Now, we assume that space-time is asymptotically flat. This implies that the vacuum expectation value of $g_{\mu\nu}$ is the Minkowski metric $\eta_{\mu\nu}$, that is,[13]

$$\langle 0|g_{\mu\nu}|0\rangle = \eta_{\mu\nu}, \tag{6}$$

where $|0\rangle$ denotes the true vacuum. Of course, b_ρ, c^ρ, and $\bar{c}_\rho$ have a vanishing vacuum expectation value. Then from (4) we have

$$\langle 0|[g_{\rho\sigma},\hat{M}^\mu{}_\nu]|0\rangle = i\delta^\mu{}_\rho\eta_{\nu\sigma} + i\delta^\mu{}_\sigma\eta_{\rho\nu}, \tag{7}$$

while $\langle 0|[b_\rho,\hat{M}^\mu{}_\nu]|0\rangle = 0$, etc. Thus the GL(4) invariance is *spontaneously broken*. Of course, the Lorentz group, which is a subgroup of GL(4), is not broken; indeed,

$$\langle 0|[g_{\rho\sigma},M_{\mu\nu}]|0\rangle = 0, \tag{8}$$

where

$$M_{\mu\nu} \equiv \eta_{\mu\lambda}\hat{M}^\lambda{}_\nu - \eta_{\nu\lambda}\hat{M}^\lambda{}_\mu. \tag{9}$$

Therefore, the degrees of freedom of the spontaneously broken symmetry are $16-6=10$. The ten components of the gravitational field are the corresponding Goldstone bosons. In particular, we see that gravitons must be massless according to the Goldstone theorem.

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U(1) as the Minimal Horizontal Gauge Symmetry

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A horizontal U′(1) gauge symmetry is proposed to distinguish two fermionic generations. By extending the notion of generation structure to the Higgs system, and using anomaly-free conditions with the requirements of lepton-number conservation and proper Cabibbo structure, we determine $\boldsymbol{Y}'$ associated with U′(1), uniquely. The model can easily be generalized to account for 2^N ($N=1,2,\ldots$) generations.

The standard left-handed Weinberg-Salam (WS)-$SU(2)_L \otimes U(1)$ model[1] has scored remarkable successes in correlating electromagnetic- and weak-interaction data. Nevertheless, there appear to exist several sets of fermions, i.e., generations, which have one and the same WS substructure and it is not yet clear what additional symmetry distinguishes them. To the lowest level, one would like the so-called horizontal symmetry to account for the natural emergence of the Cabibbo rotations[2] and the Glashow-Iliopoulos-Maiani mechanism[3] as well as to provide a reasonable explanation for the existence of several lepton numbers. A number of attempts have already been made in this direction, using discrete symmetries[4] or alternatively continuous gauge groups. However, one is still far away from a fundamental understanding of the repeated fermion structures.

In this Letter we propose to investigate the simplest gauge symmetry, namely, that corresponding to U′(1) in conjunction with the WS $SU(2)_L \otimes U(1)$ to classify fermion generations. Our main assumptions are as follows:

(a) The hypercharges Y and Y' associated with the two U(1)'s factorize; that is, they are of the form

$$Y=\lambda_i y, \quad Y'=\lambda_i' y', \tag{1}$$

where y and y' are sets of quantum numbers (QN) for the fermions in a given generation and λ_i and λ_i' are scale parameters or the so-called "seriality" numbers[6] of the ith generation.

(b) The model be anomaly free.[7] In the WS model, the anomalies are canceled separately within each generation. The additional U′(1) leads to the possibility that each generation contributes a nonvanishing anomaly, but the over all anomaly is zero.

(c) The Higgs system is minimal. We extend the concept of generation to the Higgs system as well. Associated with each fermionic generation, we assume the presence of a Higgs doublet, the vacuum expectation value (VEV) of the neutral component of which sets the mass scale for that generation. In addition we assume that there exists a Higgs singlet with zero WS hypercharge whose VEV allows us to make the mass $M(Z')$ of the new gauge boson associated with U′(1) as heavy as we please.

For the sake of definiteness, let us consider the simplest but fundamental case of two generations both of which are expected to have one and the same WS substructure. They have the usual charge and weak left-handed isospin assignments. In order to present a unified treatment of both the hypercharges, let us write (1) more explicitly, as follows:

$$Y \text{ or } Y'=\begin{cases}\lambda_1(a,a,a';A,A,A',A'') \text{ for } (\nu_L{}^e,e_L,e_R;u_L,d_L,d_R,u_R)\\ \lambda_2(a,a,a';A,A,A',A'') \text{ for } (\nu_L{}^\mu,\mu_L,\mu_R;c_L,s_L,s_R,c_R).\end{cases} \tag{2}$$

The anomaly free constraints[8] for Y and Y' are then given by

$$\sum Y_L=0\rightarrow(\lambda_1+\lambda_2)(a+3A)=0, \tag{3a}$$

$$\sum Q^2(Y_L-Y_R)=0\rightarrow(\lambda_1+\lambda_2)[a-a'+\tfrac{1}{3}(5A-A'-4A'')]=0, \tag{3b}$$

$$\sum Q(Y_L{}^2-Y_R{}^2)=0\rightarrow(\lambda_1{}^2+\lambda_2{}^2)(a^2-a'^2-A^2-A'^2+2A''^2)=0, \tag{3c}$$

$$\sum(Y_L{}^3-Y_R{}^3)=0\rightarrow(\lambda_1{}^3+\lambda_2{}^3)[2a^3-a'^3+3(2A^3-A'^3-A''^3)]=0. \tag{3d}$$

We note that two different sets $(a,\ldots,A'')$ may correspond to U(1) and U′(1). In addition, there are two more independent anomaly constraints that involve both Y and Y' which we will discuss soon. First note that Eqs. (3a)–(3d) have exactly two types of solutions:

(i) $\lambda_1+\lambda_2\neq 0$.—In this case, we obtain a unique solution for the set $(a,\ldots,A'')$ up to a common scale,

$$(a,a,a';A,A,A',A'')\sim(-1,-1,-2;\tfrac{1}{3},\tfrac{1}{3},-\tfrac{2}{3},\tfrac{4}{3}), \tag{4}$$

which are precisely the same set as in WS $SU(2)_L\otimes U(1)$ for Y of each generation. The anomalies are canceled within each generation irrespective of the values of λ_1 and λ_2.

(ii) $\lambda_1+\lambda_2=0$.—In this case Eqs. (3a), (3b), and (3d) are automatically satisfied leaving one constraint

$$a^2-a'^2=A^2+A'^2-2A''^2 \tag{5}$$

which can be satisfied in many ways different from (4). It should be noted that the special property of this type of solution is that the anomaly of one generation cancels that of the other irrespective of the explicit values for $(a,\ldots,A'')$.

It is clear that if we want to retain the WS assignments, we have to choose type (i) for Y of U(1). This leaves type (ii) for Y' if we want to distinguish the generations. The coupled anomaly equations referred to earlier are

$$(\lambda_1\lambda_1'+\lambda_2\lambda_2')k=(\lambda_1{}^2\lambda_1'+\lambda_2{}^2\lambda_2')k=0, \tag{6}$$

where

$$k=[a-a'+\tfrac{1}{3}(5A-A'-4A'')]-\tfrac{1}{2}(a+3A). \tag{7}$$

In (7), the set $(a,\ldots,A'')$ refers to Y'. We have used (4) in determining k. Now we observe the interesting fact that if $k\neq 0$, for type-(ii) solution (that is $\lambda_1'=-\lambda_2'$), the corresponding type-(i) solution has the property $\lambda_1=\lambda_2$, which implies universality in the WS model.

The Higgs doublets are degenerate with respect to WS QN. Thus if φ_i denote ith-generation Higgs doublets, its QN are given by

$$(T, Y, Y') = (\tfrac{1}{2}, 1, \lambda_i h), \quad \lambda_{1,2} = \pm 1, \tag{8}$$

where h is some nonzero number. We now require the following:

(i) *Lepton-number conservation.*—This leads to

$$a - a' = h, \quad a + a' \neq \pm h, \tag{9}$$

which implies (choosing $a = 1$ without any loss of generality)

$$h \neq 0, 1. \tag{10}$$

In deriving (9), we have required that φ_1 and φ_2 couple in the standard manner to give rise to e and μ masses, respectively. Since each generation of leptons is distinguished by Y' $(a, a, a'; -a, -a, -a')$ are Y' QN of $(\nu_L{}^e, e_L, e_R; \nu_L{}^\mu, \mu_L, \mu_R)$. In deriving (10) we have required that there exist no Higgs field which gives rise to a physical e-μ transition.[9]

(ii) *The quark sector.*—Here the important considerations concern the structures of the mass matrices in different charge sectors which leads to Cabibbo rotations. By examining the Y' quantum numbers, it is straightforward to deduce the following:

(a) Mixing in the d-s (or charge $-\frac{1}{3}$) sector, either

$$A - A' = A + A' = \pm h \Rightarrow A' = 0 \text{ and } A = \pm h \tag{11}$$

or

$$A - A' = -A - A' = \pm h \Rightarrow A = 0 \text{ and } A' = \mp h; \tag{12}$$

(b) mixing in the u-c (or charge $\frac{2}{3}$) sector, either

$$A - A'' = A + A'' = \pm h \Rightarrow A'' = 0 \text{ and } A = \pm h, \tag{13}$$

or

$$A - A'' = -A - A'' = \pm h \Rightarrow A = 0 \text{ and } A'' = \mp h. \tag{14}$$

The condition (5) combined with (9) can be written as

$$A^2 + A'^2 - 2A''^2 = h(2 - h). \tag{15}$$

Now the physical Cabibbo rotation is a combined result of both up- and down-quark sectors, and hence we may have the following situations:

(i) *Mixing in both quark sectors.*—If constraints (11)–(14) are used in (15), it follows that $h = 0, 1$ which contradicts (9). Thus the conservation of lepton number does not permit within our model Cabibbo-like rotations in both the quark sectors simultaneously.

(ii) *Mixing only in the up-quark sector.*—

$$[(A'' = 0,\ A = \pm h) \text{ or } (A'' = \pm h,\ A = 0)] \tag{16a}$$

and

$$[(A - A' = h,\ A + A' \neq \pm h) \text{ or } (A + A' = h,\ A - A' \neq \pm h)]. \tag{16b}$$

The restrictions in (16) involving A and A' assures that the mass matrix M is such that $MM^\dagger$ is diagonal in the down-quark sector, and hence no Cabibbo rotation occurs in the down-quark sector. These conditions and (15) give

$$A = -\tfrac{1}{3}, \quad A' = \pm\tfrac{2}{3}, \quad A'' = 0, \quad h = \tfrac{1}{3}; \quad a = 1, \quad a' = \tfrac{2}{3}. \tag{17}$$

(iii) *Mixing only in the down-quark sector.*—Then using (15), we obtain

$$A = -\tfrac{1}{3}, \quad A' = 0, \quad A'' = \pm\tfrac{2}{3}, \quad h = -\tfrac{1}{3}; \quad a = 1, \quad a' = \tfrac{4}{3}. \tag{18}$$

Note that prior to spontaneous symmetry breaking, in (17) u_R and c_R and in (18) d_R and s_R remain indistinguishable. Thus in both cases the Cabibbo rotation mixes those quarks whose right-handed partners have the same Q, Y, and Y'.

Since $\langle\varphi_1{}^0\rangle \equiv V_1$ and $\langle\varphi_2{}^0\rangle \equiv V_2$ give rise to e and μ masses, respectively, it is natural to assume that $V_1 \ll V_2$. It then follows that depending upon whether we choose (17) or (18), we obtain

$$\tan\theta_C \sim \eta(m_u/m_c) \text{ or } \tan\theta_C \sim \eta(m_d/m_s), \tag{19}$$

where η is independent of V_1, V_2 and depends only on the Yukawa couplings.

We do not make any assumptions concerning any of the coupling constants in the model except that they are all roughly of the same order of magnitude. From a close examination of the expressions for η we can see that its magnitude is of order unity. If we believe the present information concerning the quark masses, the Cabibbo angle is best represented by $\tan\theta_C \sim \eta(m_d/m_s)$. This leaves us with only the last alternative. Then the hierarchy $V_1 \ll V_2$ correlates the smallness in m_e with the smallness of m_u, m_d, and θ_C, and more generally $m_e : m_\mu \ldots \simeq m_u : m_c \ldots$, since the relevant mass matrices $MM^\dagger$ are diagonal. The corresponding quantum numbers are summarized in Table I.

It is interesting to notice that for $A'' = -\frac{2}{3}$, the horizontal Y' generator can be expressed as

$$Y' = \mp \tfrac{1}{3}[Y + 2(B - L)], \tag{20}$$

TABLE I. The fermion classification in a two-generation scheme.

	T_3	Y	Y'
$\nu_L{}^e, \nu_L{}^\mu$	1/2	−1	+1, −1
e_L, μ_L	−1/2	−1	+1, −1
e_R, μ_R	0	−2	+4/3, −4/3
μ_L, c_L	1/2	1/3	−1/3, +1/3
d_L, s_L	−1/2	1/3	−1/3, +1/3
d_R, s_R	0	−2/3	0, 0
u_R, c_R	0	4/3	∓2/3, ±2/3

where (∓) signs are assigned to the first and the second generations, respectively, indicating also that Y' is a linear combination of already known physical quantum numbers. However, it turns out this QN assignment leads to $k=0$ in (6). Consequently, the relation $\lambda_1=\lambda_2$ does not follow and hence must be imposed. On the other hand, the Y' assignments with $A''=+\frac{2}{3}$ imply $k=-\frac{16}{9}\neq 0$ which guarantees the WS universality. Equation (20) can still hold even for the latter $A''=+\frac{2}{3}$ case provided $u_R \leftrightarrow c_R$ in the definition of fermion generations. As mentioned before, $A''=+\frac{2}{3}$ and $A''=-\frac{2}{3}$ lead to the same $MM^\dagger$ matrix in the up-quark sector.

Horizontal gauge interactions introduce, in general, several undesirable features in the weak neutral-current sector. These include the following: (a) Flavor-changing interactions; (b) violation of e-μ universality. One can suppress these unwanted effects, however, by introducing minimally a Higgs singlet[10] Φ with $Y=0$ whose large VEV is expected to provide the heavy-mass scale for Z' without affecting the fermion mass matrix. The relevant small parameter ϵ which characterizes the magnitude of the above effects turns out to be

$$\epsilon \simeq \left(\frac{M^2(W^\pm)-\cos^2\theta_W M^2(Z)}{\cos^2\theta_W M^2(Z')}\right)^{1/2},$$

with

$$1-\cos^2\theta_W M^2(Z)/M^2(W^\pm)=O(\epsilon), \quad M^2(Z')/M^2(W^\pm)=O(1/\epsilon). \tag{21}$$

A single Z' exchange gives rise to a contribution of order $\epsilon G_F \cos^2\theta_C \sin^2\theta_C$ to the K_L-K_S mass differences.[11] Thus an upper bound for ϵ can be obtained, namely,

$$\epsilon \sim O(m_c{}^2 G_F) \Rightarrow \epsilon < 10^{-5}.$$

Correspondingly, we expect Z' mass to be at least two to three orders of magnitude heavier than that of the WS gauge bosons, and the standard WS model is fully recovered up to an ultrafine structure.

Finally, we discuss a simple generalization of our model for $n>2$ generations. Following a generalized version of Eq. (3a)–(3d), the U′(1) seriality numbers must obey

$$\sum \lambda_i' = \sum \lambda_i'^3 = 0. \tag{22}$$

For $n>3$, (22) is not sufficient to determine λ_i' uniquely, and the model becomes somewhat arbitrary. For $n=3$, although, it leads to $\lambda_{1,2,3}'=1$, 0, and −1 and the coupled anomaly equations cannot guarantee WS universality. Thus our conclusion is that a generalized WS model of SU(2) ⊗ U(1) ⊗ U′(1) type in which there exists a unique solution for the seriality numbers can be realized for two generations at most. Nevertheless, the model can be easily generalized by introducing a minimal horizontal gauge symmetry $[\mathrm{U}'(1)]^N$ to accomodate $n=2^N$ generations. This does not involve new physical quantities, and the distinction among various generations can be naturally established by using N families of the discussed seriality numbers. For $n=4$ (i.e., $N=2$), for example, we define $Y_i'=\lambda_i'(1,\ldots,\mp\frac{2}{3})$ and $Y_i''=\lambda_i''(1,\ldots,\mp\frac{2}{3})$ for ith generation, such that $\lambda_{1,2,3,4}'=1,-1,1,-1$ and $\lambda_{1,2,3,4}''=1,1,-1,-1$. In this example, the two-generation model simply repeats itself. Thus, from our point of view, the existence of a third generation suggests the existence of the fourth one as well. The interesting feature about such models are that (a) the number of generations grow very fast, and (b) a minimal number of additional gauge bosons is required. The asymptotic-freedom restriction[12] on the number of flavors ($N_f<17$) as well as the cosmological limits[13] on the number of neutrinos ($N_\nu<4$) may then determine almost uniquely the number of fermion generations.

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Detection of Weak Neutral Current Using Fission $\bar{\nu}_e$ on Deuterons

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The weak-neutral-current reaction $\bar{\nu}_e + d \to n + p + \bar{\nu}_e$ has been observed concurrently with the charged-current process $\bar{\nu}_e + d \to n + n + e^+$ using an instrumented D_2O target exposed to an $\bar{\nu}_e$ flux of 2.5×10^{13} cm^{-2} sec^{-1}. The measured neutral-current cross section, $(3.8\pm0.9)\times10^{-45}$ cm$^2/\bar{\nu}_e$, is consistent with the Weinberg-Salam model, dependent in this case only on the axial-vector contribution. The charged-current reaction cross section is $(1.5\pm0.4)\times10^{-45}$ cm$^2/\bar{\nu}_e$, in fair agreement with expectation.

We report the detection of the weak-neutral-current (NC) reaction $\bar{\nu}_e + d \to n + p + \bar{\nu}_e$ and a concurrent measurement of the charged-current reaction (CC) $\bar{\nu}_e + d \to n + n + e^+$.

This experiment was conducted at the 2000-MW fission reactor at the Savannah River Plant in the well-shielded environment used in the $\bar{\nu}_e + e^-$ experiment.[1] The drastic background reduction made possible by this shielding allowed the detection of the NC reaction with use of only the product neutron as a signature. The feasibility of this method was demonstrated in 1974,[2] when an upper limit for the weak-neutral-current reaction was determined.

Weak neutral currents have been observed with very high-energy muon-type neutrinos at CERN[3] and at the Fermi National Accelerator Laboratory.[4] Until the present work the only $\bar{\nu}_e$ reaction involving neutral currents was $\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$.[1]

Theoretical analysis of other neutral-current reactions by Hung and Sakurai[5] determines the neutral-current couplings between neutrinos and hadrons within a twofold "vector-axial ambiguity."[5,6] More recently, a large number of results on neutral-current interactions have been analyzed and the ambiguity appears to be eliminated in favor of the Weinberg-Salam model.[7]

In this experiment, using low-energy reactor $\bar{\nu}_e$'s, the NC cross section is unique in that it depends only on the axial-vector (Gamow-Teller) contribution[8] and is therefore independent of the Weinberg angle. In this case, the ambiguity is particularly easy to resolve; the Weinberg-Salam solution predicts a cross section for this reaction which is four times larger than the alternative possibility.

The detector system is shown in Fig. 1. The target consists of 268 kg of extremely pure (99.85%) heavy water. Immersed in the D_2O are ^{3}He-filled gas proportional counters[9] which detect the neutron via the reaction $^3\mathrm{He} + n \to p + {}^3\mathrm{H} + 764$ keV. The entire detector is enclosed in a