Exact Finite-Range Distorted-Wave Born-Approximation Analyses of the Reactions $^{18}O(p,t)^{16}O$, $^{48}Ca(t,p)^{50}Ca$, and $^{90}Zr(t,p)^{92}Zr$ Using Realistic Triton and Nuclear Wave Functions

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Results of exact finite-range distorted-wave Born-approximation calculations are presented for the reactions $^{18}O(p,t)^{16}O$, $^{48}Ca(t,p)^{50}Ca$, and $^{90}Zr(t,p)^{92}Zr$. The calculations employ a realistic triton wave function, and realistic nuclear-structure overlap functions. The theoretical cross sections are in good agreement with experiment which implies that the distorted-wave Born approximation provides a satisfactory description of these reactions without recourse to higher-order reaction mechanisms.

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The two-neutron transfer reactions (p,t) and (t,p) are highly useful probes of nuclear structure.[1] However, theoretical calculations of the cross sections of these reactions have been unsatisfactory. Although the shapes of angular distributions are predicted rather well, exact finite-range calculations consistently underpredicted[2-5] their absolute magnitudes. Attempts to resolve this difficulty have separately focused on either altering the reaction mechanism,[6,7] or altering the description of the nuclear structure.[8,9] The results of these latter studies seem to suggest that a complete treatment of the requisite nuclear-structure overlaps is an essential ingredient in describing (p,t) and (t,p) reactions. Recent work[10,11] has also shown that cross sections can be quite sensitive to the form of the triton wave function and of the transfer interaction. For example, the use of a realistic triton wave function alters the (p,t) selection rules and allows for the description of unnatural-parity transitions in first-order distorted-wave Born approximation (DWBA).

The question of whether contributions from higher-order processes, such as sequential transfer of nucleons, are needed to bring the theoretical cross sections into agreement with experiment cannot be answered unambiguously until first-order DWBA calculations are performed which include the important nuclear-structure features discussed above. Previous calculations[12] which employed a realistic transfer potential and triton wave function did not use sufficiently accurate nuclear overlaps. The overlap functions $\langle A+2|A\rangle$ were approximated by use of the half-separation-energy (HSE) *Ansatz* for the single-particle wave functions, together with spectroscopic amplitudes obtained from shell-model calculations. This procedure neglects important correlations between the transferred nucleons and gives form factors which are too small in the nuclear-surface and exterior regions.

In the present work we study the ground-state reactions $^{18}O(p,t)^{16}O$, $^{48}Ca(t,p)^{50}Ca$, and $^{90}Zr(t,p)^{92}Zr$ at bombarding energies for which the direct-reaction mechanism should be valid. We assume that ^{18}O, ^{50}Ca, and ^{92}Zr can each be treated as two neutrons outside a closed-shell core. With these assumptions the overlap $\langle A+2|A\rangle$ satisfies a Schrödinger equation for two interacting particles in an external potential. This equation can be solved by the extended-basis shell-model (EBSM) method, which has been discussed elsewhere.[9,13]

Our calculations are the first to make use of realistic transfer potentials, triton wave function, and nuclear overlap functions and therefore are the first serious test of the validity of the DWBA method for (p,t) and (t,p) reactions.

The full expression for the DWBA transition amplitude[11] can be written as

$$T_{fi} \propto \int d^3r_t\, d^3r_p\, \chi_t^{(-)*}(\vec{r}_t) \times \langle tA|V|pA+2\rangle \chi_p^{(+)}(\vec{r}_p), \tag{1}$$

where the reaction considered is

$$p+(A+2) \to t+A\,.$$

The functions $\chi_t^{(-)}$ and $\chi_p^{(+)}$ are the usual outgoing-triton and ingoing-proton distorted wave functions. One should note that in writing Eq. (1), we have implicitly made two approximations: (i) The distorted-wave method is assumed, and (ii) exchanges between the ingoing proton and those in the target nucleus A are neglected. In the prior representation of DWBA, the transfer potential in (1) is

$$V=V_{p,\,A+2}-U_{p,\,A+2} \approx V_{pn_1}+V_{pn_2}\,, \tag{2}$$

where $U_{p,\,A+2}$ is the proton elastic-scattering optical potential. The replacement of the transfer reaction by the terms $V_{pn_1}+V_{pn_2}$ is an approximation which has not been carefully studied for (p,t) reactions although it is commonly used. It is one of the weakest parts of our current study. We should also point out that, although EBSM is "state-of-the-art" for two-nucleon overlaps, it is nevertheless a phenomenological model whose microscopic justification is still under active study. The triton wave function used in Eq. (1) was obtained[14] as a variational solution of the three-body Hamiltonian using the Reid soft-core potential.[15] The triton wave function is expanded on a basis having definite particle-permutation orbital symmetry. A totally symmetric S state and mixed-symmetry S' and D states are included. The Reid potential is used for the transfer interaction.

In our study we analyze the reactions ^{18}O$(p,t)^{16}$O($E_{\rm lab}$=20 MeV, Refs. 16 and 17), ^{48}Ca$(t,p)^{50}$Ca ($E_{\rm lab}$=12.08 MeV, Ref. 18), and ^{90}Zr$(t,p)^{92}$Zr ($E_{\rm lab}$ =20 MeV, Ref. 19, and $E_{\rm lab}$=11.89 MeV, Ref. 20). The optical parameters for these reactions are presented in Table I. In Fig. 1 we compare the DWBA cross sections for the reactions ^{18}O$(p,t)^{16}$O and ^{48}Ca$(t,p)^{50}$Ca to the experimental data. The theoretical ^{18}O$(p,t)^{16}$O cross section is computed with the optical parameter set I in Table I. It should be noted that the theoretical calculation agrees with the experimental data in both shape and absolute magnitude. However, if parameter set II is used, a normalization factor 1.7 is needed to obtain agreement with the data. Both parameter sets I and II are equivalent representations of the elastic-scattering data. The ^{48}Ca$(t,p)^{50}$Ca reaction is evaluated using sets III and IV from

TABLE I. Optical parameters used in this work.

	Ref.	Target	Projectile	V_0	r_0	a_0	W	$4W_D$	r_I	a_I
I	16	^{18}O	p	56.0	1.17	0.75	1.7	0	1.32	0.588
		^{16}O	t	146.8	1.4	0.44	14.7	0	1.40	0.551
II	16	^{18}O	p	53.6	1.07	0.74	1.5	24.0	1.34	0.640
		^{16}O	t	170.58	1.4	0.44	9.75	0	1.4	0.551
III	21	^{50}O	p	53.0	1.25	0.65	0	30.0	1.25	0.470
		^{48}Ca	t	165.4	1.16	0.75	16.4	0	1.50	0.750
IV	2	^{48}Ca	t	144.0	1.24	0.678	30.0	0	1.45	0.841
V	19	^{92}Zr	p	48.4	1.25	0.650	0	61.6	1.25	0.470
		^{90}Zr	t	171.3	1.16	0.735	16.8	0	1.48	0.885
VI	20	^{92}Zr	p	55.37	1.20	0.592	0	45.2	1.11	0.796
		^{90}Zr	t	187.8	1.10	0.698	15.0	0	1.44	0.851

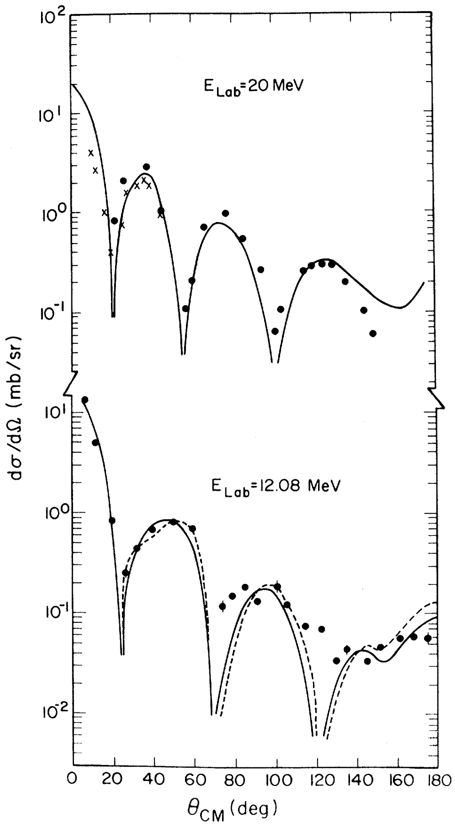

FIG. 1. Comparison of the theoretical angular distributions with experment for the reaction $^{18}O(p,t)^{16}O$ (crosses, Ref. 16; circles, Ref. 17) and also for the reaction $^{48}Ca(t,p)^{50}Ca$ (circles, Ref. 18). See text for dashed curve.

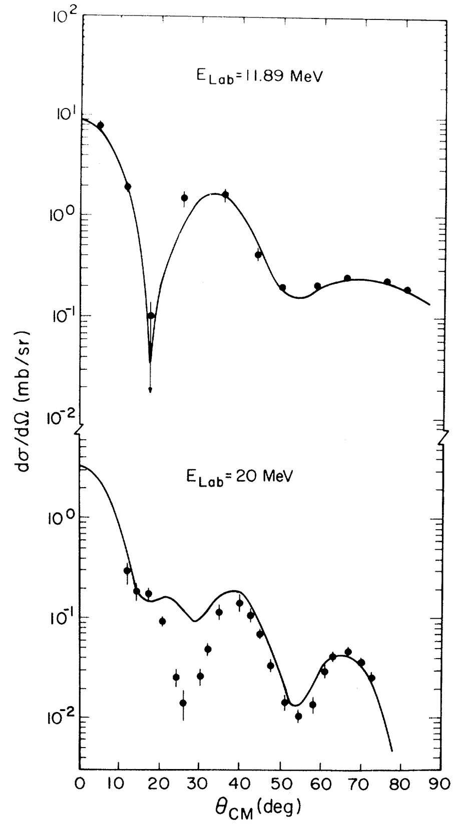

FIG. 2. Comparison of the theoretical angular distribution with experiment for the reaction $^{90}Zr(t,p)^{92}Zr$ at two different projectile energies. For $E_{lab}=11.89$ MeV, circles, Ref. 19; for $E_{lab}=20$ MeV, circles, Ref. 20.

Table I. The solid curve is the result of the present calculation and is independent of the choice of optical parameter set. The dashed curve is the result of use of the half-separation-energy method multiplied by a normalization factor of 2.7. This factor is very close to the value previously found by Bayman.[2] Again the theoretical and experimental angular distributions agree reasonably well. The reactions $^{90}Zr(t,p)^{92}Zr$ at $E_{lab}=11.89$ MeV and at $E_{lab}=20$ MeV are shown in Fig. 2 together with the experimental data. The optical parameters are sets V and VI, respectively, for the 11.89- and 20-MeV data. Both of the differential cross sections are in good agreement with the experimental data, even though they are at much different energies.

In summary we emphasize that the theoretical computations are predictions having no normalization factors or adjustable parameters. Our results show a systematic agreement between these theoretical cross sections and the experimental data both in magnitude and shape. The sensitivity of the $^{18}O(p,t)^{16}O$ angular distribution to the particular choice of the optical parameters makes the comparison to these data ambiguous, as has been noted by previous authors.[3] The other reactions studied were free from this uncertainty. Thus the present first-order DWBA calculations give a good account of the data and provide no support for the need of higher-order mechanism in these reactions. It is also clear from our study that the more correct treatment of the nuclear overlap functions is necessary, since the use of the simpler half-separation-energy model results in absolute magnitudes of angular distributions which are too small when compared to experiment.

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[1]N. Austern, *Direct Reaction Theories* (Wiley, New York, 1970).

[2]B. F. Bayman, Phys. Rev. Lett. 25, 1768 (1970); B. F. Bayman and D. H. Feng, Nucl. Phys. A205, 513 (1973).

[3]L. A. Charlton, Phys. Rev. C 12, 351 (1973).

[4]T. Takemasa, Nucl. Phys. A220, 31 (1973).

[5]T. Takemasa, T. Tamura, and T. Udagawa, Nucl. Phys. A321, 269 (1979).

[6]D. K. Olsen, T. Udagawa, T. Tamura, and R. E. Brown, Phys. Rev. Lett. 29, 1178 (1972).

[7]V. Managoli and D. Robson, Nucl. Phys. A252, 354 (1975).

[8]D. H. Feng, R. H. Ibarra, and M. Vallieres, Phys. Lett. 46B, 37 (1973), and 47B, 477 (1973).

[9]W. T. Pinkston, M. Vallieres, and D. H. Feng, Phys. Rev. C 18, 1011 (1978).

[10]M. A. Nagarajan, M. R. Strayer, and M. F. Werby, Phys. Lett. 67B, 141 (1977), and Phys. Lett. 68B, 421 (1977).

[11]M. F. Werby, M. R. Strayer, and M. A. Nagarajan, Phys. Rev. C (to be published).

[12]M. F. Werby and M. R. Strayer, J. Phys. G 3, L179 (1979).

[13]R. H. Ibarra, M. Vallieres, and D. H. Feng, Nucl. Phys. A241, 386 (1975); M. Vallieres, R. H. Ibarra, and D. H. Feng, Nucl. Phys. A256, 21 (1976).

[14]M. R. Strayer and P. U. Sauer, Nucl. Phys. A231, 1 (1974).

[15]R. V. Reid, Ann. Phys. (N. Y.) 50, 41 (1968).

[16]D. G. Fleming, A. Arima, H. W. Fulbright, and M. Blann, Phys. Rev. C 10, 445 (1974).

[17]M. Pignanelli, S. Micheletti, I. Iori, P. Guazonni, F. G. Resmini, and J. L. Escudié, Phys. Rev. C 10, 445 (1974).

[18]J. H. Bjerregaard, O. Hansen, O. Nathan, R. Chapman, S. Hinds, and R. Middleton, Nucl. Phys. A103, 33 (1967).

[19]S. S. Ipson, K. C. McLean, W. Booth, J. G. B. Haigh, and R. N. Glover, Nucl. Phys. A253, 189 (1975).

[20]E. R. Flynn, J. G. Beery, and A. G. Blair, Nucl. Phys. A218, 285 (1974).

[21]J. F. Petersen, D. A. Lewis, D. Dehnhard, H. P. Morsch, and B. F. Bayman, Phys. Rev. Lett. 36, 307 (1976).

$E2$ Strength in ^{12}C Determined by Elastic Photon Scattering

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The elastic-photon-scattering cross section for ^{12}C has been measured at 90° and 135° in the energy range from 23.5 to 39 MeV. These data disagree with the predicted scattering, derived from the measured photonuclear absorption cross section, if only $E1$ transitions are assumed. To explain the difference in these cross sections, a large component of electric quadrupole absorption between 24 and 40 MeV is inferred.

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The investigation of giant resonances, other than electric dipole, has been accomplished with various nuclear probes. Electron scattering and hadron scattering have been used most often for this purpose. The photonuclear cross sections associated with multipoles higher than dipole are very small so that photon scattering would seem an unlikely method of studying the electric quadrupole giant resonance. However, an appreciable $E1$-$E2$ interference term in the angular distribution permits the observation of $E2$ strength. In this Letter, we report on the use of elastically scattered photons to determine the location and magnitude of the $E2$ strength in ^{12}C. Scattering cross sections were measured at 90° and 135° in the energy range from 23.5 to 39 MeV and the results compared to the predicted values derived from the measured photonuclear absorption cross section.[1] The results show that for excitation energies below 24 MeV only electric dipole transitions need be considered, but in the energy range from 24 to 40 MeV, the magnitude of the electric quadrupole absorption is surprisingly large.

The elastic-scattering cross section is meas-