Tests for Two-Body Quantum-Chromodynamic Subprocesses in Dilepton Angular Correlations

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Angular distributions for dileptons formed in hadronic collisions at large mass Q^2 and large transverse momentum Q_t are calculated, assuming various two-body quantum chromodynamic subprocesses. The main results are the following: (a) The polar and azimuthal distributions are correlated. (b) For fixed Q_t^2, the distributions scale in the variable $w = Q_t^2/Q^2$, and thus are independent of total energy. (c) Each subprocess exhibits a characteristic function of w, independent of the constituent distributions.

The discovery of lepton-pair production in hadronic interactions at large transverse momentum has prompted many investigations into possible underlying "hard" quantum-chromodynamic (QCD) processes.[1] Since the transverse momentum is too large to have come from the intrinsic transverse momentum of quark partons in the usual Drell-Yan annihilation mechanism,[2] it seems reasonable to look at some two-body hard-scattering QCD processes involving gluons. For large enough transverse momentum, one might expect the strong coupling α_s to be small enough so that the lowest-order "pole" diagrams would dominate. Such calculations give reasonably shaped spectra, but unfortunately are infrared divergent for vanishing quark and gluon masses. Hence the absolute normalization at small transverse momentum is uncertain, and it is unclear on how to join this contribution to the "normal" Drell-Yan annihilation contribution. It was soon recognized, however, that the angular distribution of the dilepton pair with respect to some fixed axis gives additional information.[3] For small values of the pair transverse momentum Q_t, one expects a $1+\cos^2\theta$ distribution of the pair relative momentum in its rest frame with respect to the quark (or antiquark) axis. For small intrinsic transverse momentum (k_t) this axis should be close to the beam (or target) particle axis. The smearing of this distribution has been considered,[4] and is generally quite small for lepton-pair mass $Q^2 \gg k_t^2$. When one admits two-body processes, the angular distribution deviates from the $1+\cos^2\theta$ form, and can in general also have an azimuthal (φ) dependence. However, the extra degree of freedom in the two-body process allows contributions at fixed Q_t and Q^2 from a range of parton energies (or Feynman-x values). Thus the calculation of the angular distributions involves a complicated folding of the parton distribution functions with the hard-scattering cross sections. This has been done for some specific processes,[5] and numerical results indicate that large deviations from the $1+\cos^2\theta$ and flat φ distributions can occur in some kinematic regions. It is the purpose of this investigation to extract some properties of these distributions which are independent of the detailed parton distributions in the initial hadrons, and provide some simple experimental tests for the presence of the underlying two-body hard-scattering processes.

I use the density-matrix formalism to describe the decay of the lepton pair from a heavy photon γ^* produced in an arbitrary polarization state:

$$W(\theta,\varphi) = (3/8\pi)[1-\rho_{11}+(3\rho_{11}-1)\cos^2\theta+\rho_{1-1}\sin^2\theta\cos 2\varphi+\sqrt{2}\,\mathrm{Re}\rho_{10}\sin 2\theta\cos\varphi], \tag{1}$$

where θ and φ are the usual angles defined by a fixed axis (beam or target particle in the rest frame of the dilepton) and the production plane defined by the hadrons. The density-matrix elements ρ_{ij} refer to production-amplitude products summed over all hadron spins, with i and j the γ^* helicity along the quantization axis defined above. The individual θ and φ distributions can be written

$$W(\theta) \sim 1+\alpha\cos^2\theta, \tag{2a}$$

$$W(\varphi) \sim 1+\beta\cos 2\varphi, \tag{2b}$$

where

$$\alpha = (1-6\Delta)/(1+2\Delta) \tag{3a}$$

and

$$\beta = 2\rho_{1-1}/(3-2\Delta) \tag{3b}$$

with $\rho_{11} \equiv \frac{1}{2}-\Delta$.

I now calculate the ρ_{ij}'s for various subprocesses. The kinematics are defined by

$$[\text{beam parton } (q) + \text{target parton } (k)] \to [\text{dilepton } (Q) + \text{final parton } (p)]$$

with

$$\hat{s} = (q+k)^2, \quad \hat{t} = (q - Q^2), \quad \hat{u} = (q-p)^2.$$

The values of Δ and $\sum M^2$, the sum over all spin amplitudes, are shown in Table I. I have used the usual two-pole diagrams for each process (see Ref. 5) and omitted the coupling constants in amplitudes, since they will cancel in the ratio which gives Δ, α, and β. I have considered the usual QCD subprocesses $q\bar{q} \to \gamma^* G$ and $qG \to q\gamma^*$ (note the different Δ for q or G as quantization axis) as well as the constituent-interchange process $q + M \to q + \gamma^*$, which has been claimed to possibly fill the gap between present laboratory energies and truly asymptotic energies.[6] A common feature of *all* these processes is that $\rho_{11} + \rho_{1\,-1} = \frac{1}{2}$, or $\rho_{1\,-1} = \Delta$. Thus we can write

$$\beta = \frac{2\Delta}{3 - 2\Delta} = \frac{1-\alpha}{4(\alpha+2)}, \tag{4}$$

and the angular distributions in θ and φ are correlated. Observation of this correlation would be strong evidence that one or a combination of two-body subprocesses are dominant at Q_t large enough to make the pole approximation valid.

Now turn to the calculation of α, through an averaging of Δ over beam and target parton distribution functions at fixed Q_t. If we write the Feynman-x fractions of the beam parton momentum $q = (x_1\sqrt{s}/2)$ and target $k = (x_2\sqrt{s})/2$, where $\sqrt{s}$ is the usual total hadronic center-of-mass energy, then $s = x_1 x_2 \hat{s} \equiv z\hat{s}$. The Q_t constraint gives two possible $\hat{t}$ values at each $\hat{s}$ (corresponding to different signs of dilepton longitudinal momentum). I write $\hat{t} \equiv \eta(Q^2 - \hat{s})$,

$$\eta_\pm = \tfrac{1}{2} + [\tfrac{1}{4} - z x_t^{\,2}/4(z-\tau)^2]^{1/2} \tag{5}$$

with the usual

$$\tau \equiv Q^2/s, \tag{6a}$$

$$x_t^{\,2} \equiv 4Q_t^{\,2}/s. \tag{6b}$$

The average Δ can then be written

$$\bar{\Delta} = \frac{\int dx_1\, f_1(x_1) \int dx_2\, f_2(x_2) g(z) \sum M^2 \Delta(z, \eta_\pm)}{\int dx_1\, f_1(x_1) \int dx_2\, f_2(x_2) g(z) \sum M^2(z, \eta_\pm)}, \tag{7}$$

where f_1 and f_2 are the parton distribution functions. The factor $g(z) = z^{-1}[(\tau - z)^2 - z x_t^{\,2}]^{-1/2}$ weights the Δ to events in a constant $Q_t^{\,2}$ interval ($d\sigma/dQ_t^{\,2}$ rather than $d\sigma/d\Omega$). The range of integration is $0 \leq x_1, x_2 \leq 1$, but with a minimum value of z to ensure enough energy to produce the lepton pair,

$$z_{\min} = \tau + \tfrac{1}{2} x_t^{\,2} + [\tau x_t^{\,2} + \tfrac{1}{4} x_t^{\,4}]^{1/2}; \tag{8}$$

and since $z_{\min} \leq 1$, another kinematic restriction $x_t^{\,2} \leq (1-\tau)^2$ is required. The averaging integral is now considered separately for each subprocess.

(A) For $q\bar{q} \to \gamma^* G$, from Table I and (5) we calculate

$$\sum M^2(\eta_+) = \sum M^2(\eta_-) = \frac{4}{z x_t^{\,2}}(2z^2 + 2\tau^2 - z x_t^{\,2}), \tag{9}$$

$$\Delta(\eta_+) + \Delta(\eta_-) = \frac{4\tau x_t^{\,2}}{(x_t^{\,2} + 4\tau)^2} = \frac{w}{(1+w)^2} \equiv \Delta_1(w), \tag{10}$$

with

$$w \equiv x_t^{\,2}/4\tau = Q_t^{\,2}/Q^2. \tag{11}$$

Since both beam and target axis quantization give the same result for this process, we can write

$$\bar{\Delta}_{q\bar{q}} = \tfrac{1}{2}\Delta_1(w), \quad \text{or} \quad \alpha_{q\bar{q}} = \frac{1 - w + w^2}{1 + 3w + w^2}. \tag{12}$$

This result has three noteworthy features. First, it is entirely independent of the g and $\bar{g}$ distribution functions and the identity of target or beam particles. Second, it not only scales with τ and $x_t^{\,2}$, but it is *only* a function of their *ratio* w, i.e., independent of initial hadron energy. Finally, a characteristic function of w, Eq. (12), is predicted for the α coefficient in the angular distribution. Note that it is essential for this result to include both $\eta_\pm$, i.e., to sum over all lepton-

TABLE I. Values of Δ and ΣM^2 from pole diagrams for various subprocesses. The first constituent is taken as the quantization axis, and $\hat{t}$ is the momentum transfer between that constituent and the dilepton.

	Δ	ΣM^2
$q\bar{q}$	$\dfrac{Q^2\hat{s}\hat{t}\hat{u}}{(\hat{t}-Q^2)^2[(\hat{t}-Q^2)^2+(\hat{u}-Q^2)^2]}$	$\dfrac{2(\hat{u}-Q^2)^2+2(\hat{t}-Q^2)^2}{\hat{t}\hat{u}}$
qG	$\dfrac{Q^2\hat{s}\hat{t}\hat{u}}{(\hat{t}-Q^2)^2[(\hat{s}-Q^2)^2+(\hat{t}-Q^2)^2]}$	$\dfrac{2(\hat{s}-Q^2)^2+2(\hat{t}-Q^2)^2}{-\hat{s}\hat{t}}$
Gq	$\dfrac{2Q^2\hat{s}\hat{t}\hat{u}}{(\hat{t}-Q^2)^2[(\hat{s}-Q^2)^2+(\hat{u}-Q^2)^2]}$	$\dfrac{2(\hat{s}-Q^2)^2+2(\hat{u}-Q^2)^2}{-\hat{s}\hat{u}}$
qM	$\dfrac{Q^2\hat{s}\hat{t}\hat{u}}{(\hat{s}+\hat{t})^2(\hat{t}-Q^2)^2}$	$\dfrac{(\hat{s}+\hat{t})^2}{-\hat{s}\hat{t}}$
Mq	0	$\dfrac{(\hat{s}+\hat{u})^2}{-\hat{s}\hat{u}}$

pair longitudinal momentum.

(B) For $qM \to q\gamma^*$, we calculate

$$\sum M^2(\eta_+)+\sum M^2(\eta_-) = (4/x_t^2)(z-\tau)-3-\tau/z, \qquad (13)$$

$$\Delta(\eta_+)=\Delta(\eta_-)=\Delta_1(w). \qquad (14)$$

But for the meson as the quantization axis, we get $\Delta=0$, with the same $\sum M^2$. Hence for identical target and beam, we get $\bar{\Delta}_{qM}=\frac{1}{2}\Delta_1(w)$, the same as for the $q\bar{q}$ case. But for unequal target and beam particles, one gets $\bar{\Delta}_{qM}=\epsilon\Delta_1(w)$, where ϵ is the fraction of total events in which q comes from the beam and M comes from the target. Note that, in general, ϵ will depend on both τ and x_t^2 and not just on their ratio, so that the resulting α_{qM} will not exhibit the simple scaling behavior of $\alpha_{q\bar{q}}$. One can, however, recover this scaling behavior by extracting Δ from distributions about both the beam and target axes. One expects

$$\Delta_{qM}(\text{beam axis})+\Delta_{qM}(\text{target axis})=\Delta_1(w) \qquad (15)$$

even for unequal beam and target particles. A distinction between the $q\bar{q}$ and qM subprocesses will only be evident in the unequal separate beam- and target-axis distributions for the qM case.

(C) For $qG\to q\gamma^*$, from Table I we see that neither $\sum M^2$ nor Δ in symmetric $\eta_\pm$ combination has a simple z-independent form. Thus the $\bar{\Delta}$ expression has many factors:

$$\bar{\Delta}_{qG}=\Delta_1(w)\frac{\langle z-\tau-\frac{3}{4}x_t^2-\frac{1}{4}x_t^2\tau/z\rangle_1+\langle\frac{1}{2}x_t^2+\frac{3}{2}(x_t^2\tau/z)+(2\tau^2/z)-(2\tau^3/z^2)\rangle_2}{\langle z+\frac{1}{4}x_t^2-3\tau+\frac{3}{4}(x_t^2\tau/z)+(4\tau^2/z)-(2\tau^3/z^2)\rangle_{1+2}}, \qquad (16)$$

where the average $\langle\ \rangle_1$ is over $f_{q/\text{beam}}f_{G/\text{target}}$ and $\langle\ \rangle_2$ is over $f_{q/\text{target}}f_{G/\text{beam}}$. For target = beam, we get a partial simplification:

$$\bar{\Delta}_{qG}=\tfrac{1}{2}\Delta_1(w)\left[1+\frac{2(1-w)\langle\tau-\tau^2/z\rangle}{\langle z-3\tau(1-\frac{1}{3}w)+(4\tau^2/z)(1+\frac{3}{4}w)-2\tau^3/z^2\rangle}\right], \qquad (17)$$

where again the average is over the q-G distribution function product.

Note that this $\bar{\Delta}$ will have a *single* point $w=1$ at which it is energy independent. Numerical calculation with some typical distribution functions verifies this property. However, deviations from this scaling even for $w\neq 1$ is typically less than 2%, for fairly wide ranges $0.05\leq x_t^2$, $\tau\leq 0.5$, of the individual scaling variables. One can see why this is true by examining the behavior of a typical distribution function. One uses

$$f(x)\sim(1-x)^\alpha x^\beta/x, \qquad (18)$$

where typically $0<\beta<1$, and $3<\alpha<7$ for quarks and gluons in protons. Hence the integrals are weighted heavily toward small x, and the $z=x_1x_2$ values toward $z_{\min}$. From (5) and (8) we see that in this region $\eta_+=\eta_-=\frac{1}{2}$, and

$$\langle z^n\rangle\approx z_{\min}{}^n=\tau^n(1+2w+2[w+w^2]^{1/2})^n. \qquad (19)$$

Then (17) yields

$$\bar{\Delta}_{qG}\approx\tfrac{1}{2}\Delta_1(w)\left[1+\frac{2(1-w)}{1+5y}\right] = \frac{3}{2}\frac{w}{(1+w)(1+5w)}\equiv\Delta_2(w). \qquad (20)$$

And, not surprisingly, this is precisely the function of w which was obtained by an empirical fit to the numerical calculations for a wide range of distribution-function parameters. Thus we predict

$$\alpha_{qG}\approx(1-3w+5w^2)/(1+9w+5w^2), \qquad (21)$$

along with the corresponding β from (4). For beam $\neq$ target, one would expect, in general, some x_t^2- and τ-dependent factor times $\Delta_2(w)$. Again, one can extract $\bar{\Delta}$ values from distributions about both beam and target axes, and use (16) to predict

$$\bar{\Delta}_{qG}(\text{beam axis})+\bar{\Delta}_{qG}(\text{target axis})=2\Delta_2(w). \qquad (22)$$

The situation is thus similar to that for $q\bar{q}$ and qM: One expects distributions (suitably symmetrized if beam $\neq$ target) which are independent of the quark or gluon distribution functions, energy independent, and having a characteristic functional dependence (21) on the scaling variable $w=Q_t^2/Q^2$. These characteristic $\alpha(w)$ functions are shown in Fig. 1, along with the β values for the φ distributions. Note that α drops to values substantially below unity for quite small $w=Q_t^2/Q^2$, with the lower values (for small w) occurring for the quark-gluon subprocess.

For fixed-y lepton pairs, only one of the $\eta_\pm$ values comes in, and the symmetry is lost. However, one can combine events with all fixed $+y$ plus all fixed $-y$ to recover the previous con-

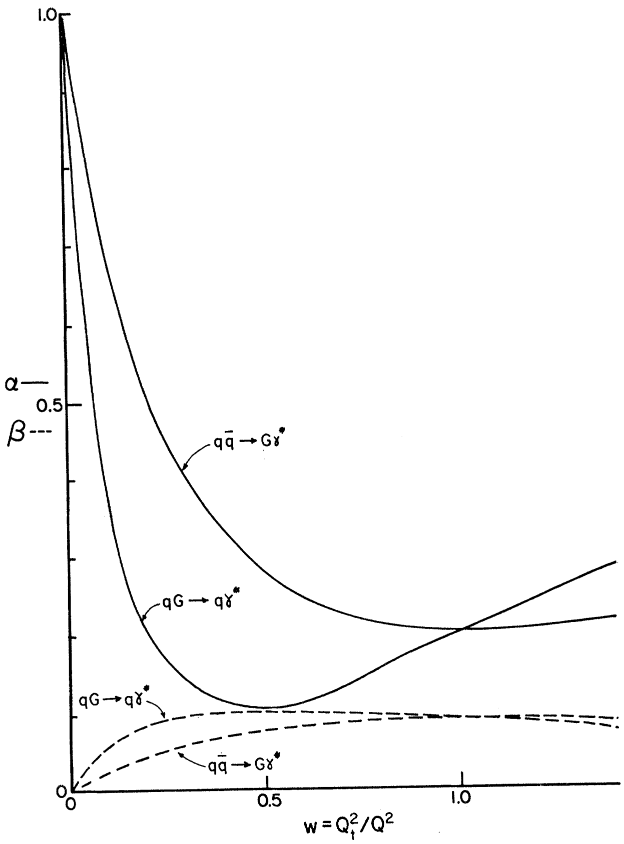

FIG. 1. Variation of coefficients α and β for θ and φ angular distributions with the scaling variable w, for various subprocesses.

clusions, with coefficients also independent of the particular fixed $-y$ value.[7]

The main conclusions of these calculations can be restated as follows: Typical two-body subprocesses predict correlated θ and φ distributions according to Eqs. (2a), (2b), and (4). The coefficients should be independent of constituent distribution functions, be independent of energy, and exhibit a characteristic dependence on the new scaling variable $w = Q_t^2/Q^2$ according to (12) or (21). These results hold for fixed[8] $-Q_t$ with either y-integrated distributions or symmetrized positive- plus negative-y distributions. In the latter case, the coefficients are also independent of y. For unequal beam and target particles, a symmetrization between beam and target axes must also be performed.

[1]G. Altarelli, G. Parisi, and R. Petronzio, Phys. Lett. 76B, 351, 356 (1978); H. Georgi, Phys. Rev. D 17, 3010 (1978); H. Fritzsch and P. Minkowski, Phys. Lett. 73B, 80 (1978); F. Halzen and D. M. Scott, Phys. Rev. Lett. 40, 1117 (1978); K. Kajantie and R. Raitio, Nucl. Phys. B139, 72 (1978).

[2]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. 25, 316, 902(E) (1970).

[3]Minh Duong-van, SLAC Report No. SLAC-PUB-1819, 1977 (to be published).

[4]J. C. Collins and D. E. Soper, Phys. Rev. D 16, 2219 (1977).

[5]K. Kajantie, J. Lindfors, and R. Raitio, Phys. Lett. 74B, 384 (1978); J. Cleymans and M. Kuroda, University of Bielefeld Report No. BI-TP-78/24, 1978 (to be published).

[6]Christopher M. Debeau and Dennis Silverman, Phys. Rev. D 18, 2435 (1978).

[7]R. L. Thews, to be published.

[8]John C. Collins, Phys. Rev. Lett. 42, 291 (1979), has derived Eq. (12) in the Collins-Soper frame for the $q\bar{q}$ subprocess at fixed Q_t. The present analysis shows that it also is valid in the beam or target frames.