, \quad A_\varphi=F/2\pi\rho. \tag{1}$$

This potential, though it gives zero fields outside, yields observable effects distinct from zero potential; the usual argument of gauge invariance fails because the transformation $\exp[ie\varphi F/(2\pi)]$ necessary to go to zero potential is not allowed for $eF/2\pi \neq$ integer, as it would yield a multivalued wave function. Another important example is the potential due to a pair of Dirac strings[9] along directions $\vec{n}_1$ and $\vec{n}_2$,

$$A_0=0, \quad \vec{A}=\frac{g}{r}\left(\frac{\vec{r}\times\vec{n}_1}{r-\vec{r}\cdot\vec{n}_1}-\frac{\vec{r}\times\vec{n}_2}{r-\vec{r}\cdot\vec{n}_2}\right), \tag{2}$$

which yields zero field except on $\vec{r}=r\vec{n}_1$ and $\vec{r}=r\vec{n}_2$ (where the wave function vanishes) and leads to observable effects except for special values of g.

In contrast, Mandelstam, De Witt, and Belinfante have independently claimed[10] that electrodynamics may be formulated entirely in terms of field strengths. If they allow inaccessible fields also to appear, there is no direct physical contradiction with the claim of Aharonov and Bohm. Nevertheless, I suggest that their results do not hold for the potentials (1) and (2). I obtain sufficient conditions on a vector potential to guarantee that its physical effects depend on accessible fields only. For the class of potentials obeying these conditions there is no room for Abelian gauge-field copies.[5,11]

My physical results derive from the following elementary mathematical proposition. Let $z_\mu(x,\xi)$ ($\mu=1$ to 4, $z_4=iz_0$) be single-valued differentiable functions of the coordinates x_μ and of a real parameter ξ, $-\infty<\xi\leq 0$, obeying

$$\begin{aligned} &z_\mu(x,0)=x_\mu, \\ &\lim_{\xi\to-\infty} z_\mu(x,\xi)=\text{spatial infinity.} \end{aligned} \tag{3}$$

Let $A_\mu(x)$ be single-valued functions obeying the conditions

$$\begin{aligned} &\int_{-\infty}^0 A_\mu(z)\frac{\partial z_\mu}{\partial\xi}d\xi<\infty, \quad \lim_{\xi\to-\infty} A_\nu(z)\frac{\partial z_\nu}{\partial x_\mu}=0; \\ &\int_{-\infty}^0 d\xi\, F_{\rho\nu}(z)\frac{\partial z_\rho}{\partial\xi}\frac{\partial z_\nu}{\partial x_\mu}<\infty, \quad F_{\rho\nu}\equiv\partial_\rho A_\nu-\partial_\nu A_\rho, \end{aligned} \tag{4}$$

where the integrand involving $F_{\rho\nu}$ is continuous in ξ and x_μ, and the convergence of the corresponding integral is uniform in the relevant region of x_μ. Then

$$\begin{aligned} &\frac{\partial}{\partial x_\mu}\left(\int_{-\infty}^0 d\xi\, A_\nu(z)\frac{\partial z_\nu}{\partial\xi}\right) \\ &\quad =A_\mu(x)+\int_{-\infty}^0 d\xi\, F_{\rho\nu}(z)\frac{\partial z_\nu}{\partial\xi}\,\frac{\partial z_\rho}{\partial x_\mu}. \end{aligned} \tag{5}$$

Note that conditions (4) validate differentiation under integral sign[12] and a subsequent partial integration needed to derive (5). I deduce the following physical results.

Let a charged particle be confined to a region R (which may be multiply or simply connected) and experience a single-valued, unquantized potential A_μ.

Proposition 1. If there exists a single-valued and differentiable path $z_\mu(x,\xi)$ lying in R for every x_μ in R such that conditions (3) and (4) are obeyed, then physical effects on the particle are completely determined by field strengths in R alone.

This follows from (5) when one notes that the integral on the left-hand side is single valued and hence the usual gauge-invariance argument establishes the physical equivalence of A_μ with the potential

$$A_\mu'\equiv\int_{-\infty}^0 d\xi\, F_{\nu\rho}(z)\frac{\partial z_\nu}{\partial\xi}\,\frac{\partial z_\rho}{\partial x_\mu} \tag{6}$$

which involves field strengths in R alone.

Remark 1. Examples of paths z_μ obeying (3) are (see Belinfante[10])

$$\begin{aligned} &z_0=x_0, \quad \vec{z}=\vec{x}+\xi\vec{n}, \\ &\vec{n}^2=1, \quad \vec{n} \text{ independent of } x_\mu, \end{aligned} \tag{7}$$

and

$$z_0=x_0, \quad \vec{z}=\vec{x}(1-\xi). \tag{8}$$

When R is the outside of an infinite cylinder the paths lie entirely in R for every x_μ in R, for the choice (8) with origin of coordinates inside the cylinder, and also for the choice (7) with $\vec{n}$ parallel to axis of cylinder.

Remark 2. The new feature of proposition (1) over earlier work[10] is condition (4). For example, De Witt[10] considered a charged Dirac particle with wave function $\psi(x)$ in a potential A_μ which vanishes at infinity and he claimed that the gauge-invariant wave function

$$\Psi\equiv\exp\left[-ie\int_{-\infty}^0 d\xi\, A_\mu(z)\frac{\partial z_\mu}{\partial\xi}\right]\psi \tag{9}$$

obeys

$$\gamma_\mu\left(\frac{\partial}{\partial x_\mu}-ieA_\mu'\right)\Psi+m\Psi=0, \tag{10}$$

with A_μ' defined by (6). It is important to recognize that Eq. (10) does not hold for the potentials (1) and (2). For example, with the choice (8) when R is the outside of a cylinder, $F_{\rho\nu}=0$ in Eq. (5) which implies $A_\mu'=0$, and hence Eq. (10) would imply that the potential (1) has no effect. This is false[1-3,5] (unless one modifies continuity requirements on the wave function[7]). Further, the derivation of (5) suggests that Eq. (10) is invalid for the potential (1) for all the choices (7) and (8), since condition (4) is violated; note that the appearance of inaccessible fields in A_μ' for $\vec{n}$ not parallel to axis of cylinder does not help to satisfy (4).

Proposition 2. If two potentials $A_\mu^{(1)}$ and $A_\mu^{(2)}$ with $\Delta A_\mu=A_\mu^{(1)}-A_\mu^{(2)}$ obey

$$\Delta_\mu(\Delta A_\nu)-\partial_\nu(\Delta A_\mu)=0 \text{ in } R, \tag{11a}$$

$$\int_\infty^x \Delta A_\mu(z)\frac{\partial z_\mu}{\partial\xi}d\xi<\infty, \tag{11b}$$

and

$$\lim_{\xi \to -\infty} \frac{\partial z_\nu}{\partial x_\mu} \Delta A_\nu(z) = 0 \tag{11c}$$

for at least one single-valued and differentiable $z_\mu(x, \xi)$ obeying (3) and lying in R for every x_μ in R, then the two potentials are physically equivalent.

This follows from (5) with $A_\mu \to \Delta A_\mu$, which shows that ΔA_μ is the derivative with respect to x_μ of a single-valued function, and hence is equivalent to zero potential.

Remark 3. The advantage of this proposition over proposition 1 is that the condition involving integrals of $F_{\rho\nu}$ is removed.

In electrostatics with localized charges, magnetostatics with localized currents, and radiation fields of localized oscillating sources,[13] the following asymptotic behaviors of $A_\mu(x)$ are adequate ($r \equiv |\vec{x}|$): For electrostatics,

$$A_0 \to \text{const}/r, \quad \vec{A} = 0; \tag{12}$$

for magnetostatics,

$$A_0 = 0, \quad \vec{A} \to \vec{m} \times \vec{x}/r^3; \tag{13}$$

and for oscillating sources,

$$A_0 \to \text{const}/r, \quad \vec{A} \to (\vec{c}/r)\cos(kr - \omega t). \tag{14}$$

For choice (8) in cases (12) and (13) condition (4) holds. In the oscillating case (14) we may use Chartier's test[14] to show the validity of (4) for the choice (7) of z_μ with $\vec{n}$ parallel to the axis of the excluded cylinder if any. We conclude that at least for R being the outside of a cylinder, and for localized sources, electromagnetism may be determined by accessible fields only.

In a typical electron interference experiment, the distance from electron source to screen is about 50 cm, and the excluded cylinder (e.g., solenoid of Möllenstedt and Bayh[4]) of length less than 1 cm and radius a few microns. For a solenoid of zero radius extending from $z = -L$ to $z = +L$ and linked by flux F at $z = 0$, the vector potential at (ρ, z, φ) is given by[15] $A_0 = A_\rho = A_z = 0$ and

$$A_\varphi = \frac{F}{4\pi\rho}\left(\frac{\operatorname{sgn}(L-z)}{\{1+[\rho/(L-z)]^2\}^{1/2}} + \frac{\operatorname{sgn}(L+z)}{\{1+[\rho/(L+z)]^2\}^{1/2}}\right). \tag{15}$$

Any electron path from source to screen contains a large region where $\rho/(L \pm z)$ cannot be neglected and (15) cannot be approximated by the Aharonov-Bohm expression (1). The potential (15) because of good asymptotic behavior [and unlike (1)] obeys condition (4) with the choice (8) for z_μ. Proposition 1 shows that physical effects of this potential and therefore results of experiments[4] on the Aharonov-Bohm effect are completely determined by accessible fields only.

This argument is more general than that of Pryce (see Chambers[4] and Fowler *et al.*[4]) which applies to fields due to tapering whiskers, rather than uniform solenoids. My remarks seem logically independent of Refs. 6–8 which consider the potential (1). For example, Strocchi and Wightman[6] attribute experimental results seen to the solution of the Schrödinger equation having a tail which runs into the cylinder, i.e., to a breakdown of the ideal of multiple connectedness. In contrast, my point is that a solenoid of finite length, even if surrounded by an infinite impenetrable cylinder, yields a potential of such asymptotic behavior that it excludes effects of inaccessible fields.

I gratefully acknowledge the contributions bordering on authorship of V. Singh to the ideas in this paper. I also wish to thank warmly R. Cowsik, S. Guha, S. S. Jha, P. K. Kabir, S. K. Mitra, L. K. Pandit, T. R. Ramadas, and P. Roy for their critical comments.

Note added.—A. S. Goldhaber has communicated that he has an alternate proof of proposition 1. Further, he and P. K. Kabir have made stimulating observations on the possibility of an Aharonov-Bohm-type effect when electrons are confined outside a toroidal magnetic field. Such an effect is not ruled out by proposition 1 and is under investigation.

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[12]See, e.g., E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (Cambridge Univ. Press, Cambridge, England, 1963), 4th ed., p. 74.

[13]See, e.g., J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., pp. 136, 181, and 393.

[14]Reference 12, p. 72.

[15]See, e.g., Ref. 13, p. 178, which gives potentials due to a current loop. Elementary integrations yield those due to a soleniod.

Measurement of High-Energy Direct Photons in ψ Decays

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(Received 5 November 1979)

The inclusive γ and π^0 momentum distributions at the ψ have been measured. Using these data and estimates of η production, it is found that $(4.1 \pm 0.8)\%$ of ψ decays contain a direct photon with energy greater than 60% of the beam energy. The expected momentum distribution for direct photons calculated to lowest order in quantum chromodynamics is qualitatively different from that observed in the data.

First-order quantum chromodynamics (QCD) calculations predict that a significant fraction of the hadronic decays of heavy quark-antiquark $^3S_1$ resonances (such as the ψ) result in the production of direct γ's (i.e., γ's not coming from secondary decays of π^0's or η's).[1] We have measured the inclusive γ and π^0 momentum distributions at the ψ, and have made estimates of the η momentum distribution from the data. We observe γ production in excess of the expected contributions from π^0 and η decay.

The data were collected with the Mark II magnetic detector at the SLAC e^+e^- storage ring facility SPEAR at energies near the peak of the ψ(3095) resonance. The detector has been described in detail elsewhere,[2] and only a brief description of the particle detection will be presented in this Letter.

