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Interpretation of Electric-Field-Induced Oscillations in the Cross Section of a One-Electron Atom above the Ionization Limit

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Exact calculations of the photoionization cross sections of ground-state H in the presence of an external electric field are presented. The oscillations obtained above the zero-field ionization limit for π polarization do not arise from specific properties of $m=0$ states, but from cancellation effects in the oscillator strengths due to the symmetry properties of the wave functions and of the light. The dependence of the cross section upon the light polarization is thoroughly explained and agrees with recent experiments on Rb.

Recently strong oscillations have been observed in the π-photoionization spectrum from the ground state of Rb in the presence of an electric field $\mathcal{F}$, not only for energies just above the classical field-ionization threshold[1,2] $E_c = E_0 - 2\sqrt{\mathcal{F}}$ (in a.u.) but also above the ionization potential of the unperturbed atom, E_0.[3] This phenomenon is observed only with π polarization. In order to explain the origin of the differences between spectra obtained for π and σ polarizations two different properties are to be discussed. First, does this structure depend on specific properties of upper states with magnetic quantum number $m \neq 0$ which are excited from the ground state with π-polarized light? $m=0$ states have two peculiar characteristics. On the one hand, in contrast to $m \neq 0$ states, their wave functions do not possess a nodal line in the direction of the field $\mathcal{F}$ (z axis); so the electron has a larger probability to escape through the bottleneck of the potential surface. On the other hand, $m=0$ states are composed of low-l states with high quantum defects, and thus the nonhydrogenic field-ionization process similar to autoionization[4] could be prevailing for such states. Second, is the observed structure related to the relative symmetry properties of the exciting light and of the electric field? In this case, structures could be observed with different polarizations or for $m \neq 0$ states.

From the study of the classical motion of an electron in the potential $V = -1/r + \mathcal{F}z$, Freeman *et al.*[3] did show that even for energies greater than E_0 there exist nearly closed orbits which correspond to motion tightly constrained in the vicinity of the positive z axis. Thus these authors argue for the existence of quantized $m=0$ states. These cannot be excited from the ground state, except with π polarization. Replacing the potential V by the one-dimensional model potential $V_{\text{eff}} = -1/z + \mathcal{F}z$ for $z>0$, $V_{\text{eff}} = \infty$ for $z<0$, these authors found that this model accounts for equally spaced states near E_0, with a spacing as a function of $\mathcal{F}$ in very good agreement with the experimental results.

More recently, Rau[5] showed that this pattern of equally spaced structures stems from a "strong-field mixing" effect, the bounded motion of the electron in the ξ direction[6,7] determined equally by the Coulomb field and by the electric field. However, a periodic structure is obtained for any m, and so the dependence of the experimental data upon the polarization of the light is not clearly explained.

In the present Letter we report exact calculations of photoionization cross sections of hydrogen in the presence of an external electric field and we show that the observed structures are related to the Δm value for the studied transition rather than to the m value of the upper states, thus explaining thoroughly the polarization dependence.

The Stark spectrum of hydrogen is not a dis-

crete spectrum. Rather, every state has a finite width related to its lifetime with respect to the field-ionization process. Consequently, a significant depth of modulation in the density of states is expected to appear when the width of a state is much smaller than the spacing between two neighboring states. The structure of the density of states in the Stark spectrum of hydrogen can be exactly calculated by a method described elsewhere.[8] This method can be used to study energies greater than E_0 or very strong field strengths for which perturbation and WKB methods are no longer valid.[6] We present here only the results and a qualitative interpretation of the overall characteristics of the calculated structures. A stationary quantum state is entirely defined by E, m, and n_1.[9] The last quantum number is a good quantum number for hydrogen only.[4]

Since we are concerned with the photoabsorption spectrum from the ground state of hydrogen, we are mainly interested in the behavior of the wave functions not too far from the nucleus, where the overlap between ground- and excited-state wave functions is important. In this region a stationary wave function behaves as $\Psi_{n_1}{}^m(E,\vec{r}) \simeq [\mathcal{C}_{n_1}{}^m(E)/2\pi]^{1/2}\rho^{|m|}\exp(\mathrm{Im}\varphi)$, where $\rho^2 = x^2 + y^2$. For a normalized wave function the constant $\mathcal{C}$ is unambiguously determined.[6] $\mathcal{C}_{n_1}{}^m(E)$ is equal to the weight of the stationary state E, m, n_1 in the time development of a wave function corresponding to an electron located at the nucleus at $t=0$. Thus, $\mathcal{C}_{n_1}{}^m(E)$ is the partial—with respect to m, n_1—density of states. The total density of states with fixed E and m, $\mathcal{C}^m(E)$, is the sum over all n_1 contributions. In Fig. 1 the partial and total densities of states with $|m|=0$ or 1 are reported. For $E<E_0$ quasidiscrete resonances corresponding to the bound states of hydrogen are observed. Besides these sharp structures $\mathcal{C}_{n_1}{}^m(E)$ exhibits only a very broad bump, which extends between two limits called $Ec_{n_1}{}^m$ and $Ed_{n_1}{}^m$; this "broad level" with a width $\Gamma \simeq E_{n_1}{}^m - Ec_{n_1}{}^m$ is to be compared with the discrete states of the semiclassical models.[3,5]

We now explain the origin of this structure for $|m|=1$ states. The constant $\mathcal{C}$ depends on the probability of the electron being at the nucleus; thus significant values for $\mathcal{C}$ are obtained when the effective kinetic energy T of the electron is positive simultaneously at $\xi=0$ and $\eta=0$, i.e., when the electron can classically be found at the nucleus. For the n_1, $|m|=1$ level, and for the η motion, T is positive for $E \geq Ec_{n_1}{}^1$. $Ec_{n_1}{}^1$ is the ionization threshold for the studied level, since

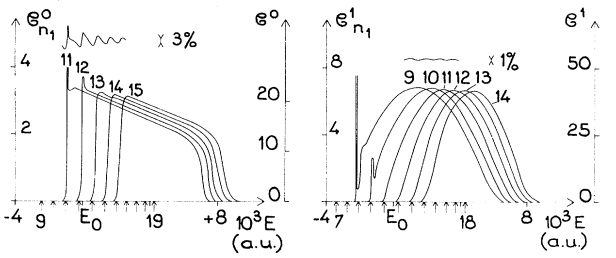

FIG. 1. Energy dependences of the partial densities of states $\mathcal{C}^{|m|}_{n_1}(E)$ for $|m|=0$ and 1, for different n_1 ($\mathcal{F} = 1.5\times10^{-5}$ a.u.). The ionization thresholds are indicated by an arrow. The total densities of states $\mathcal{C}^{|m|}(E)$ are drawn at the top of the figure; the depth of the modulation does not exceed 3% and 1% for $|m|=0$ and 1, respectively.

for $E > Ec_{n_1}{}^1$ the potential barrier in the η motion disappears. For n_1 values such that $Ec_{n_1}{}^1 > E_0$, the energy $E = Ec_{n_1}{}^1$ is associated with the value $Z_1 = 1$ for any n_1. But when $Ec_{n_1}{}^1 < E_0$, for $E = Ec_{n_1}{}^1$, Z_1 satisfies $Z_1 = 1 - (Ec_{n_1}{}^1 - E_0)^2/4\mathcal{F}$.[6] For the ξ motion T is positive for $E \leq Ed_{n_1}{}^1$. The energy $E = Ed_{n_1}{}^1$ corresponds to $Z_1 = 0$. Since the energies $E = Ec_{n_1}{}^1 > E_0$ and $E = Ed_{n_1}{}^1$ are associated with fixed values of Z_1, they can be considered as the eigenvalues of quantized motions in the ξ coordinate. The case $Z_1 = 1$ is identical to the potential model of Freeman.[3] Thus for $|m|=1$ states, the latter may be used to predict exactly the position of the thresholds higher than E_0, while for other states the predictions will be approximate.

For any m value, each partial density $\mathcal{C}_{n_1}{}^m(E)$ corresponds to a maximum in the total density of states. The depth of the modulation in $\mathcal{C}^m(E)$ increases with $\mathcal{F}$,[10] and is only a few percent at the studied strength (77 kV/cm). In conclusion, the oscillations observed in the π-photoionization cross section of Rb cannot be explained from the existence of quantitized $m=0$ states in hydrogen, as was previously done,[3,5] since the depth of the modulation in the density of states is much smaller than that recorded. Furthermore, we have shown here that similar structures exist for any value of m.

Actually, the photoionization cross section does not depend directly on $\mathcal{C}^m(E)$ but is related to the total density of oscillator strengths $df(i;m,E)/dE$ in the absorption from the lower state i, towards upper m states. It is a sum over all partial contributions $df(i;n_1,m,E)/dE$ corresponding to upper states with different n_1 values and is proportional to the nondiagonal quantity $|\langle\Psi_i|\vec{r}|\Psi_{n_1}{}^m(E)\rangle|^2$. Thus, the energy dependence

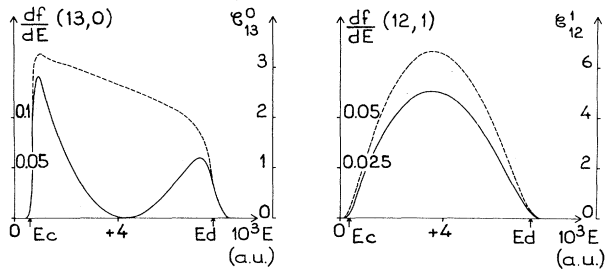

FIG. 2. Energy dependences of the partial density of states $\mathcal{C}_{n_1}^{|m|}(E)$ (dashed line) and the partial density of oscillator strengths $df(n_1,|m|,E)/dE$ (solid line) in the photoabsorption spectrum from the ground state of hydrogen ($\mathcal{F}=1.5\times10^{-5}$ a.u.).

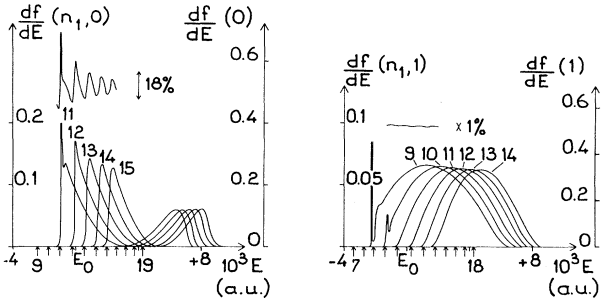

FIG. 3. Energy dependences of the partial densities of oscillator strengths $df(n_1,|m|,E)/dE$ for $|m|=0$ and 1 in the photoabsorption spectrum from the ground state of hydrogen ($\mathcal{F}=1.5\times10^{-5}$ a.u.). The total densities of oscillator strengths $df(|m|,E)/dE$ are presented at the top of the figure; the depth of the modulation at the zero-field ionization limit E_0 is equal to 18% or 1% for $|m|=0$ or 1, respectively.

of $\mathcal{C}$ and df/dE can be different, particularly when there exist symmetry properties in the wave functions, as is discussed below. For $\mathcal{F}=0$, the hydrogen wave function in parabolic coordinates $\Psi_{n_1n_2}{}^m$ has no particular symmetry property with respect to the $z=0$ plane, except for the case $n_1=n_2$ where it is symmetrical.[6] Then there exists only one strict selection rule for electric-dipole transitions: $(n_1n_2m)\rightarrow(n_1'n_2'm')$ is forbidden for $m=m'$, $n_1=n_2$, and $n_1'=n_2'$, since the wave functions are symmetrical and the transition operator antisymmetrical under reflection through the $z=0$ plane.[6] For $\mathcal{F}\neq0$, any level n_1, m presents such a symmetry property, confined however to the neighborhood of the nucleus, in the energy range such that $Z_1\simeq Z_2$. Indeed the effective kinetic energies in the ξ and η motions are then equal. Thus the partial density of oscillator strengths for transitions from the ground state ($n_1=n_2=m=0$) towards upper levels n_1, $m'=0$ vanishes at an energy approximately midway between $Ec_{n_1}{}^0$ and $Ed_{n_1}{}^0$ through total cancellation of the transition matrix element. Consequently $df(i;n_1,0,E)/dE$ exhibits a relatively narrow resonance located at $Ec_{n_1}{}^0$. This property is shown in Fig. 2. For $|m|=1$ states no cancellation occurs and $\mathcal{C}_{n_1}{}^1(E)$ and $df(i;n_1,1,E)/dE$ are almost proportional. In Fig. 3 the partial and total densities of oscillator strengths in the photoionization spectra from the ground state of hydrogen are reported for π and σ polarizations. An important modulation (18%) is obtained in the π spectrum in contrast to the almost structureless σ spectrum: A rapid increase in $df(i;0,E)/dE$ is observed at each threshold $Ec_{n_1}{}^0$. Consequently the spacing of the oscillations in the density of oscillator strengths at $E>E_0$ can be calculated from the spacing between two thresholds—i.e., from the potential model of Freeman.[3]

However, the depth of the modulation observed in the π spectrum of Rb is much smaller than the value calculated in H. Indeed it is related to nondiagonal quantities specific to a given spectrum. In a previous study[11] of the photoabsorption spectrum at $E<E_c$ we have already shown that the positions of the Stark patterns can be interpreted in the hydrogenic model, but that the intensities are characteristic of the non-Coulombic part of the atomic potential.

The dependence of the recorded spectra on the polarization of the exciting light does not arise from specific properties of the m value of the upper state, but is related to the Δm value of the transition—i.e., to the respective symmetry properties of the light and of the electric field. Consequently we predict that similar structures could be observed in the photoionization spectra of alkali-metal atoms in different experimental conditions. Indeed, oscillations associated with $m\neq0$ upper states could be obtained for a π polarization in the absorption from a lower l, m zero-field state with $l+m$ even, since the wave function of the latter state is symmetrical with respect to the $z=0$ plane. Furthermore, oscillations could be observed even with σ polarization for antisymmetrical lower states ($1+m$ odd) because the transition operator is then symmetrical. Such experiments, which could be carried out by multistep excitation, would present interest in the understanding of the field-ionization properties of atomic Rydberg states.

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[6]H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer, Berlin, 1957).

[7]The Stark Hamiltonian for hydrogen is separable in parabolic coordinates $\xi = r+z$, $\eta = r-z$. There are two motions, one bounded (in ξ) and one free (in η), coupled through the separation constants Z_1 and Z_2 ($Z_1 + Z_2 = 1$). Z_1 is the eigenvalue of the ξ motion and depends not only on m, but also on E.

[8]E. Luc-Koenig and A. Bachelier, to be published.

[9]For fixed values of E and m, Z_1 takes on an infinite number of discrete values labeled by n_1, the number of nodes in the bounded function.

[10]The energy spacing Δ between two "broad levels" is given at E_0 by Rau's law [Ref. 5, Eq. (5)]. The width of the first "broad level" above E_0 is $\Gamma = (5.82\ \mathrm{cm}^{-1}) \times (\mathcal{F}\ \mathrm{V/cm})1/2$. The theoretical depth of the modulation $\gamma = \Delta/\Gamma$ increases as $\mathcal{F}^{1/4}$ and is only equal to 0.0728 for $\mathcal{F} = 10$ kV/cm.

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Enhancement of Stimulated Brillouin Scattering due to Reflection of Light from Plasma Critical Surface

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We consider the effect of light reflected from the plasma critical surface on stimulated Brillouin scattering. For an expanding plasma, there will be a certain Mach number where the reflected light acts as a noise source for stimulated-Brillouin-scattering exponentiation. We show calculations and plasma simulations that exhibit significantly enhanced levels of scattering.

Stimulated Brillouin scattering (SBS) is of great interest to laser fusion, since a high level of scattering may leave insufficient laser energy to drive the fusion reaction. Previous analyses of SBS[1,2] did not consider the effects of a partially reflecting boundary, such as the critical surface in a realistic plasma expansion. Generally, these analyses found that the instability exponentiates in space from some small noise level, α, usually taken to be in the range 10^{-4}–10^{-2}. Thus the effective interaction length, which is set by the finite length of the plasma or by density or velocity gradients, must be much greater than the growth length for SBS to be appreciable. In this paper, we discuss a mechanism by which a reflecting surface can greatly increase the effective noise level, so that only a few growth lengths are required for significant scatter.

Consider a mass of plasma moving with respect to the reflecting surface at a speed $v = Mc_s$ toward the laser where M is the Mach number and c_s is the sound speed. In the rest frame of the plasma, the incident light is Doppler shifted up to $\omega_i = \omega_0 + k_0 M c_s$, where k_0 is the free-space wave number and ω_0 is the laser frequency, while the light reflected from the surface is Doppler shifted down to $\omega_r = \omega_0 - k_0 c_s (M - 2M_c)$, where M_c is the Mach number of the critical surface. At Mach number $M = M_c + \sqrt{\epsilon}$ (ϵ is the plasma dielectric constant), $\omega_r = \omega_i - \omega_s$, where $\omega_s = 2k_0 c_s \sqrt{\epsilon}$ is the local ion acoustic frequency. Thus, the reflected light has the same frequency as SBS-scattered light and will serve as a source for SBS exponentiation. In this case $\alpha \sim 1$ and only one growth length is required for a large amount of scatter.

In order to make the analysis of this effect more clear we first recapitulate the analysis of SBS with no reflecting surface. A good discussion of the full equations for SBS and their subsequent reduction to the slow-time and space equations may be found in Ref. 1. Since a typical exponentiation time is much shorter than the laser pulse width, we consider only the time-asymptotic case. Further, since we are interested in the enhancement of direct backscatter, one space dimension is sufficient. The model equations we