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[1]P. W. Anderson and W. F. Brinkman, Phys. Rev. Lett. 30, 1108 (1973).
[2]D. Rainer and J. W. Serene, Phys. Rev. B 13, 4745 (1976); J. W. Serene and D. Rainer, J. Low Temp. Phys. 34, 589 (1979).
[3]R. A. Webb, T. J. Greytak, R. T. Johnson, and J. C. Wheatley, Phys. Rev. Lett. 30, 210 (1973).
[4]K. Andres and S. Darack, Physica (Utrecht) 86–88B, 1071 (1977).
[5]S. F. Shields and J. M. Goodkind, J. Low Temp. Phys. 27, 259 (1977).
[6]T. A. Alvesalo, T. Haavasoja, P. C. Main, M. T. Manninen, J. Ray, and Leila M. M. Rehn, Phys. Rev. Lett. 43, 1509 (1979).
[7]T. A. Alvesalo, T. Haavasoja, and M. T. Manninen, to be published.
[8]D. N. Paulson, M. Krusius, J. C. Wheatley, R. S. Safrata, M. Kolác, T. Tethal, K. Svec, and J. Matas, J. Low Temp. Phys. 34, 63 (1979), and 36, 721(E) (1979).
[9]E. R. Grilly, J. Low Temp. Phys. 4, 615 (1971).
[10]J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975).
[11]W. P. Halperin, F. B. Rasmussen, C. N. Archie, and R. C. Richardson, J. Low Temp. Phys. 31, 617 (1978).
[12]C. N. Archie, T. A. Alvesalo, J. D. Reppy, and R. C. Richardson, Phys. Rev. Lett. 43, 139 (1979).
[13]D. N. Paulson, M. Krusius, and J. C. Wheatley, J. Low Temp. Phys. 25, 699 (1976).

Characteristic Radiation from Channeled Electrons

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Sharp lines in the bremsstrahlung spectrum have been observed for 1.5–4-MeV electrons channeled along a ⟨111⟩ direction in silicon. The lines originate in transitions between bound states in the axial transverse potential and may be described as characteristic radiation from a two-dimensional atomlike system moving with relativistic velocity in the ⟨111⟩ direction.

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Recently, sharp lines in the bremsstrahlung spectrum for positrons[1] and electrons[2] were observed, which could be associated with transitions between bound states in the transverse motion of particles channeled along crystal planes. This radiation is closely connected[3] with the well-known phenomenon of coherent bremsstrahlung[4,5] but exhibits special features associated with the governing of the particle motion by the crystal potential.[6] As for the coherent bremsstrahlung, the strongest effects are expected for particles moving nearly parallel to a low-index crystal axis, but until now, only a broad, featureless enhancement at low photon energies has been observed for this case (see Ref. 2, and references therein). We have measured the radiation from 1.5- to 4-MeV electrons channeled along a ⟨111⟩ axis in silicon and observed sharp lines from transitions between bound states in the axial transverse potential. The identification of the lines has been confirmed by measuring the variation of the intensity of the lines with the angle of incidence of the electron beam with respect to the ⟨111⟩ axis.

An electron beam from the 5-MV Van de Graaff accelerator in Aarhus was transmitted through a (111)-oriented, 1.2-μm-thick silicon crystal and magnetically deflected into a Faraday cup. The angular spread of the beam was measured to be $<0.05°$ full width at half maximum (FWHM). Radiation in the forward direction was detected in a proportional counter with a 2-μm aluminized Mylar window and filled with an Ar-CH_4 mixture at 1 atm. The acceptance solid angle was 0.6×10^{-6} of 4π, and the efficiency was $\epsilon \gtrsim 40\%$ in the energy range 1–8 keV.

Photon energy spectra are shown in Fig. 1 for beam energies between 1.5 and 4 MeV. Each of them is compared with a "random" spectrum, obtained for incidence away from the axis and ma-

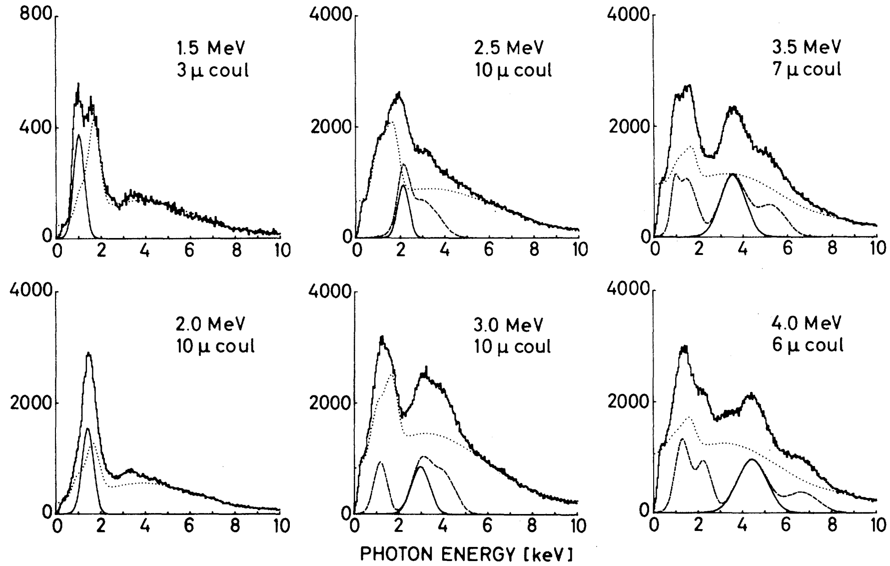

FIG. 1. Radiation intensity (counts/40 eV) for incidence parallel to ⟨111⟩. Beam energy and charge are indicated.

jor planes, which has been normalized to fit the high-energy tail. The peak at the low-energy end of the "random" spectrum is due to Si K x rays and to absorption of bremsstrahlung just above the K edge in silicon (1.8 keV). The position and width of the prominent peaks in the spectra have been determined from least-squares fits with a superposition of the normalized "random" spectrum (dotted curves) and a few Gaussian peaks, which are indicated separately (dashed curves). At the lowest beam energies, only one line is observed. The energy of this line (solid curve in all spectra) increases with beam energy, additional lines appear, and for 4 MeV electron energy, four lines can be distinguished.

In a classical picture, the radiation is emitted by electrons spiraling around the ⟨111⟩ atomic strings, but the peak structure must be interpreted within the quantum theory of channeling. The basic concept is motion in an effective transverse crystal potential with conservation of transverse energy, which has been established in both a classical[7] and a quantal[8] treatment. For electrons, there are strong potential minima at the atomic strings, and for incidence at a small angle to the axis, electrons may be captured into localized bound states, which are eigenstates of the Hamiltonian:

$$[p_\perp^2/2m + U(\vec{r})]\psi_\nu(\vec{r}) = E_{\perp,\nu}\psi_\nu(\vec{r}) \qquad (1)$$

Here $\vec{p}_\perp$ and $\vec{r}$ are the projections of the momentum and the position on the plane perpendicular to the axis, and U is the crystal potential averaged in the axial direction (z direction). The mass m is the relativistic mass, $m \simeq \gamma m_0$, with $\gamma = (1-\beta^2)^{-1/2}$ and $\beta = v/c$. Because of the relativistic increase of m, the number of bound states localized at one atomic string increases[9] approximately proportional to γ.

The discrete radiation from channeled electrons originates in transitions between these states. Relativistic effects are very important[6] and may be included by calculating the frequency and intensity of the radiation in the "rest frame" of the electrons, where the velocity in the z direction is transformed to zero. The transverse coordinates and momentum components are invariant under the transformation, but the mass is reduced, $m \to m/\gamma = m_0$, and the potential is increased, $U \to \gamma U$, as a result of the Lorentz contraction of the atomic strings. (Since the velocity is nearly parallel to the z axis, we may use the approximation $\beta_z \simeq \beta$ and $\gamma_z \simeq \gamma$.) The eigenfunctions ψ_ν are therefore invariant, but the corresponding energies are larger by a factor γ in the rest frame, $E_{\perp,\nu}{}^R = \gamma E_{\perp,\nu}$. The energy of the characteristic radiation associated with a transition between levels ν and μ is $\hbar\omega^R = E_{\perp,\nu}{}^R - E_{\perp,\mu}{}^R$. In the laboratory frame, one obtains in the forward direction from the Doppler transformation

$$\hbar\omega^L = \hbar\omega^R\gamma(1+\beta) \simeq 2\gamma\hbar\omega^R. \qquad (2)$$

Radiation from axially channeled electrons has

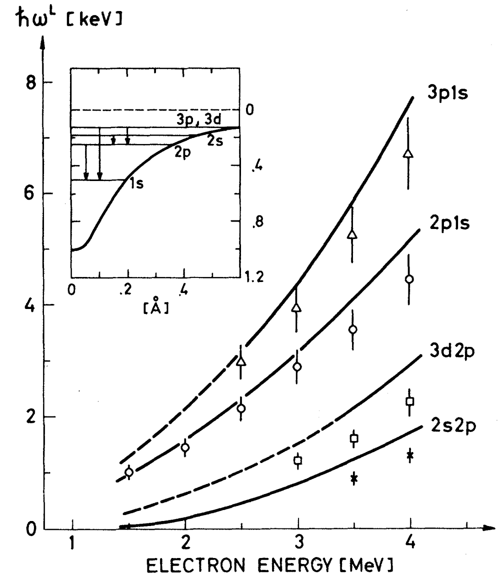

FIG. 2. Comparison of measured line energies with calculations (Ref. 13). Estimated error ± 10%. Dashed curves for transitions from free states. Inset shows transitions between levels in ⟨111⟩ axial potential at 4 MeV. The vertical scale gives the binding energy $-E_\perp^R$ in kiloelectronvolts.

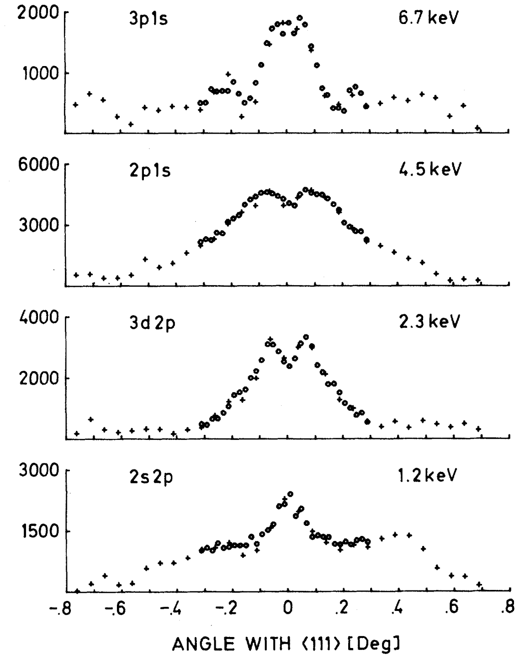

FIG. 3. Intensity scans through ⟨111⟩ at ~ 15° from a {110} plane. Beam energy 4 MeV and charge 0.6 μC per point. Circles and crosses indicate results of two scans with different steps in angle.

been discussed[10, 11] on the basis of analytical solutions to Eq. (1) for a simple approximate potential, $U(\vec{r}) \propto 1/r$. In the inset in Fig. 2, a more realistic ⟨111⟩ axial potential[12] is shown. The lowest level has been estimated from a harmonic approximation to the bottom of the potential, and the higher levels have been determined from the line energies in Fig. 1 (4 MeV), converted to the rest frame according to Eq. (2). The line energies are compared in Fig. 2 with results of a many-beam calculation[13] using this potential.

The identification of the lines may be checked by measuring their intensity as a function of incidence angle φ. If mixing of the populations of different levels can be ignored, the intensity scans can be given a simple interpretation. The population of a bound state ν is determined at the surface by matching to the incident wave,[9]

$$P_\nu \propto |\langle \psi_\nu(\vec{r}), \exp(i\vec{k}_\perp \cdot \vec{r})\rangle|^2, \qquad (3)$$

where $\hbar k_\perp \simeq p\varphi$ is the transverse momentum of the incident electrons. The scans therefore reflect directly the density of the initial-state wave functions in transverse-momentum space.

Intensity scans through the ⟨111⟩ axis of the four lines in the 4-MeV spectrum in Fig. 1 are shown in Fig. 3. The intensities were determined as peak areas in a series of energy spectra recorded for different angles of incidence. The results are consistent with the interpretation of the lines in Fig. 2. For example, only the 2*s* state has a maximum at zero momentum, and for the 3*p* state, there is an indication of a node in the distribution. Also, the widths of the momentum distributions are consistent with the energy levels indicated in Fig. 2.

It is evident that the observation of "characteristic" radiation can be a powerful tool for detailed studies of electron channeling. The individual states are singled out, and their energy and even their wave function may be determined. Also, information about the stability of the states may be derived from the width of the lines. The 2*p*-1*s* line at 4 MeV electron energy has been determined in detail with a Si-Li detector with resolution ~ 300 eV. The line shape is Lorentzian, with $\Gamma = 0.93$ keV. The corresponding coherence length is $l \simeq 2\gamma^2\hbar c/\Gamma \simeq 330$ Å. This may be compared with an estimate of the broadening of the 1*s* state due to inelastic (thermal) scattering.[9] The mean free path for thermal scattering in silicon is ~ 12 000 Å for relativistic electrons, but the concentration of the 1*s* state in a region of high atomic density reduces the mean free path by a large factor. We estimate a concentration factor of ~50 at 4 MeV, leading to a coherence length $l \sim 240$ Å, in reasonable agreement with the measured value.

As has been emphasized in earlier publications,[6,10] the channeling radiation is potentially interesting as a radiation source. In this connection, also the observed intensities are of interest. The yields of bremsstrahlung for random incidence are close to theoretical estimates.[4] The yield in the forward direction from a transition between states ν and μ may be estimated[10,11] from the standard expression for a dipole transition in the rest frame, Doppler shifted to the laboratory,

$$N = \tfrac{1}{2}\alpha c^{-3}\gamma^{-2}(\omega^L)^3 t |r_{\nu\mu}|^2 (\delta\Omega/4\pi)\epsilon \quad \text{[photons/electron]}. \tag{4}$$

Here α is the fine-structure constant, $\alpha = 1/137$, t is the crystal thickness, $\epsilon\delta\Omega$ the effective solid angle, and $r_{\nu\mu}$ the radial matrix element of r between states ν and μ, which are assumed to satisfy the selection rule $\Delta l = \pm 1$. For the $2p$-$1s$ transition at 4 MeV electron energy, this estimate (with $|r_{\nu\mu}| \simeq 0.1$ Å) is consistent with experiment if the maximum population of the $2p$ state is ~5%.

In itself, the spectroscopic study of bound states in one- and two-dimensional potentials of continuously variable strength is a fascinating subject, which remains to be studied in detail. We have in this Letter concentrated on the analysis of transitions between bound states in an axial potential, but also planar radiation and radiation due to transitions between unbound states and from free to bound states have been studied. These types of radiation connect the channeling radiation to the previously studied coherent bremsstrahlung.

[1] M. J. Alguard, R. L. Swent, R. H. Pantell, B. L. Berman, S. D. Bloom, and S. Datz, Phys. Rev. Lett. 42, 1148 (1979).

[2] R. L. Swent, R. H. Pantell, M. J. Alguard, B. L. Berman, S. D. Bloom, and S. Datz, Phys. Rev. Lett. 43, 1723 (1979).

[3] J. U. Andersen, in Proceedings of the Conference on Atomic Collisions in Solids, Hamilton, Canada, 1979 (to be published).

[4] M. L. Ter-Mikaelian, *High-Energy Electromagnetic Processes in Condensed Media* (Wiley, New York, 1972).

[5] G. Giambrini Palazzi, Rev. Mod. Phys. 40, 611 (1968).

[6] M. A. Kumakhov, Phys. Lett. 57, 17 (1976).

[7] J. Lindhard, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. 34, No. 14 (1965).

[8] P. Lervig, J. Lindhard, and V. Nielsen, Nucl. Phys. A96, 481 (1967).

[9] J. U. Andersen, S. K. Andersen, and W. M. Augustyniak, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. 39, No. 10 (1977).

[10] R. W. Terhune and R. H. Pantell, Appl. Phys. Lett. 30, 265 (1977).

[11] M. A. Kumakhov and R. Wedell, Phys. Status Solidi (b) 84, 581 (1977).

[12] B. R. Appleton, C. Erginsoy, and W. M. Gibson, Phys. Rev. 161, 330 (1967).

[13] S. K. Andersen and K. R. Eriksen, private communication. See Ref. 9 for description of calculations.