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Effect of Flavor Mixing on Proton Decay in SU(5) Grand Unified Theories

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I discuss the effect of flavor mixing on proton-decay Hamiltonian in SU(5) grand unified theories with arbitrary number of fermion families and show that in the simplest model with only one $\{\underline{5}\}$ Higgs multiplet, all fermion interactions (with Higgs as well as gauge bosons) are describable in terms of a single mixing matrix. For the more complicated case of several $\{\underline{5}\}$ multiplets, the above feature persists for gauge-boson interactions though not for Higgs bosons.

Gauge theories based on the local SU(5) symmetry group have received much attention recently as a possible candidate for grand unification of weak, electromagnetic, and strong interactions.[1-3] At low energies (i.e., $E \lesssim 10^2$ GeV), only the $SU(3) \otimes SU(2)_L \otimes U(1)$ subgroup of the grand unifying group is supposed to manifest itself and the associated gauge bosons $V_\mu{}^i$ ($i = 1, \ldots, 8$), $W_\mu{}^\pm$, Z, and the photon are believed to mediate strong, weak, and electromagnetic interactions of the fermions (quarks and leptons). The remaining twelve gauge bosons (denoted by complex fields $X_i^{(+4/3)}$ and $Y_i^{(1/3)}$, $i = 1, \ldots, 3$) mediate interactions which do not conserve either baryon or lepton numbers and are responsible for proton decay. Then, of course, there are the Higgs bosons interacting with fermions. Thus, the complexion of flavor interactions in this model is quite rich. When one includes different generations of fermions in this model in a sequential fashion, mixings between different generations will add further complications to the interactions. In fact, given that normal charged-current weak interactions are characterized by generalized Cabibbo-like mixing angles θ_i (θ_i small), it is *a priori* not obvious that baryon-number–nonconserving interactions would also be described by the same set of angles. In general, one would expect them to depend on the mixing angles for $I_{3W} = \pm\frac{1}{2}$ fermion sectors separately. It is then conceivable (and indeed has been suggested[4]) that the structure of proton-decay Hamiltonian could be very different from expectations based only on one generation of fermions and that the most crucial prediction of the model concerning the proton lifetime could possibly be affected by inclusion of these mixings.

In this Letter, I study this question of flavor mixing for an arbitrary number of generations of fermions and its impact on the nature of gauge boson as well as Higgs-boson interactions with fermions. The main result is that in the simplest SU(5) model with a single $\{\underline{5}\}$ Higgs multiplet, *all* fermion interactions (both Higgs as well as gauge boson) are characterized by a single mixing matrix. As a consequence, the dominant part of the proton-decay Hamiltonian is the same as with only a single generation of fermions (i.e., u, d, e, ν). For the more complicated case of several $\{\underline{5}\}$ Higgs multiplets coupling to fermions, the above feature persists for the gauge-boson interactions though not for the Higgs bosons. Thus, the predictions for the proton lifetime are not affected by flavor mixing in the presence of several generations of fermions and Higgs bosons. I now proceed to outline the proof of this result.

To establish notation, let us take M families of fermions, each family prior to mixing, being denoted by

$$(p_i, n_i, L^0, \epsilon^-)_a, \quad a=1,\ldots,M, \tag{1}$$

where i is the color index, L^0 stands for neutrino-type leptons, and ϵ stands for charged leptons. The physical fermion fields which are eigenstates of the mass matrix will be denoted by (u_i, d_i, ν_e, e^-), $(c_i, s_i, \nu_\mu, \mu^-)$, $(t_i, b_i, \nu_\tau, \tau^-)$, $\ldots$. Assignment of fermions under SU(5) is given for each family by

$$\psi^a = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ \epsilon^+ \\ L^0 \end{pmatrix}_R^a; \quad \chi^a = \begin{pmatrix} 0 & p_3^c & -p_2^c & p_1 & n_1 \\ -p_3^c & 0 & p_1^c & p_2 & n_2 \\ p_2^c & -p_1^c & 0 & p_3 & n_3 \\ -p_1 & -p_2 & -p_3 & 0 & \epsilon^+ \\ -n_1 & -n_2 & -n_3 & -\epsilon^+ & 0 \end{pmatrix}_L . \tag{2}$$

We are now ready to write down the flavor gauge-boson interaction of fermions. Using the standard notation[3] for the bosons, we obtain, for the charged-current part of the interaction Lagrangian,

$$\mathfrak{L}_I = i2^{-1/2} g W_\mu^+ (\overline{Q}_{1L}\gamma_\mu Q_{2L} + \overline{M}_{1L}{}^c \gamma_\mu M_{2L}{}^c) + i2^{-1/2} g_X X_{\mu i}^- (\epsilon_{ijk} \overline{Q}_{1kL}{}^C \gamma_\mu Q_{1jL} + \overline{Q}_{2i}\gamma_\mu M_1)$$
$$+ i2^{-1/2} g_Y Y_{\mu i}^- (\epsilon_{ijk} \overline{Q}_{1kL}{}^C \gamma_\mu Q_{2jL} - \overline{Q}_{1iL}\gamma_\mu M_{1L} + \overline{Q}_{2iR}\gamma_\mu M_{2R}) + \text{H.c.}, \tag{3}$$

where Q and M are column vectors for unmixed like-charge fermions of different generations (and $\psi^C = C\overline{\psi}^T$):

$$\begin{aligned} Q_{1i} &= (p_i{}^1, p_i{}^2, \ldots), \\ Q_{2i} &= (n_i{}^1, n_i{}^2, \ldots), \\ M_1 &= ((\epsilon^+)^1, (\epsilon^+)^2, \ldots), \\ M_2 &= ((L^0)^1, (L^0)^2, \ldots) \end{aligned} \tag{4}$$

(superscripts stand for generations and subscripts for color).

The fermion mass term in the Lagrangian subsequent to spontaneous breakdown can be written as

$$\mathfrak{L}_m = \overline{Q}_{1L} M^{(+)} Q_{1R} + \overline{Q}_{2L} M^{(-)} Q_{2R} + \overline{M}_{1L} M^{(-)} M_{1R}. \tag{5}$$

The above mass matrices will in general be complex and can be diagonalized by the following biunitary transformations:

$$U_L M^{(+)} U_R{}^\dagger = D^{(+)}, \tag{6a}$$

$$V_L M^{(-)} V_R{}^\dagger = D^{(-)}. \tag{6b}$$

The physical fermions are then given by

$$\begin{aligned} P_L &= U_L Q_{1L}, \\ N_L &= V_L Q_{2L}, \\ E^+{}_L &= V_L M_{1L}, \end{aligned} \tag{7}$$

and similarly for right-handed helicity components, where

$$P = (u, c, t, \ldots),$$
$$N = (d, s, b, \ldots),$$
$$E^+ = (e^+, \mu^+, \tau^+, \ldots),$$

and

$$D^{(+)} = \mathrm{diag}(m_u, m_c, m_t, \ldots), \quad D^{(-)} = \mathrm{diag}(m_d, m_s, m_b, \ldots). \tag{8}$$

In terms of the physical fermion fields, Eq. (3) can be rewritten as

$$\begin{aligned} \mathfrak{L}_I = {} & i2^{-1/2} g\, W_\mu^{\,+} (\overline{P}_L \gamma_\mu U_L V_L^{\,\dagger} N_L + \overline{E}^+{}_R \gamma_\mu E^0{}_R) \\ & + i2^{-1/2} g_X X_{\mu i}^{\,-} (\epsilon_{ijk} \overline{P}_{kL}^{\,C} \gamma_\mu U_R{}^* U_L^{\,\dagger} P_{jL} + \overline{N}_i \gamma_\mu E^+) \\ & + i2^{-1/2} g_Y Y_{\mu i}^{\,-} (\epsilon_{ijk} \overline{P}_{kL}^{\,C} \gamma_\mu U_R{}^* V_L^{\,\dagger} N_{jL} - \overline{P}_{iL} \gamma_\mu U_L V_L^{\,\dagger} E^+{}_L + \overline{N}_{iR} \gamma_\mu E^0{}_R) + \mathrm{H.c.}, \end{aligned} \tag{9}$$

where $E^0{}_R = V_R M_{2R}$. Note that in this model since all neutrinos are massless, $E^0{}_R$ is also an eigenstate of the Hamiltonian. From Eq. (9), we find that charged-current weak interactions are described by the matrix $U_L V_L^{\,\dagger}$ which for two families can be made completely real by suitable redefinitions of the phases of the fermion fields. For three families, it can be written in the form first suggested by Kobayashi and Maskawa.[5] The interaction of X and Y bosons, however, contain mixing matrices like $U_R{}^* U_L^{\dagger}$ or $U_R{}^* V_L^{\,\dagger}$, etc., which will involve mixing angles very different from those in $U_L V_L^{\,\dagger}$. However, we now show that the SU(5) model possesses an intrinsic symmetry for the mass matrix of the $I_{3W} = +\frac{1}{2}$ quarks, which enables us to simplify Eq. (8) further. To see this, note that, the Yukawa coupling leading to $M^{(+)}$ is

$$\sum_{a,b} h_{ab} \epsilon^{pqrst} (\chi^T)_{pq}{}^a \varphi_r C^{-1} \chi_{st}{}^b, \tag{10}$$

where C is the Dirac charge-conjugation matrix. An examination of Eq. (10) reveals that the mass matrix $M^{(+)}$ following from it has the following property,

$$(M^{(+)})_{ab} = v(h_{ab} + h_{ba}) = (M^{(+)})_{ba}, \tag{11}$$

where $v \equiv \langle \varphi_5 \rangle$. It the follows that

$$U_L = K U_R{}^*, \tag{12}$$

where K is a diagonal unitary matrix. Using Eq. (12), we find that

$$U_R{}^* U_L^{\,\dagger} = K^{-1}$$

and

$$U_R{}^* V_L^{\,\dagger} = K^{-1} U_L V_L^{\,\dagger}. \tag{13}$$

Thus, up to a diagonal unitary matrix K, all gauge-boson interactions in Eq. (9) are expressible in terms of a single mixing matrix $U_L V_L^{\,\dagger}$. From this, it follows that, since the mixing angles between the light-mass quarks ($u, d, \ldots$) and the heavier ones ($c, s, \ldots$), etc., are small, the proton-decay Hamiltonian is not substantially altered in the presence of more generations of fermions. This is the main result of this Letter.

I now proceed to show that for the simplest Higgs system with one $\{\underline{5}\}$ and one $\{\underline{24}\}$ Higgs multiplet, even the Higgs-boson–fermion interactions become very simple. As is well known, only the $\{\underline{5}\}$ multiplet φ interacts with fermions prior to spontaneous breakdown:

$$\mathfrak{L}_Y = \sum_{a,b} f_{ab} \overline{\chi}^{a,pq} \varphi_p \psi_q + \sum_{a,b} h_{ab} \epsilon^{pqrst} (\chi^T)_{pq}{}^a \varphi_r C^{-1} \chi_{st}{}^b + \mathrm{H.c.} \tag{14}$$

We write

$$\varphi_p = \begin{pmatrix} H_i \\ \varphi^4 \\ \varphi^5 \end{pmatrix},$$

where H_i are the heavy, colored, fractionally charged Higgs mesons that couple to baryon-number–nonconserving currents. We will concern ourselves with their interactions with the physical fermions. From straightforward algebra, it follows that their interaction with physical fermions can be written as

$$\mathcal{L}_Y = -H_i(\epsilon_{ijk}\bar{P}_{kL}{}^C U_R{}^*\mathfrak{F}V_R{}^\dagger N_{jR} + \bar{P}_{iL}U_L\mathfrak{F}V_R{}^\dagger E^+{}_R + \epsilon_{ijk}(P^T)_{jL}C^{-1}U_L{}^*\mathcal{H}V_L{}^\dagger N_{kL} + \epsilon_{ijk}(N^T)_{jL}C^{-1}V_L{}^*\mathcal{H}U_L{}^\dagger P_{kL} + (P^T)_{ijL}{}^C C^{-1}U_R\mathcal{H}V_L{}^\dagger E^0{}_L) + \text{H.c.} \tag{15}$$

where $(\mathfrak{F})_{ab}=f_{ab}$; $(\mathcal{H})_{ab}=h_{ab}$. But note that $\mathfrak{F}=v^{-1}M^{(-)}$ and $\mathcal{H}=M^{(+)\dagger}v^{-1}$. Therefore, using Eq. (6) and (12), we find

$$U_R{}^*\mathfrak{F}V_R{}^\dagger = K^{-1}(U_LV_L{}^\dagger)D^{(-)}v^{-1}, \tag{16a}$$

$$U_L\mathfrak{F}V_R{}^\dagger = (U_LV_L{}^\dagger)D^{(-)}v^{-1}, \tag{16b}$$

$$U_L{}^*\mathcal{H}V_L{}^\dagger = D^{(+)}K^{-1}(U_LV_L{}^\dagger)v^{-1}, \tag{16c}$$

$$V_L{}^*\mathcal{H}U_L{}^\dagger = (U_LV_L{}^\dagger)^TK^{-1}D^{(+)}v^{-1}, \tag{16d}$$

$$U_R\mathcal{H}V_L{}^\dagger = D^{(+)}(U_LV_L{}^\dagger)v^{-1}. \tag{16e}$$

Thus, all Higgs baryon-number–nonconserving interactions become also expressible in terms of the same mixing matrix $U_LV_L{}^\dagger$ that describes the charged-current weak interactions. Therefore, the Higgs contribution to the proton decay can also be safely estimated using the model with a single generation of fermions. It is now obvious that if there are more than one $\{\underline{5}\}$ Higgs multiplets coupling to fermions, Eqs. (16a)–(16e) do not hold and, therefore, the nature of the baryon-number–nonconserving Higgs-boson interactions becomes more complicated.

We would now like to comment on the impact of adding Higgs fields belonging to $\{\underline{45}\}$ representation[7] of SU(5), denoted by $\Sigma_{qr}{}^p$. This will generate new kinds of Yukawa couplings involving $\{\underline{5}^*\}$ and $\{\underline{10}\}$ as well as those involving only the $\{\underline{10}\}$ representations of fermions. These interactions will be given by $\mathcal{L}_Y'$:

$$\mathcal{L}_Y' = \sum_{a,b} f_{ab}'\bar{\chi}^{a,pq}\psi_r\Sigma_{pq}{}^r + \sum_{a,b} h_{ab}'\epsilon^{pqrst}(\chi^T)_{pq}{}^aC^{-1}\chi_{lr}{}^b\Sigma_{st}{}^l + \text{H.c.} \tag{17}$$

The Σ fields acquire vacuum expectation values as follows: $\langle\Sigma_{45}{}^4\rangle = -3\langle\Sigma_{i5}{}^i\rangle$ (no sum over i). By substituting $\langle\Sigma\rangle$ into Eq. (17), it becomes clear that the contribution of $\mathcal{L}_Y'$ to the up-quark mass matrix $M^{(+)}$ does not satisfy Eq. (11). This enables us to draw the following conclusions: (a) If the $\{\underline{45}\}$ Higgs Σ contributes *only* to the down-quark mass matrix $M^{(-)}$ (as in the model of Georgi and Jarlskog[7]), my conclusions about the dominant structure of the proton-decay Hamiltonian *remain valid*.[8] (b) On the other hand, if Σ contributes to $M^{(+)}$, then the situation will be very different and my conclusions will not hold in general.

In summary, I show that increasing the number of fermion generations and inclusion of flavor-mixing effects do not change the estimates of proton lifetime calculated on the basis of a single generation of fermions. Crucial to this conclusion was the observation that the mass matrix for the up quarks in SU(5) model is necessarily symmetric for $\{\underline{5}\}$ Higgs mesons.

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