Melting of Two-Dimensional Vortex Lattices

B. A. Huberman
Xerox Palo Alto Research Center, Palo Alto, California 94304

and

S. Doniach
Department of Applied Physics, Stanford University, Stanford, California 94305

(Received 30 May 1979)

The existence of a new phase boundary for thin-film superconductors in an applied magnetic field and in rotating films of superfluid helium is predicted. The melting temperature is shown to lie below and scale with the zero-field vortex plasma transition temperature.

At sufficiently low temperatures, an external magnetic field of magnitude $H_{c_1}(\text{film}) < H < H_{c_2}$ applied normal to a thin-film superconductor will induce a stable lattice of superfluid vortices in the film. An analogous lattice, now rotating with the sample, will be induced by the rotation of a thin film of superfluid ^{4}He about an axis normal to the film plane.

The purpose of this paper is to point out that, for sufficiently thin films of arbitrary area, the energy needed to nucleate dislocation pairs in the vortex lattice will become low enough for thermal excitation of finite densities of dislocation pairs to become appreciable at temperatures (T) below the superfluid–normal-fluid transition temperature. Because of the dependence of the strain energy induced by dislocation pairs on the logarithm of their separation, it is expected that in the thin-film limit the rigidity modulus will drop to zero at a critical temperature, T_M, at which the lattice melts.

The principal result of this paper is that this critical melting temperature lies below and scales with the temperature T_{2D}, above which the zero-field vortex plasma phase appears.[1] Thus, in finite applied fields, the phase diagram for the thin-film superconductor contains at least one additional phase boundary (see Fig. 1) between the low-temperature state and the vortex plasma state predicted by Doniach and Huberman.[1] Above the melting temperature of the vortex lattice, there may or may not exist an additional "hexatic" or liquid-crystal–like phase,[2] in which, although the positional correlation function of the vortex lattice decays exponentially, the correlation function for orientational order of the lattice persists at large distances. The stability of the hexatic phase for $T > T_M$ depends on the magnitude of the Franck constant which is unknown for the vortex lattice.

The stress-induced interaction energy between a pair of dislocations in the vortex lattice, whose separation is large compared with the lattice constant, may be calculated in terms of the rigidity modulus, μ, of the lattice. Following Fetter and Hohenberg,[3] Fiory[4] and Conan and Schmid[5] have shown that μ is given for superconducting films by

$$\mu = B\varphi_0/64\pi^2\lambda_\perp, \tag{1}$$

where φ_0 is the flux quantum $hc/2e$, B is the applied field, and $\lambda_\perp = \lambda^2/d$ is the thin-film penetration depth. For sufficiently thin films, the presence of a gas of thermally excited dislocation pairs leads to a discontinuous drop of the ridigity modulus at a temperature T_M given by[6]

$$k_B T_M = \mu a^2/8\pi(1-\nu), \tag{2}$$

where a is the lattice spacing and ν the Poisson's ratio. Since[3] the vortex lattice is incompressible,

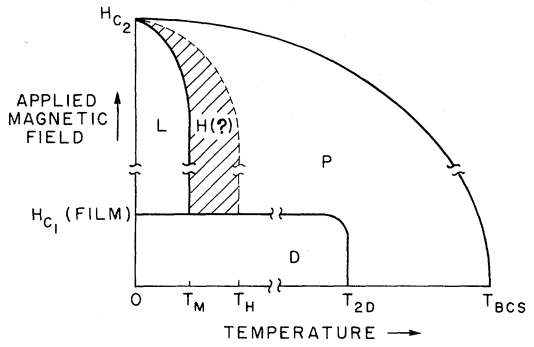

FIG. 1. Phase diagram of a two-dimensional superconductor in the magnetic-field–temperature plane. D denotes the weakly diamagnetic Meissner phase with no vortices present. L denotes the rigid–vortex-lattice phase and P the vortex plasma phase. The shaded area H indicates a possible hexatic phase.

Poisson's ratio is given by $\nu = \frac{1}{2}$.

Thus T_M may be expressed directly in terms of the zero-field transition temperature T_{2D} for onset of the vortex plasma state valid for superconducting samples of size $R \lesssim \lambda_\perp$ which is given by[1,7,8]

$$k_B T_{2D} = (\varphi_0/4\pi)^2(1/2\lambda_\perp). \quad (3)$$

For a triangular lattice $a^{-2} = \sqrt{3}n/2$, where n is the vortex density ($n = B/\varphi_0$); this leads to a relationship between T_M and T_{2D} which is independent of applied field, and hence of the density of the vortex lattice in the thin-film limit:

$$T_M/T_{2D} = 1/4\pi\sqrt{3}. \quad (4)$$

For superfluid helium films, the rigidity modulus, in the rotating frame in which the vortex lattice is stationary,[9] may be written as

$$\mu = (h\Omega d/8\pi)n_{He}, \quad (5)$$

where Ω is the angular velocity and n_{He} the volume density of the He atoms. Using the formula $k_B T_{2D} = \frac{1}{2}\pi\hbar^2 n_s^{2D}/M_{He}$ valid for films of infinite extent[6] and identifying $a^{-2} = (\sqrt{3}/2)(2M_{He}\Omega/h)$, the relationship given in Eq. (4) is seen also to be valid for thin superfluid He films. It should be noted that in the case of He films, Eq. (4) is independent of sample size R. For superconducting films, on the other hand, Eq. (3) is only applicable in the limit $R \ll \lambda_\perp$. However, since the logarithmic nature of the interaction between dislocations is independent of the form of the vortex-vortex interaction, we expect the melting transition to occur at T_M, as given in Eq. (2), independently of sample size in the thin-film limit even in the superconducting case.

As the film thickness is increased, Eq. (3) for the 2D superconductor is no longer proportional to film thickness but saturates as $T_{2D} \to T_{BCS}$. T_M will then continue to increase to increase with film thickness until it approaches T_{BCS}. For very thick films the dislocation core energy E_c will eventually become so large that the mean dislocation pair density for $T \sim T_M$, of order $a^{-2}\exp(-E_c/k_B T)$ will become extremely small, of order R^{-2}, and so the concept of two-dimensional melting will no longer be a useful one.

As the applied field is increased towards H_{c2}, Eq. (1) for the rigidity modulus is no longer valid. In this regime, Conen and Schmid[5] have shown that

$$\mu = C(H_{c_b}^2/8\pi)(1 - B/H_{c2})^2, \quad (6)$$

where H_{c_b} is the thermodynamic critical field and C is a constant with value $C = 0.353$ for thin films. Hence, as $H \to H_{c2}$, T_M/T_{2D} approaches zero quadratically in B (see Fig. 1).

For $T > T_M$, Halperin and Nelson[2] have shown that the hexatic-phase melting temperature is given by

$$k_B T_H = K_A(T_H)/72 \quad (7)$$

when $K_A(T)$ is the renormalized value of the Franck constant at temperature T given approximately by $K_A \sim 2E_c a^2$, where E_c is the core energy of a disclination. Our present lack of knowledge of E_c for a vortex lattice means we are unable to say whether T_H is greater than T_M, and hence do not know whether the hexatic phase exists. Concerning the experimental observations of this phenomenon, we point out that the melting of the vortex lattice should have a significant influence on the effect of pinning on flux flow in thin superconducting films. For $T \to T_M$ the existence of a lattice with a finite rigidity modulus implies that a small concentration of pinning centers can inhibit motion of the entire vortex lattice. Above T_M, on the other hand, the lack of rigidity modulus means that pinning centers will be relatively ineffective at impeding flux flow. Therefore, measurements of the pinning current J_p as a function of field and temperature can be used to determine the value of μ. These experiments could also determine the softening of the shear modulus as T_M is approached from below, since larger local fluctuations of the vortex lattice lead to an enhancement[4] of J_p. This lack of pinning in thin superconducting films at temperatures approaching T_{2D} appears to be a common observation.[10]

The alternative possibility would be a direct measurement of the shear rigidity modulus of the vortex lattice using the method invented by Fiory.[4] This technique is based on the fact that the dynamic fluctuations of the moving vortex lattice in a superconducting film couple rf and dc motions and currents, allowing for a determination of μ as a function of field and temperature. Since Fiory has shown that the existence of a finite rigidity modulus manifests itself in the appearance of steplike transitions in the field dependence of the resistivity, the new phase boundary could in principle be determined by the abrupt disappearance of the steps at T_M.

We have profited from useful discussions with A. L. Fetter, M. R. Beasley, and L. Susskind.

[1]S. Doniach and B. A. Huberman, Phys. Rev. Lett. 42, 1169 (1979).
[2]B. I. Halperin and D. R. Nelson, Phys. Rev. Lett. 41, 121, 519(E) (1978), and to be published.
[3]A. L. Fetter and P. C. Hohenberg, Phys. Rev. 159, 330 (1967).
[4]A. T. Fiory, Phys. Rev. B 8, 5039 (1973).
[5]E. Conen and A. Schmid, J. Low Temp. Phys. 17, 331 (1974).
[6]J. M. Kosterlitz and D. J. Thouless, J. Phys. C 6, 1181 (1973).
[7]M. R. Beasley, J. E. Mooij, and T. P. Orlando, Phys. Rev. Lett. 42, 1165 (1979).
[8]B. I. Halperin and D. R. Nelson, to be published.
[9]V. K. Tkachenko, Zh. Eksp. Teor. Fiz. 50, 1573 (1966) [Sov. Phys. JETP 23, 1049 (1966)].
[10]P. Horn and R. D. Parks, Phys. Rev. B 4, 2178 (1972).

Effects on Photoemission of the Spatially Varying Photon Field at a Metal Surface

Harry J. Levinson and E. W. Plummer
Department of Physics and Laboratory for Research on the Structure of Matter, University of Pennsylvania, Philadelphia, Pennsylvania 19104

and

Peter J. Feibelman
Sandia Laboratories, Albuquerque, New Mexico 87185

(Received 18 July 1979)

Enhancement of photoionization cross sections due to spatially varying photon fields at a metal surface has been observed in the normal-emission cross sections for the surface state and Fermi level of Al(100) at photon energies between 9 eV and 23 eV. The data for Fermi-level photoexcitation are in excellent agreement with theoretical results for jellium. Below $\hbar\omega_p$, the predominant contribution to the photoionization matrix element comes from the spatially varying fields, which provide the momentum required for photoexcitation.

In this Letter we present unambiguous experimental evidence which shows that the spatially varying photon field at a metal surface must be taken into account in order to explain the magnitude and frequency dependence of energy- and angle-resolved photoionization cross sections. As seen in our data, this effect can be the dominant mechanism for photoexcitation. Since energy and momentum cannot be simultaneously conserved in photoexcitation from a translationally invariant electron gas, photoemission is usually attributed to the presence of a surface (the "surface photoeffect") or the lattice potential (the "bulk photoeffect"), either of which breaks translational symmetry and can provide the momentum necessary to overcome the kinematic restriction. However, the structure in our data cannot be explained in terms of either the surface barrier or lattice potential, and it is necessary to consider the spatially varying photon field at the surface[1-6] as a source of momentum.

The discussion of the photoeffect given above can be made precise by examining all of the contributions to the cross section for photoexcitation from an initial state ψ_i to a final state[7] ψ_f,

$$\frac{d\sigma}{d\Omega\, d\omega}=\frac{e^2}{\hbar c}\frac{\pi^2 k}{|\vec{A}_0|^2 m\hbar\omega}\left|\int \vec{j}_{fi}(\vec{r})\cdot\vec{A}(\vec{r})d^3r\right|^2, \qquad (1)$$

where $\vec{j}_{fi}(\vec{r})=\psi_f^*(\vec{r})\nabla\psi_i(\vec{r})-\psi_i(\vec{r})\nabla\psi_f^*(\vec{r})$ is the transition current density and $\vec{A}(\vec{r})$ is the vector potential of the photon field.[8] In approximating Eq. (1) it is usually assumed that the spatial dependence of $\vec{A}(\vec{r})$ can be neglected since the wavelength of the light is long compared with atomic dimensions. This leads to the familiar dipole matrix element which in a single-particle model may be written in terms of the crystal potential[7] $V(\vec{r})$:

$$\frac{d\sigma}{d\Omega\, d\omega}=\frac{e^2}{\hbar c}\frac{4\pi^2\hbar^2 k}{m\hbar\omega(E_f-E_i)^2}\left|\langle f|\vec{e}_A\cdot\nabla V(\vec{r})|i\rangle\right|^2. \qquad (2)$$

What ∇V does is to furnish the required momentum so that both energy and momentum can be conserved in photoexcitation. The assumptions leading to Eq. (2) are incorrect when the dielectric response of the solid to the incident electro-

