Coherent Quark-Gluon Jets

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With use of the coherent-state formalism, general expressions are presented for arbitrary massless quark and gluon jet cross sections. Also, normalized probability distributions are given for the jet transverse momentum which take due account of correlations imposed by the momentum conservation. These results are used to discuss successfully the SPEAR data of Hanson *et al.* between 3 and 7.8 GeV.

Much attention has been directed recently to study theoretically the jet structure in quantum chromodynamics (QCD) perturbation theory.[1] In the present work, we describe the results of an approach based on the coherent-state formalism.[2,3] A simple formula is obtained for a general jet cross section in terms of an infinite number of gluons and massless quarks. We also obtain probability distributions for the jet transverse momentum which include correlations induced by the momentum conservation. A phenomenological extrapolation of our jet K_1 distribution in the nonperturbative region is then presented. This assumes that the hadronic transverse-momentum distribution follows that of the QCD radiation[4] and uses a parametrization of $\overline{\alpha}(k_\perp)$ obtained in an earlier work.[5] We find excellent agreement with the accurate SPEAR data for the two-jet process $e^+e^- \to q\overline{q} \to$ hadrons, for total energies 3–7.8 GeV.[6]

In Ref. 2, coherent states $|\overline{i}\rangle$, consisting of an indefinite number of soft gluons, corresponding to a "pure" state $|i\rangle$ were constructed to obtain ir-finite inclusive cross sections in QCD. For example, for a quark in a color-singlet potential, the inclusive cross section was found to be[2]

$$\sigma_{\text{inc}} \propto |\langle \overline{f}|S|\overline{i}\rangle|^2 = \exp\{\int d^3k\,(2k)^{-1}[j_\mu{}^c(k)^\dagger]\overline{g}_{\text{YM}}{}^2(k)\}\,|\langle f|S|i\rangle|^2, \tag{1}$$

where $j_\mu{}^c(k)$ is the classical color current operator and $\overline{g}_{\text{YM}}(k)$ is the effective coupling constant for pure Yang-Mills theory. The finite corrections in Eq. (1) after the cancellation of ir singularities can be written as

$$\exp\left[-\frac{C_F}{2\pi^2}\int_{\Delta\omega}^{E}\left(\frac{dk}{k}\right)\int_{m^2}^{-q^2}\frac{dk_\perp{}^2}{k_\perp{}^2}\overline{g}_{\text{YM}}{}^2(k)\right], \tag{2}$$

where $C_F = (N_C{}^2-1)/2N_C$ for SU(N_C) color, m and $\Delta\omega$ are the mass and energy loss of the quark, and q^2 is the momentum transfer.

To discuss the massless limit $m \to 0$ new coherent states have to be defined[3] which take into account the collective effect of hard and collinear gluons, i.e., the set of states degenerate with the initial and final states.[7] As discussed in Ref. 3 the mass singularities exactly cancel between soft and hard contributions provided $g_{\text{YM}}(k)$ in the above Eq. (2) is replaced by the full coupling constant $\overline{g}(k_\perp)$, since, as $m \to 0$, quark loops in addition to the gluons must be included.

Thus, for $e^+e^- \to \overline{q}q$ we have ir- and mass-singularity–free "super" inclusive cross sections given by

$$d\sigma_{\text{super}}{}^{(2q)} = d\sigma_0 \exp\left[-\frac{1}{\pi^2}\int_{\Delta\omega/E}^{1} dx\,P_{gq}(x)\int_{k_{\perp1}{}^2}^{k_{\perp2}{}^2}\frac{(d^2k_\perp)}{k_\perp{}^2}\overline{\alpha}(k_\perp)\right], \tag{3}$$

where the gluon distributions due to the quarks is[8]

$$P_{gq}(x) = C_F[1+(1-x)^2]/x. \tag{4}$$

In Eq. (3) $k_{\perp1} = E\delta$, $k_{\perp2} = Q/2 = E$, E is the quark energy, δ is the half angle of the jet cone, and $d\sigma_0$ is the "pointlike" cross section.

The ratio $d\sigma_{\text{super}}/d\sigma_0$ in Eq. (3) gives the probability of finding a fraction $\epsilon = \Delta\omega/Q$ of the total energy Q outside a pair of oppositely directed cones of half angle δ. First order expansion of Eq. (5) directly gives the result of Sterman and Weinberg[1] up to terms of order $\epsilon\ln\delta$. A more accurate kinematical analysis shows that the upper limit in x should be restricted to $1-\Delta\omega/E=1-2\epsilon$. This gives

$$I_q(\epsilon)\equiv\int_{2\epsilon}^{1-2\epsilon}dx\,P_{gq}(x)=-C_F[2\ln2\epsilon+\tfrac{3}{2}-2\epsilon+4\epsilon^2+O(\epsilon^3)],\tag{5}$$

in agreement with Stevenson.[1] From now, we exponentiate with the right kinematics and certainly our results will remain valid for small ϵ but they may not account for all ϵ terms.

The generalization of Eq. (3) to gluon jets is obtained as follows. The probability $P_{gq}(x)$ in Eq. (4) has to be replaced by

$$P_{gg}(x)+N_fP_{qg}(x)=N_C\left[\frac{x}{1-x}+\frac{1-x}{x}+x(1-x)\right]+N_f\frac{x^2+(1-x)^2}{2},\tag{6}$$

corresponding to the sum of probabilities that a gluon radiates gluons radiates gluons or $N_f(q\bar{q})$ pairs. Then Eq. (5) is replaced by

$$I_g(\epsilon)\equiv\int_{2\epsilon}^{1-2\epsilon}dx[P_{gg}(x)+N_fP_{qg}(x)]$$
$$=-N_C\left[2\ln2\epsilon+\left(\frac{11}{6}-\frac{1}{3}\frac{N_f}{N_C}\right)-4\epsilon\left(1-\frac{N_f}{2N_C}\right)+8\epsilon^2\left(1-\frac{N_f}{2N_C}\right)+O(\epsilon^3)\right].\tag{7}$$

The logarithmic and constant terms (in ϵ) in Eq. (7) agree with the explicit perturbative calculations of Shyzuya and Tye, Einhorn and Weeks, and Smilga and Vysotsky.[1] Finite order terms in ϵ have, as yet, not been calculated in perturbation theory. Notice that our distributions in ϵ for $I_q(\epsilon)$ and $I_g(\epsilon)$ have been obtained without any recourse to singular distributions.[8] This difference arises because we do not integrate to $x=1$.

The above results generalize for n_q and n_g quark and gluon jets, respectively, to

$$d\sigma_{\text{super}}^{(n_q,n_g)}=d\sigma_0\exp\left\{-\frac{1}{\pi}[n_qI_q(\epsilon)+n_gI_g(\epsilon)]\int_{k_{\perp1}}^{k_{\perp2}}\left(\frac{dk_\perp}{k_\perp}\right)\bar{\alpha}(k_\perp)\right\},\tag{8}$$

where for simplicity the same kinematical limits $k_{\perp1}$ and $k_{\perp2}$ are used for all jets. The curly bracket in Eq. (8) gives the leading terms in ($\ln\delta$). All nonleading terms in δ are included in $d\sigma_0$ and have to be calculated, if need be, for each process separately in perturbation theory. First-order expansion of Eq. (18) agrees with the results of Smilga and Vysotsky.[1]

From now on we only consider $q\bar{q}$ jets in e^+e^- annihilation. From Eq. (3), we can define a probability distribution $dP(K_\perp)$,

$$\int_0^{K_{\perp1}}\left(\frac{dP}{dK_\perp}\right)dK_\perp\equiv\exp\left[-\frac{2}{\pi}I(\epsilon)\int_{K_{\perp1}}^{Q/2}\frac{dk_\perp}{k_\perp}\bar{\alpha}(k_\perp)\right],\tag{9}$$

so that

$$\frac{dP}{dK_\perp}\simeq\frac{2I(\epsilon)}{\pi}\left[\frac{\bar{\alpha}(K_\perp)}{K_\perp}\right]\left[\frac{\bar{\alpha}(Q/2)}{\bar{\alpha}(K_\perp)}\right]^{I(\epsilon)/\pi b},\tag{10}$$

where $b=(33-2N_f)/12\pi$ for three colors, $I(\epsilon)\equiv Iq(\epsilon)$ and we have used the asymptotic-freedom formula for $\bar{\alpha}(k_\perp)$. Thus, Eq. (10) should be valid only for $\Lambda\ll K_\perp\ll Q/2$.

So far, we have omitted any correlation in transverse momentum, since it was assumed that the individual emissions were statistically independent. We can impose transverse-momentum conservation in a manner similar to that of Greco, Pancheri-Srivastava, and Srivastava.[9] This leads us to the following results, the details of which shall be presented elsewhere. We obtain

$$\frac{d^2P}{d^2K_\perp}=\frac{1}{2\pi}\int_0^\infty x_\perp dx_\perp J_0(x_\perp K_\perp)\exp\left\{-\frac{2I(\epsilon)}{\pi}\int_0^{Q/2}\left(\frac{dk_\perp}{k_\perp}\right)\bar{\alpha}(k_\perp)[1-J_0(x_\perp k_\perp)]\right\},\tag{11}$$

and

$$P(\epsilon,K_{\perp1})=K_{\perp1}\int_0^\infty dx_\perp J_1(x_\perp K_{\perp1})\exp\left\{-\frac{2}{\pi}I(\epsilon)\int_0^{Q/2}\left(\frac{dk_\perp}{k_\perp}\right)\bar{\alpha}(k_\perp)[1-J_0(x_\perp k_\perp)]\right\}.\tag{12}$$

Equation (12) gives the probability of finding a fraction $1-\epsilon$ of the total energy inside two opposite cones of maximum transverse momentum $K_{\perp 1}$, and replaces our earlier Eq. (3) for $(d\sigma_{\rm super}/d\sigma_0)$. Similarly, Eq. (11), which is properly normalized to 1 for any fixed ϵ, replaces the uncorrelated emission result [Eq. (10)]. From it various moments can be derived. For example,

$$\langle K_\perp^2\rangle = (I(\epsilon)/\pi)\int_0^{Q^2/4} dk_\perp^2\,\overline{\alpha}(k_\perp), \qquad (13)$$

which for large Q becomes

$$\langle K_\perp^2\rangle \simeq \text{const} + \frac{I(\epsilon)}{4\pi}Q^2\overline{\alpha}(\tfrac{1}{2}Q)\left[1+O\left(\frac{1}{\ln Q}\right)\right]. \qquad (14)$$

The constant term can only be obtained upon a knowledge of $\overline{\alpha}(k_\perp)$ in the nonperturbative region $(K_\perp \lesssim C\Lambda)$. Using the parametrization discussed below, we find it negligible for a wide range of C values $(C \gtrsim 1)$. A comparison of quark and gluon jets then gives, for fixed ϵ and Q,

$$\frac{\langle K_\perp^2\rangle_{g\ \rm jet}}{\langle K_\perp^2\rangle_{q\ \rm jet}} = \frac{I_g(\epsilon)}{I_q(\epsilon)} \xrightarrow[\epsilon\to 0]{} \frac{2N_C^2}{N_C^2-1}, \qquad (15)$$

which predicts gluon jets broader by a factor of two than the quark jets. Similar results have been obtained in the recent literature.[1,4]

A few remarks are in order concerning the validity of the above results, since, as is clear from Eqs. (11-12), the range of the $k_\perp$ integrals extends over a nonperturbative region $(k_\perp \lesssim C\Lambda)$ where the leading logarithmic approximation is questionable. The influence of this region is negligible for $K_\perp \gg C\Lambda$, which is the region of interest for a direct test of QCD, as can be seen by approximating $J_0(z)$ by $\theta(1-z)$ and $J_1(z)$ by $\delta(1-z)$ in Eqs. (11-12) which then reduce to the naive expressions (3) and (10). Numerical results which explicitly verify this fact, as well as giving our predictions for jet physics at the forthcoming very high-energy machines, will be presented elsewhere.

We now present an interesting result obtained by extrapolating smoothly our result (11) to the region $K_\perp \lesssim C\Lambda$. Assuming that the above $K_\perp$ distribution obtained for the radiation of a $q\overline{q}$ jet also describes the $k_\perp$ behavior of a single hadron[4] and using a simple parametrization for $\overline{\alpha}(k_\perp)$, directly suggested[5] by all the data on $R \equiv \sigma(e\overline{e} \to \text{hadrons})/\sigma(e\overline{e}\to\mu\mu)$, we find that Eq. (11) reproduces the experimental data quite well. More in detail, we limit the maximum transverse momentum allowed for a single hadron to a value $\sim Q/2\langle n\rangle$, where $\langle n\rangle$ is the average multiplicity.

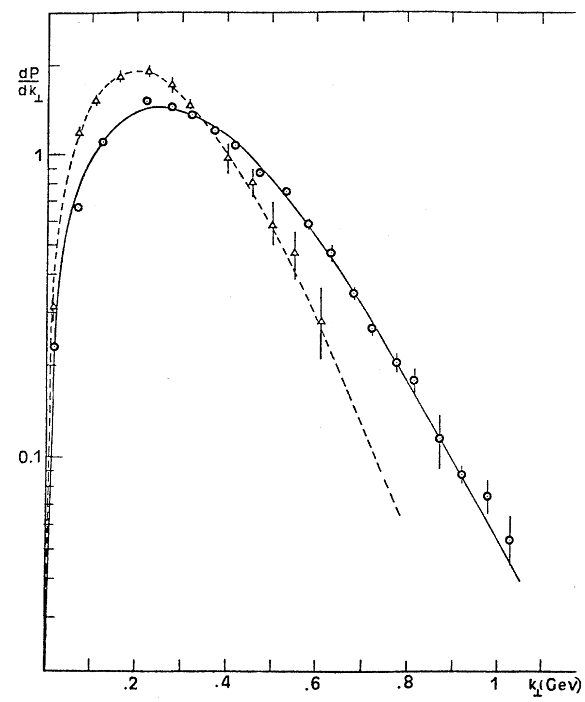

FIG. 1. Normalized $\sigma^{-1}d\sigma/dk_\perp$ vs $k_\perp$ for $Q=3$ GeV (triangles) and 7.5 GeV (circles) SPEAR single inclusive data from Ref. 6. Our results are given by the dashed line for 3 GeV and by the solid line for 7.5 GeV.

This automatically restricts the $k_\perp$ range in Eq. (11) to values $\lesssim 2$-3 GeV for present and near future e^+e^- beam energies. For $\overline{\alpha}(k_\perp)$ we use the following parametrization:

$$\overline{\alpha}(k_\perp) = \begin{cases} \dfrac{6\pi}{(33-2N_f)\ln C}, & k_\perp \leq C\Lambda \\[2ex] \dfrac{6\pi}{(33-2N_f)\ln(k_\perp/\Lambda)}, & k_\perp \geq C\Lambda, \end{cases} \qquad (16)$$

with $C\simeq 4$ and $\Lambda \simeq 0.7$ GeV. This effective $\overline{\alpha}$ has been shown[5] to bring the QCD prediction $R = \sum_s Q_i^2[1+\overline{\alpha}(S)/\pi]$ in excellent agreement with the average value $R_{\rm av} \equiv \overline{S}^{-1}\int_{S_{\rm th}} ds\, R_{\rm exp}(S)$, extracted from *all* data below $\overline{S}\simeq 60$ GeV^2.

With the above modifications in Eq. (11) our predictions are plotted in Fig. (1) for $Q=3$ and 7.5 GeV, and compared with the SLAC data[6] on $\sigma^{-1}d\sigma/dk_\perp$, suitably normalized to unity. Being unable to extract ϵ directly from the data, we have fixed $I(\epsilon)$ by requiring the same $\langle k_\perp\rangle$ as given by the data at $Q=7.5$ GeV. This leads to $2\epsilon \simeq 0.025$, which is also used for $Q=3$ GeV. The data in Fig. (1) agree very well with our expressions and thus support our extrapolation of the

QCD $K_\perp$ distribution to the nonperturbative region.

A useful analytical approximation to Eq. (11), valid for $\overline{\alpha}$ constant, is given by

$$\frac{dP}{dk_\perp} \simeq \frac{2\beta}{a\Gamma(1+\beta/2)}\left(\frac{k_\perp}{a}\right)^{\beta/2} K_{1-\beta/2}\left(\frac{2k_\perp}{a}\right), \tag{17}$$

where $a = Q/2$, $\beta = 2\overline{\alpha}I(\epsilon)/\pi$, and $K_\nu(z)$ is the modified Bessel function. This result was derived earlier in Ref. 4. Equation (17) gives the same $\langle k_\perp \rangle$ and $\langle k_\perp^2 \rangle$ as the original distribution (11).

In conclusion, we have presented a general formalism for quark and gluon jets and their $k_\perp$ distributions. A successful analysis of single-hadron $k_\perp$ distributions at SPEAR ($Q = 3-7.8$ GeV) indicates a rather smooth extrapolation of QCD for predictions to the nonperturbative region as also found[5] for the total cross section.

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Particle-Gamma Coincidence Study of Gross-Structure Peaks in $^{12}C + ^{12}C$ Inelastic Scattering

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Particle-γ angular correlations with forty ^{12}C angles for a fixed γ angle have been measured for the reaction $^{12}C(^{12}C, ^{12}C)^{12}C^*(4.44\text{ MeV})$ over the energy range $14.5 \le E_{c.m.} \le 32$ MeV. Satisfactory agreement with a large body of experimental data is obtained using a simple distorted-wave Born-approximation calculation without introducing the idea of "quasimolecular" resonances.

The recent discovery[1,2] of pronounced periodic structure in the excitation curves for inelastic scattering of two ^{12}C nuclei has stimulated considerable interest. Various interpretations of this periodic structure have been advanced.[2-6] Among these, two extreme points of view may be noted. First, one can assume that these anomalies are manifestations of quasibound 24-nucleon "molecular" states with definite spin and parity. Alternatively, it is possible that the gross structure in the inelastic scattering excitation curves is a phenomenon only indirectly related to the ^{24}Mg system and is primarily the consequence of simple kinematic relationships involving the dominance of surface partial waves.

In several of the theoretical models involving nuclear quasimolecules these states form one or more rotational bands with $E(J)$ proportional to $J(J+1)$; these bands are characterized by a moment of inertia roughly given by two carbon nuclei rotating about each other. The simplicity of such a picture makes it attractive, but if it is to