Superunification in SU(5|1)

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This paper presents a superunified model in six dimensions using the superalgebra SU(5|1) with the vector and Higgs mesons being the various components of the gauge fields. These are fixed by the principle of infinitesimal gauge invariance to give the standard symmetry-breaking mechanism. Superheavy vector-meson masses are obtained by an additional scalar matter 24-plet. Fermions are placed in the $\underline{10}\oplus\underline{5}$ representation, and their masses examined; the theory is shown to be anomaly free.

Various attempts have been made recently to gauge the superalgebra SU(2|1) in various extra dimensions,[1-5] so as to obtain naturally the Weinberg angle $\theta_W = 30°$. It is natural to extend these discussions to include quarks in a superunified model; I propose to do that here. Salam has suggested[6] that the proper extension of SU(2|1) is to SU(5|1), with the fermions in the $\underline{15} = \underline{10}\oplus\underline{5}$ representation of SU(6). I have worked out here the consequences of this suggestion. It is indeed natural to choose initially the superunified group SU(5)[7] as the simplest possible. In so doing we have automatically the value $\sin^2\theta_W = 3/8$, but standard renormalization effects from superbosons will bring this value down closer to 0.2 by the usual arguments.

Arguments have been given elsewhere[4] against the use of the "natural" invariant for a superalgebra, the graded trace, because of its negative-energy features for SU($n|m$) and the loss of the kinetic energy terms for the Higgs sector (the gauge potential in the higher dimensions). The only suitable expression with which to commence appears to be[4]

$$L = -(1/2g^2)\,\mathrm{Tr}F_{MN}F^{MN}, \qquad (1)$$

where

$$F_{MN} = \partial_M A_N - \partial_N A_M + i[A_M, A_N]$$

is the gauge field strength constructed from the potential A_M and the bracket is that arising in the superalgebra. Following arguments given in Ref. 4 we take for a generic potential $A = \begin{pmatrix} a & ib \\ ib^\dagger & c\end{pmatrix}$, where a is a Hermitian 5×5 matrix, $\mathrm{Tr}c = \mathrm{Tr}a$, and b is a complex quintuplet. For

$$A_0 = \begin{pmatrix} a & 0 \\ 0 & c\end{pmatrix}, \quad A_1 = \begin{pmatrix} 0 & ib \\ ib^\dagger & 0\end{pmatrix},$$

then

$$[A, B] = [A_0, B_0]_- + [A_0, B_1]_- + [A_1, B_0] + i[A_1, B_1]_+ .$$

Under the graded gauge transformation

$$\delta_u A_M = i[A_M, u] + \partial_M u,$$
$$\delta_u(AB) = (\delta_u A)B + (-1)^{au}A\,\delta_u B$$

(where $au = 1$ only if A and u are odd), then[4]

$$\delta_u F_{MN} = i[F_{MN}, u],$$
$$\delta_u L = (1/g^2)\,\mathrm{Tr}(u_1[F_{MN0}, F^{MN}{}_1]_+). \qquad (2)$$

From (2) it is clear that $\delta_u L$ will be nonzero in general. I impose the requirement of "inifinitesimal" gauge invariance by requiring that the A_M be chosen so that $\delta_u L = 0$; thus we need $F_{MN1}F^{MN}{}_0 = 0$. This can be solved[4] by taking

$$F_{\mu\nu 1} = 0, \quad A_{M0} = gM_m\lambda_{6'}, \qquad (3)$$

for $1 \le \mu,\ \nu \le 4,\ m \ge 5$ (m denoting the extra components of space-time), where M_m are constants, and $\lambda_{6'} = \mathrm{diag}(1,1,1,1,1,5)$. The noninvariance of L under odd gauge transformations is now clear, since the form of these potentials is not then preserved. We thus have to consider the condition $\delta_u L = 0$ as a prescription for fixing the Higgs sector; since it leads to numerous experimental predictions[3] it can undoubtedly be checked over the next few years.

We evaluate the Lagrangian (1) in two extra timelike dimensions,[3] using the values of the fields and potentials specified by (3) and

$$A_\mu = \frac{g}{\sqrt{2}}\sum_{a=1}^{24} A_\mu{}^a\lambda_a + \frac{g}{2\sqrt{15}}B_\mu\lambda_{6'},$$
$$A_{51} = \frac{g}{\sqrt{2}}\begin{pmatrix} 0 & i\varphi_5 \\ i\varphi_5{}^\dagger & 0\end{pmatrix}, \quad A_{61} = \frac{g}{\sqrt{2}}\begin{pmatrix} 0 & i\varphi_6 \\ i\varphi_6{}^\dagger & 0\end{pmatrix}, \qquad (4)$$
$$M_5 = M_6 = \tfrac{1}{4}M.$$

The Lagrangian (1) then becomes

$$L = -\tfrac{1}{4}\,\mathrm{Tr}[F_{\mu\nu}(\underline{24})]^2 - \tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + \tfrac{1}{2}|D_\mu\varphi_5|^2 + \tfrac{1}{2}|D_\mu\varphi_6|^2 - V, \qquad (5)$$

where

$$V = g^2\{4|(\varphi^\dagger\varphi)|^2 + 4|(\psi^\dagger\psi)|^2 + 2(\varphi^\dagger\varphi)(\psi^\dagger\psi) + 6(\varphi^\dagger\psi)(\psi^\dagger\varphi) - 4M^2(\varphi^\dagger\varphi)\}, \quad (6)$$

$\varphi_5 = \varphi + \psi$, $\varphi_6 = \psi - \varphi$, and $F_{\mu\nu}(\underline{24})$ is the SU(5) gauge-invariant field strength arising from the 24-plet $\sum_\mu{}^\alpha\lambda_\alpha$. The minimum of V in (6) occurs for $\psi = 0$, $(\varphi^\dagger\varphi) = \frac{1}{2}M^2$. Then ψ and the fifth component of Reφ are massive with mass $2Mg = 2M_W$, where W are the usual SU(2)⊗U(1) massive W mesons; this predicted mass is the same as for the original case of SU(2|1).[3] It is interesting to note that the same ratio of the Higgs mass to the W-meson mass occurs in any SU($n|m$) theory in six dimensions. Three further components of φ are absorbed by the W's so leaving six real massless W fields.

To give mass to these latter fields and also to bring in the superheavy-mass scale I introduce a further scalar 25-plet as a matter field, $\Phi = \sum\varphi^a\lambda^a + \varphi\lambda_{6'}$. It would have been possible to have introduced this multiplet as a higher-dimensional gauge-field component, but since it is even it would have been necessary to use at least two such components, say A_7 and A_8, with $A_{70} = \Phi_7$, $A_{71} = 0$, $A_{80} = \Phi_8$, $A_{81} = 0$. The resulting potential term in L of (1) would then have been $\mathrm{Tr}([\Phi_7, \Phi_8])^2$. The stationary value of such a term is at $\Phi_7 = \Phi_8$, with value zero, so that no further spontaneous symmetry breaking (SSB) can occur in this manner. Thus I introduce an additional term $\frac{1}{2}\mathrm{Tr}(D_M\Phi)^2 - V(\Phi)$ in L, with $V(\Phi)$ being of form

$$V(\Phi) = -\tfrac{1}{2}\mu^2\,\mathrm{Tr}(\Phi^2) + P_4(\Phi), \quad (7)$$

where $P_4(\Phi)$ is a positive fourth-order polynomial in Φ and μ is a mass of order 10^{15} GeV. It may not be necessary to introduce P_4 explicitly, since it may arise by a radiative-correction mechanism[8]; in either case we expect an SSB value of $\langle\Phi\rangle_0 \sim \mu$. We have to note in addition the further scalar self-interaction term from $\frac{1}{2}\mathrm{Tr}[(D_5\Phi)^2 + (D_6\Phi)^2]$ of value

$$-\tfrac{1}{2}[\varphi_5{}^\dagger\Phi^2\varphi_5 + \varphi_6{}^\dagger\Phi^2\varphi_6] \quad (8)$$

in V. The total expression (6), (7), and (8) has a minimum with ψ and φ parallel. Following Buras *et al.*[9] there will be expected to be a minimum of the combined potential with $\psi = 0$, the masses of the $\underline{24}$ and $\underline{5}$ all being large except for the familiar SU(2)⊗U(1) Higgs particle. Thus the model will be satisfactory in the meson sector as is SU(5).[10]

Let us now turn to the fermion sector. This is described by a $(\underline{5}\times\underline{5})_a$ and a $\underline{5}$ placed together to make the SU(5|1) spinor-valued matrix

$$\Psi = \begin{pmatrix} (\underline{5}\times\underline{5})_a & \underline{5}/\sqrt{2} \\ -\underline{5}/\sqrt{2} & 0 \end{pmatrix} \quad (9)$$

with the standard assignments

$$\underline{5} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \\ \nu \end{pmatrix}_R, \quad (\underline{5}\times\underline{5})_a = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u_3{}^c & -u_2{}^c & -u_1 & -d_1 \\ -u_3{}^c & 0 & u_1{}^c & -u_2 & d_2 \\ u_2{}^c & -u_1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & -u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L,$$

where I neglect the Cabibbo rotation. I also set to zero the extra left-handed antineutrino which really should have been included in the Ψ of (9) as $\nu'\lambda_{6'}$ to give the complete $\underline{16}$ of SU(5|1), but decouples from the theory due to its chirality and singlet character under SU(5). The fermion kinetic term in the Lagrangian will thus be $i\,\mathrm{Tr}(\bar\Psi \not{D} \Psi)$, where D_M is the usual covariant derivative, $\not{D} = D_M\Gamma_M$, the Γ_M being the Dirac matrices in the higher dimensions, and $\bar\Psi = \Psi^\dagger\Gamma_0 = \Psi^\dagger\gamma_0$. It is possible to show the invariance of this term under infinitesimal graded gauge transformations by methods similar to that of the third paper in Ref. 3 only provided the grading in Ψ again follows chirality as it did in the case of the $\underline{3}$ for leptons in SU(2|1). Thus

$$\delta_u\,\mathrm{Tr}(\bar\Psi\not{D}\Psi) = \mathrm{Tr}\{2i\bar\Psi\not{D}\Psi u_1 - 2i[\bar\Psi_0, u_1]_-\not{D}\Psi - \bar\Psi_0[\Psi_1, \not{\partial}u_1]_+\}$$

where Ψ_0, Ψ_1, are the even and odd parts of Ψ, respectively. Only provided Ψ_0 and Ψ_1 have opposite chiralities does the right-hand side vanish, as can be seen by inspection. The fermion mass term is therefore

$$\mathrm{Tr}\{\bar\Psi(\Gamma_5[\langle A_5\rangle_0, \Psi] + \Gamma_6[\langle A_6\rangle_0, \Psi])\},$$

which leads to the mass values $m_d = m_e$, $m_u = 0$, as for the original SU(5) model.[7] Extension of the theory to higher dimensions, following the first paper of Ref. 3, would lead to $m_c = m_\mu$, $m_s = 0$, $m_b = m_\tau$, $m_t = 0$. It is not possible to include the term $\text{Tr}\bar{\Psi}[\varphi,\Psi]$ since this is not invariant under the infinitesimal elements of the algebra for the graded choice of Ψ given above (this would also exclude φ from being a Higgs scalar among the gauge components in the extended dimensions).

The mass values above could be modified by the further addition of a scalar quintuplet coupled to the fermions, if so desired, though radiative corrections will also be relevant here; I will not give details.

Finally I turn to the problem of anomalies. This question has been analyzed by Elias,[11] who, on the suggestion of Salam, analyzed the anomalies assuming that the fermion $\underline{5} \oplus \underline{10}$ belong to a $\underline{15}$ of SU(6). He found there to be an anomaly arising from the generator T_{35}. For SU(5|1) this anomaly changes sign as a result of the graded tracelessness of the corresponding generator $T_{35}{}^1$; the anomaly still persists. It is not required to gauge $T_{35}{}^1$, and so I will not do so here; thus my theory will be anomaly free. It is relevant to point out that there will be an anomaly in SU(2|1) since the equivalent generator to $T_{35}{}^1$, $T_8{}^1$, has to be gauged to keep the $U(1)_{L+R}$ nontrivial. This anomaly arises since the relevant quantity $\text{Tr}[[\lambda_8{}^1,\lambda_8{}^1]_+,\lambda_8{}^1]_+ \neq 0$. Thus if superalgebras are being used, but with the ordinary trace in Lagrangians, it is in any case necessary to go to a larger group which is anomaly free, such as SU(5|1). It is clear that a Lagrangian built on the graded trace would still be preferable; I plan to return to this elsewhere.

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Vanishing Graphs, Planarity, and Reggeization

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An infinite class of nonplanar skeleton graphs is found to vanish in any non-Abelian gauge theory. Thus, the dominance of planar graphs is enhanced, particularly in processes where some momenta are very large.

Non-Abelian gauge theories have surprising properties which enhance the interest of the topological expansion. The techniques used in the present perturbative computations are known, yet the interplay of the group properties with the high-energy limit of the space-time factors seems very interesting. I shall then list the main results and sketch the derivations.

Every Feynman graph in a non-Abelian gauge theory is conveniently written as a product of a group weight factor, W_G, times a space-time factor, M_G. Diagrammatic methods which efficiently compute W_G were described by Cvitanović.[1] The first results (A) and (B) in this Letter easily follow from that paper.

(A) *In every non-Abelian gauge theory there exists an infinite class of skeleton graphs with vanishing group weight factor.* The lowest-order vanishing graph in Fig. 1 of order g^5. W_G vanishes

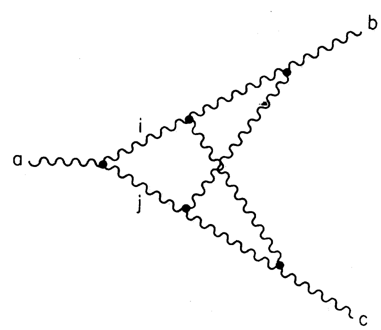

FIG. 1. The lowest-order graph with vanishing group weight.