the term $\mathrm{Tr}\bar{\Psi}[\varphi,\Psi]$ since this is not invariant under the infinitesimal elements of the algebra for the graded choice of Ψ given above (this would also exclude φ from being a Higgs scalar among the gauge components in the extended dimensions).

The mass values above could be modified by the further addition of a scalar quintuplet coupled to the fermions, if so desired, though radiative corrections will also be relevant here; I will not give details.

Finally I turn to the problem of anomalies. This question has been analyzed by Elias,[11] who, on the suggestion of Salam, analyzed the anomalies assuming that the fermion $\underline{5}\oplus\underline{10}$ belong to a $\underline{15}$ of SU(6). He found there to be an anomaly arising from the generator T_{35}. For SU(5|1) this anomaly changes sign as a result of the graded tracelessness of the corresponding generator $T_{35}{}^1$; the anomaly still persists. It is not required to gauge $T_{35}{}^1$, and so I will not do so here; thus my theory will be anomaly free. It is relevant to point out that there will be an anomaly in SU(2|1) since the equivalent generator to $T_{35}{}^1$, $T_8{}^1$, has to be gauged to keep the $\mathrm{U}(1)_{L+R}$ nontrivial. This anomaly arises since the relevant quantity $\mathrm{Tr}[[\lambda_8{}^1,\lambda_8{}^1]_+,\lambda_8{}^1]_+ \neq 0$. Thus if superalgebras are being used, but with the ordinary trace in Lagrangians, it is in any case necessary to go to a larger group which is anomaly free, such as SU(5|1). It is clear that a Lagrangian built on the graded trace would still be preferable; I plan to return to this elsewhere.

The author would like to thank A. Salam for help in the execution of these ideas.

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Vanishing Graphs, Planarity, and Reggeization

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An infinite class of nonplanar skeleton graphs is found to vanish in any non-Abelian gauge theory. Thus, the dominance of planar graphs is enhanced, particularly in processes where some momenta are very large.

Non-Abelian gauge theories have surprising properties which enhance the interest of the topological expansion. The techniques used in the present perturbative computations are known, yet the interplay of the group properties with the high-energy limit of the space-time factors seems very interesting. I shall then list the main results and sketch the derivations.

Every Feynman graph in a non-Abelian gauge theory is conveniently written as a product of a group weight factor, W_G, times a space-time factor, M_G. Diagrammatic methods which efficiently compute W_G were described by Cvitanović.[1] The first results (A) and (B) in this Letter easily follow from that paper.

(A) *In every non-Abelian gauge theory there exists an infinite class of skeleton graphs with vanishing group weight factor.* The lowest-order vanishing graph in Fig. 1 of order g^5. W_G vanishes

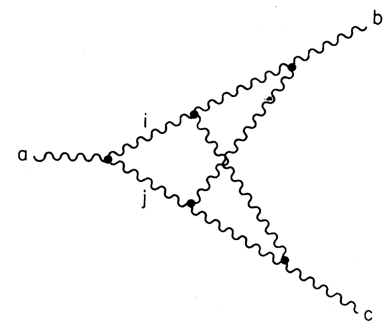

FIG. 1. The lowest-order graph with vanishing group weight.

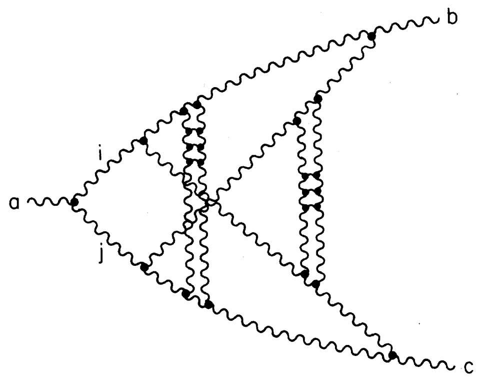

FIG. 2. A general class of graphs with vanishing group weight.

since it results from contracting an antisymmetric tensor C_{aji} with a tensor T_{ijbc} symmetric in the exchange $i \leftrightarrow j$. A more general vanishing graph is in Fig. 2. Again one checks that the tensor T_{ijbc} is symmetric in $i \leftrightarrow j$, provided the two ladders have the same number of rungs.[2] Next if one notices that in a pure gauge theory (no fermions and no Higgs particles) the group weight factor of every 2-point function is proportional to δ_{ab} and for a 3-point function to C_{ijk}, then each graph in Fig. 1 or 2 may be understood as a skeleton graph, where every three-gluon vertex is replaced by an arbitrary (planar or otherwise) 3-point function and every gluon line may be replaced by arbitrary 2-point function. By use of diagrammatic methods, one sees that (A) holds properly in any non-Abelian theory with a compact Lie group. All vanishing graphs are nonplanar.[3] They are many, yet negligible when compared with nonvanishing ones,[4] but still they seem to have intriguing consequences, described in property (C).

(B) *In the non-Abelian SU(N) theory, without fermions, the group weight factor of a graph is a polynomial in N^2, i.e., the topological expansion is a power series[5] in $1/N^2$ rather than $1/N$.*

Indeed to compute the group weight factor W_G one performs two steps[1]: (a) Re-express all three-gluon vertices in terms of the defining representation

$$iC_{ijk} = 2\,\mathrm{Tr}(T_i T_j T_k - T_k T_j T_i), \tag{1}$$

and (b) replace internal gluon lines with gluon projection operators:

$$2(T_i)_b{}^a (T_i)_d{}^c = \delta_d{}^a \delta_b{}^c - N^{-1}\delta_b{}^a\delta_d{}^c. \tag{2}$$

However, because of the trilinear nature of the coupling[6] the singlet term $-N^{-1}\delta_b{}^a\delta_d{}^c$ of step (b) is seen to cancel and it can be ignored. That is, one may use, for the internal gluon lines, the simpler replacement, proper for the U(N) theory,

$$2(T_i)_b{}^a (T_i)_d{}^c = \delta_d{}^a\delta_b{}^c. \tag{3}$$

For 2-point and 3-point functions, where there is just one basic tensor (δ_{ab} and f_{abc}, respectively), the group weight W_G of the generic Feynman graph is

$$W_G = \delta_{ab}(Ng^2)^s \sum_{P=0}^{[s/2]} c_P (N^2)^{-P} \quad \text{at order } g^{2s}, \tag{4}$$

$$W_G = f_{abc}\, g (Ng^2)^s \sum_{P=0}^{[s/2]} c_P (N^2)^{-P} \quad \text{at order } g^{2s+1}, \tag{5}$$

where the leading coefficient c_0 is different from zero if and only if the graph is planar.[7]

For the 4-point function one has six basic tensors,[8] three of which (A, B, C) have one boundary[5] and three (D, E, F) have two boundaries. At order g^{2s+2} one finds

$$W_G = g^2(g^2N)^s \Big[A \sum_0^{[s/2]} a_P(N^2)^{-P} + B \sum_0^{[s/2]} b_P (N^2)^{-P} + C\sum_0^{[s/2]} c_P(N^2)^{-P} + \frac{1}{N}\Big(D\sum_0^{[s/2]} d_P (N^2)^{-P} + E \sum_0^{[s/2]} e_P (N^2)^{-P} + F\sum_0^{[s/2]} f_P (N^2)^{-P}\Big)\Big]. \tag{6}$$

Higher n-point functions have weights W_G expressed in the same form after one has taken care of the N factors associated with the number of boundaries of the basic tensors.

One may remark that the properties (A) and (B) hold for every $N \geq 2$, every value of the coupling constant g, and every dimensionality (complex too) of the space-time dimensions. They also hold for spontaneously broken theories, provided the local gauge group still survives as a global symmetry.

(C) *There are kinematical (asymptotic, leading-logarithm) regions of the Lorentz invariants where the large-N expansions is exact (graphs with nondominant weight are not leading-logarithm dominant).*

The property (C) will be shown here by quoting some results of a new study[9] of Reggeization in non-Abelian gauge theory. The high-energy be-

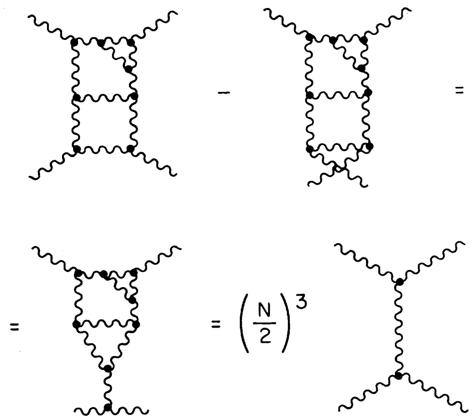

FIG. 3. An example of the relation $T_1 - T_2 = (N/2)^n T_{\rm Ad}$, pertinent to the leading ln$s$ planar graphs.

havior (large s, fixed t) of the elastic scattering amplitude has been computed in a pure-gauge (without fermions or Higgs particles) SU(N) theory. The main differences with previous investigations[10] are (a) dimensional regularization is used, instead of the Higgs bosons, (b) an improved treatment of the large-energy limit of Feynman integrals, by which the numerators of relevant Feynman integrals are decomposed into sums of terms, each of which may be associated to a contracted *scalar* Feynman integral. The asymptotic behavior of the latter is then computed by counting the number and length of the shortest t paths.[11] The main results of previous studies[10] are reproduced: In the t channel with the quantum numbers of the gluon (adjoint representation t channel) there is just one Regge pole, the Reggeized gluon, with the trajectory[12]

$$\alpha(t) = 1 + \tfrac{1}{2}g^2 N t K_{d-2}(t) + O((g^2N)^2), \tag{7}$$

while in the Pomeron channel the perturbative results are consistent with the Froissart-bound-violating fixed cut previously found.[10] Yet the improved treatment of the space-time factor[9,13] shows a different mechanism leading to these results. The set of leading (i.e., leading ln$s$) planar graphs is divided into two sets, the strictly planar graphs[14] and the set of graphs obtained from the first set after the exchange $s \leftrightarrow u$. At order g^{2n+2} the leading ln$s$ contribution of the first set in the t channel of the adjoint representation is

$$C = g^2 T_1 s(t)^{n-1}[g^2 K_{d-2}(t)\ln s]^n/n!, \tag{8}$$

while the contribution of the second set ($s \leftrightarrow u$ interchanged) is

$$\tilde{C} = g^2 T_2(-s)t^{n-1}[g^2 K_{d-2}(t)\ln s]^n/n!. \tag{9}$$

Here the group weight tensors T_1 and T_2 have the property $T_1 - T_2 = (N/2)^n T_{\rm Ad}$, where $T_{\rm Ad}$ is the projection operator of the adjoint representation. An example is given in Fig. 3, where use is made of Jacobi identity and triangle contraction. The relevant (leading ln$s$) nonplanar graphs are also divided into two sets such that the second set may be obtained from the first set after $s \leftrightarrow u$ permutation. Each graph of the first set gives the (leading ln$s$) contribution

$$D = (T_P + N^{-1}T_3 + T_4)g^4(g^2N)^{p-1} s f(t)(\ln s)^p, \tag{10}$$

where T_P is the projection operator for the Pomeron channel, T_3 is a tensor contributing to the adjoint channel, and T_4 is a tensor contributing to other channels. Furthermore, T_P, T_3, and T_4 are symmetric under the interchange $s \leftrightarrow u$. Then after summing the contribution of the second set of nonplanar graphs (see an example of the cancellation in Fig. 4, the last term vanishes as it contains Fig. 1 as a subgraph), the contribution of nonplanar graphs is of the order $g^4 s f(t)(Ng^2 \times \ln s)^{p-1}$ in the Pomeron channel and next to the leading ln$s$ in the adjoint representation t channel. In other words, to prove Reggeization of the vector mesons, one does not need the nonplanar graphs. Furthermore, because the nonleading N terms in planar graphs cancel (Fig. 3) one would obtain the correct results, in the leading ln$s$ approximation, by computing only the leading N contribution of the group weight factors W_G, rather than their complete value.

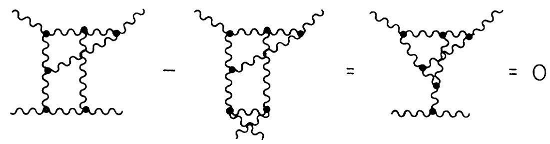

FIG. 4. An example of the cancellation that occurs in leading ln$s$, in nonplanar graphs, related to the existence of the vanishing graphs.

I would like to thank G. Marchesini for involving me in this computation, E. Witten for explaining the topological expansion, P. Butera and M. Enriotti for many discussions, and R. Blankenbecler for the kind hospitality of the Stanford Linear Accelerator Center theory group.

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[2]It was already found in Ref. 1 that the group weight factor W_G vanishes for the graph in Fig. 1, which is topologically the complete bipartite $K_{3,3}$ [after adding the external vertex—see N. Nakanishi, *Graph Theory and Feynman Integrals* (Gordon and Breach, New York, 1971), Sect. 4.] and the following one, at order g^9, which is topologically the Petersen graph, and is obtained in Fig. 2 by deleting all the rungs of the two ladders and one side of each ladder. Also some generalizations were noticed, which were obtained by replacing a gluon line with a fermion line or by including such graphs as subgraphs in other larger graphs.

[3]Planarity of Feynman graphs is related to the definition of planarity in graph theory. See Nakanishi, Ref. 2.

[4]The class of vanishing skeletons is discussed in P. Butera, G. Cicuta, and M. Enriotti, SLAC Report No. SLAC-PUB-2376, 1979 (unpublished). At large order n in pertubation theory, the number of vanishing skeletons is shown to be roughly $(n/2)!$.

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[6]It is well known that the four-gluon couplings may always be replaced by a sum of three-gluon couplings.

[7]All sums in Eqs. (4)-(6) extend up to $[s/2]$, which denotes the integral part of $s/2$.

[8]I am here using the notation of P. Yeung, Phys. Rev. D 13, 2306 (1976).

[9]G. M. Cicuta and G. Marchesini, to be published.

[10]The list of previous investigations is long and this is a subset: M. Grisaru, H. Schnitzer, and H. Tsao, Phys. Rev. D 8, 4498 (1973); H. T. Nieh and Y. P. Yao, Phys. Rev. D 13, 1082 (1976); L. N. Lipatov, Yad. Fiz. 23, 642 (1976) [Sov. J. Nucl. Phys. 23, 338 (1976)]; E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Zh. Eksp. Teor. Fiz. 72, 337 (1977) [Sov. Phys. JETP 45, 199 (1975)]; B. M. McCoy and T. T. Wu, Phys. Rev. D 13, 1076 (1976); L. Tyburski, Phys. Rev. D 13, 1107 (1976); C. Y. Lo and H. Cheng, Phys. Rev. D 15, 2959 (1977); H. Cheng *et al.*, Phys. Lett. 76B, 129 (1978); C. Y. Lo, to be published (I thank him for making his unpublished work available to me); J. Bartels, Phys. Lett. 68B, 258 (1977); J. B. Bronzan and R. L. Sugar, Phys. Rev. D 16, 466 (1977).

[11]See for instance, R. J. Eden *et al.*, *The Analytic S-Matrix*, (Cambridge Univ. Press, Cambridge, England, 1966), Chap. 3.

[12]To compare the present results with the previous ones, one should imagine the computation with double infrared regularization, that is, the SU(N) model with Higgs particles (Ref. 10) in d-dimensional space-time To work with dimensionless coupling constant, in d-dimensional space-time, one usually replaces $g \to g(m^2)^{2-d/2}$. However, in the Regge trajectory there is an effective coupling $\gamma = g^2N/2$ which may be replaced by a dimensionless coupling constant $\gamma \to \gamma(m^2)^{2-d/2}$. One would then obtain the Reggeized gluon with the trajectory

$$\alpha(t) = 1 + (m^2)^{2-d/2}(t-\mu^2)K_{d-2}(t),$$

where $K_{d-2}(t)$ is the usual bubble graph in an Euclidean $(d-2)$-dimensional space-time,

$$K_{d-2}(t) = \frac{1}{2\pi}\Gamma\left(\frac{d-2}{2}\right)\Gamma\left(3-\frac{d}{2}\right) \times \int_0^1 \frac{d\alpha}{[\alpha(1-\alpha)t+\mu^2]^{3-d/2}}.$$

This trajectory smoothly approaches Eq. (7) by removing the Higgs particles or it goes smoothly into the usual $\alpha(t) = 1 + g^2(N/2)(t-\mu^2)K_2(t) + \ldots$ by removing the dimensional regularization, that is, by replacing $d = 4$. Either way, one has a singular limit if one removes the second regularization. Still my way may have some advantage because by letting d approach 4 from above in Eq. (7), the slope of the trajectory becomes infinite while the intercept is always 1, while in letting $\mu \to 0$ in the customary Yang-Mills-Higgs system, the slope becomes infinite while the intercept is 1 only in the limit. I thank A. White for a discussion on this and for mentioning this possible advantage for a supercritical Pomeron. See, A. White, CERN Reports No. TH 2592, 1978 (unpublished) and No. TH 2629 1979 (unpublished). Equation (7) may be written more explicitly as

$$\alpha(t) = 1 + \gamma\left(\frac{t}{m^2}\right)^{d/2-2}\frac{\sqrt{\pi}}{2^{d-6}} \times \frac{\Gamma(d/2-1)}{(3-d/2)[\sin(\pi d/2)]\,\Gamma(d-5/2)}.$$

The unusual behavior of the trajectory for $d < 4$ is perhaps related to the inadequacy of the lowest-order expansion of the trajectory in a situation in which the infrared divergence becomes more severe. I thank R. L. Sugar for comments about it and encouragement.

[13]It has long been known that the infinite-momentum techniques usually employed in Ref. 10 do not correctly evaluate the asymptotic behavior of each Feynman graph, but may produce the correct result for well-chosen sets of graphs.

[14]Here strictly planar graphs indicate the planar Feynman graphs that have the double spectral function $\rho(s,t)$.