Compatibility of Cabibbo-Suppressed D^0 Decays with Weak-Interaction Mixing Angles

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The new measurement of the $D^0 \to K^-\pi^+$, K^-K^+, and $\pi^-\pi^+$ branching ratios is compared with the range of values allowed by weak-interaction mixing angles of heavy quarks. The possibility of accommodating this measurement in the standard gauge model is discussed.

The branching ratios were determined recently for two-body decays of D^0.[1] Theoretically, these decay amplitudes obey one simple sum rule involving mixing angles that is based only on SU(3) symmetry of strong interactions and the absence of flavor-changing weak neutral currents.[2] I examine the sum rule to see whether it is compatible with the values of weak-interaction mixing angles allowed in the standard sequential $SU(2)\otimes U(1)$. SU(3) symmetry is assumed for strong interactions.

I will write the charged weak current as

$$J_\lambda = (\bar{d}, \bar{s}, \bar{b}, \dots)\, U\gamma_\lambda(1-\gamma_5)\begin{pmatrix} u \\ c \\ t \\ \vdots \end{pmatrix}, \qquad (1)$$

where $U=(U_{ij})$ is a unitary matrix of quark mixing. Hereafter, I write the upper-left (2×2) submatrix as

$$\begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} = \begin{pmatrix} \cos\theta_{ud} & -\sin\theta_{cd} \\ \sin\theta_{us} & \cos\theta_{cs} \end{pmatrix}. \qquad (2)$$

The experimental and theoretical information that are included in this analysis are the following:

(1) A sum rule for $D^0 \to K^-K^+$, $\pi^-\pi^+$, and $K^-\pi^+$ amplitudes[2]:

$$\frac{M(D^0\to K^-K^+) - M(D^0\to\pi^-\pi^+)}{\cos\theta_{ud}\sin\theta_{cd} + \sin\theta_{us}\cos\theta_{cs}} = \frac{M(D^0\to K^-\pi^+)}{\cos\theta_{cs}\cos\theta_{ud}}. \qquad (3)$$

The sum rule results from U-spin (λ_7) rotation of SU(3) and it remains valid however many heavy quarks may participate in the decays. $M(D^0 \to K^-K^+)$ and $M(D^0\to\pi^-\pi^+)$ generally have different phases of final-state interactions even in the SU(3) limit,

$$\begin{aligned} M(D\to K^-K^+) &= a_1A_1\exp(i\delta_1) + a_0A_0\exp(i\delta_0), \\ M(D\to\pi^-\pi^+) &= -a_1A_1\exp(i\delta_1) + a_0A_0\exp(i\delta_0), \end{aligned} \qquad (4)$$

where $a_1 = (\cos\theta_{ud}\sin\theta_{cd} + \cos\theta_{cs}\sin\theta_{us})/2$ and $a_0 = (\cos\theta_{ud}\sin\theta_{cd} - \cos\theta_{cs}\sin\theta_{us})/2 +$ [contributions from $(\bar{u}b)(\bar{b}c)\dots$]. The subscripts 1 and 0 denote the U spins of the final mesons. Without having detailed knowledge of strong interactions, I leave the phase difference as a free parameter.

With the definitions

$$r = [\Gamma(D^0\to K^-K^+)p_K^{-1}]/[(D^0\to\pi^-\pi^+)p_\pi^{-1}], \qquad (5)$$

$$\epsilon = [\Gamma(D^0\to\pi^-\pi^+)p_\pi^{-1}]/[\Gamma(D^0\to K^-\pi^+)p_{K\pi}^{-1}], \qquad (6)$$

$$\Theta = \frac{\cos\theta_{ud}\sin\theta_{cd} + \cos\theta_{cs}\sin\theta_{us}}{\cos\theta_{cs}\cos\theta_{ud}}, \qquad (7)$$

one finds, by eliminating the phase difference $\Delta = \delta_1 - \delta_0$, that

$$\epsilon(r^{1/2}-1)^2 \leq \Theta^2 \leq \epsilon(r^{1/2}+1)^2 \qquad (8)$$

must hold. If $\Delta = 0$, either of the upper and lower bounds is realized. The data give $\Gamma(D^0\to K^-K^+)/\Gamma(D^0\to K^-\pi^+) = (11.3\pm 3.0)\%$ and $\Gamma(D^0\to\pi^-\pi^+)/\Gamma(D^0\to K^-\pi^+) = (3.3\pm 1.5)\%$.[1]

(2) The values of $\cos\theta_{ud}$ and $\sin\theta_{us}$ as determined from the existing data. I use the results of Shrock and Wang[3]:

$$\begin{aligned} \cos\theta_{ud} &= 0.9739 \pm 0.0025, \\ \sin\theta_{us} &= 0.219 \pm 0.002. \end{aligned} \qquad (9)$$

From unitarity,

$$\begin{aligned} \textstyle\sum_i U_{iu}{}^*U_{ic} &= \textstyle\sum_i U_{di}{}^*U_{si} = 0, \\ \textstyle\sum_i |U_{iu}|^2 &= \textstyle\sum_i |U_{ic}|^2 = \textstyle\sum_i |U_{di}|^2 \\ &= \textstyle\sum_i |U_{si}|^2 = 1. \end{aligned} \qquad (10)$$

Generally, $\cos\theta_{cs}$ contains a phase causing CP nonconservation when $\cos\theta_{ud}$, $\sin\theta_{us}$, and $\sin\theta_{cd}$ are chosen to be real.[4] In this analysis, however, I ignore the CP-nonconservation phase in $\cos\theta_{cs}$ (not necessarily in U_{iu} or U_{ic} with $i = b, \dots$) since the phase factor enters U_{ic} with a small coefficient and the phase itself is expected to be very small. From (10) one obtains by the Schwarz inequality

$$(1-\cos^2\theta_{ud}-\sin^2\theta_{us})(1-\cos^2\theta_{cs}-\sin^2\theta_{cd}) \geq (-\sin\theta_{cd}\cos\theta_{ud}+\sin\theta_{us}\cos\theta_{cs})^2. \quad (11)$$

In the six-quark model, the equality in (11) should hold.

(3) The K_L-K_S mass difference, the lifetime of the D mesons, and the nonleptonic enhancement factors. *One should be fully aware that they depend on dynamical models of strong interactions.* Since an established theoretical model exists for each of them, however, I will add this information to my analysis for the sake of comparison. For the K_L-K_S mass difference, I will compare my results with the results of Barger *et al.*[5] and Shrock *et al.*[5] based on improved box diagrams,

$$0.5 < \Delta M_{K-M}/\Delta M_{\rm exp} < 2, \quad (12)$$

in the six-quark model. For the lifetime, I combine with the observed semileptonic branching ratio of the D mesons ($\simeq 10\%$), the decay-rate estimate based on the four-fermion interactions corrected with gluon emission and loops.[6] The theoretical estimate is

$$\frac{0.4\times 10^{-12}}{\cos^2\theta_{cs}+\kappa\sin^2\theta_{cd}}\ \text{sec} \quad (1<\kappa<2) \quad (13)$$

as compared with the experimental value of $\leq 0.7 \times 10^{-12}$ sec.[7] I will compare the result of my analysis with nonleptonic enhancement factors of different origins.

The constraints imposed on $\cos\theta_{cs}$ and $\sin\theta_{cd}$ are plotted in Fig. 1(a). The unitarity constraint (11) requires both values to be in the inside of the very flat ellipse. The measured branching ratios can be reproduced as long as $\cos\theta_{cs}$ and $\sin\theta_{cd}$ lie between the two straight lines bounded by (8). The distance of a point in the allowed area from the circumference of unit circle is the upper limit of the strength of the charmed-quark coupling to any of heavy quarks of charge $-\frac{1}{3}$. If only six quarks enter the charged weak current, only the values on the arc of the ellipse are allowed. If the relative phase of final-state interactions is 0 or π, only the values on the straight lines are allowed. It should be cautioned, however, that the two straight lines are subject to ambiguities due to the measurement errors in the branching ratios.[1] A typical ambiguity is plotted at the point of intersection of the ellipse and one of the straight lines.

The K_L-K_S mass difference (12) restricts the angles to the upper right-hand corner of the ellipse. The boundary for $\mathrm{Re}\cos\theta_{cs}$ is drawn for $m_t > 15$ GeV in the six-quark model with the CP phase. A similar restriction is obtained in models with more than six quarks *if* one bars an accidental cancellation among heavy-quark contributions. The lifetime (13) eliminates the region close to the origin. After all the restrictions, there is still some area left for the mixing angles if final-state interactions are favorable. However, the situation does not look so optimistic when we look into the relative enhancement factor of the $U = 0$ rate versus the $U = 1$ rate that is needed to realize the measured branching ratios. Dropping

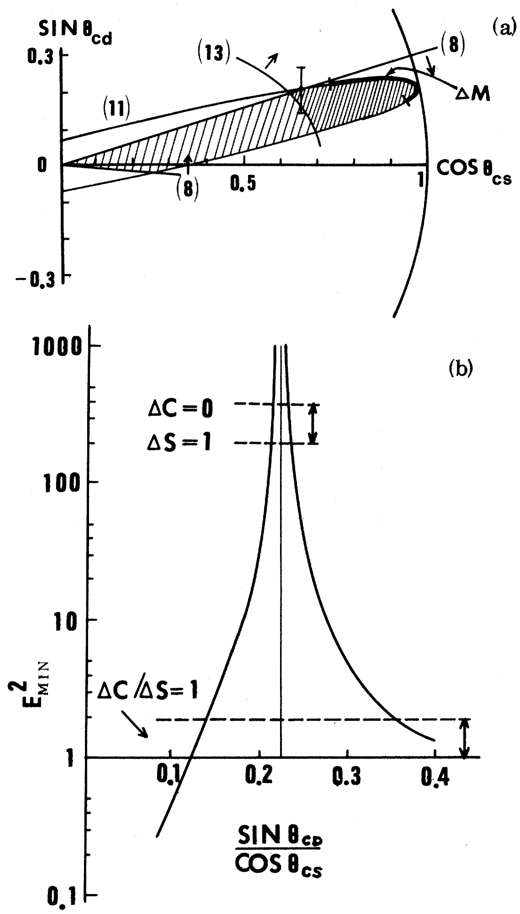

FIG. 1. (a) Consistency of the constraints imposed by the Cabibbo-suppressed D^0 decays with those imposed by other experimental and theoretical information. The shaded region is commonly allowed. See the text for further details. (b) The minimum enhancement factor of the $U=0$ rate vs the $U=1$ rate in the SU(3) symmetry limit. Effective $U=1$ amplitudes due to SU(3) breaking are not negligible particularly near $\sin\theta_{cd}/\cos\theta_{cs} = 0.23$. For the sake of comparison, the enhancement factors are plotted for the strange-particle decays and the Cabibbo-favored decays of the charmed particles.

the $U=0$ amplitude versus the $U=1$ amplitude as the contributions from the $(\bar{u}b)(\bar{b}c)$-type interactions involving heavy quarks of charge $-\frac{1}{3}$ for the moment, one obtains the enhancement factor E for

$$\frac{r^{1/2}-1}{r^{1/2}+1} \leqslant \left|\frac{\cos\theta_{ud}\sin\theta_{cd}-\cos\theta_{cs}\sin\theta_{us}}{\cos\theta_{ud}\sin\theta_{cd}+\cos\theta_{cs}\sin\theta_{us}}\right| E \leqslant \frac{r^{1/2}+1}{r^{1/2}-1}. \tag{14}$$

Near $\sin\theta_{cd}/\cos\theta_{cs}=\sin\theta_{us}/\cos\theta_{ud}$, we need a very large enhancement. I have plotted $E_{\min}{}^2$, the minimum enhancement factor in rate that is needed even with the most favorable final-state interaction phases. In most of the allowed region, particularly in the likely region of $\sin\theta_{cd}\gtrsim 0.17$, the needed enhancement for the $U=0$ processes is quite large. Can we find an origin of such a large enhancement for $U=0$, but not for $U=1$? The standard short-distance enhancement does not provide a solution to this problem.[8] Looking at the $U=0$ and $U=1$ pieces of the interactions

$$[(\bar{u}d)(\bar{d}c)\pm(\bar{u}s)(\bar{s}c)](\cos\theta_{ud}\sin\theta_{cd}\mp\cos\theta_{cs}\sin\theta_{us}), \tag{15}$$

one finds that the so-called "penguin diagrams" can contribute to the $U=0$ amplitudes, but not to the $U=1$ amplitudes in the SU(3) limit.[9] If they are really important in the Cabibbo-suppressed decays of the charmed mesons, one can readily explain the large enhancement of the $U=0$ amplitudes, and therefore, the large $K^-K^+/\pi^-\pi^+$ ratio of the D^0 decay. The heavy-quark contributions to $U=0$ that have been left out also produce the penguin diagrams. Since the penguin diagrams are not of short-distance nature, we are unable to estimate them numerically accurately. However, because they are not short-distance processes, there is a good chance that they produce a sizable enhancement even in the heavy-quark decay. Enhancement due to a $(C=0)$ resonance at the D^0 mass is unlikely since the phase-shift analyses of $\pi\pi$ and $K\bar{K}$ channels do not indicate any conspicuous resonance in that mass region.[10] However, SU(3) breaking effects of nonresonant nature should be examined carefully.[11] Near $\sin\theta_{cd}/\cos\theta_{cs}=\sin\theta_{us}/\cos\theta_{ud}$, effective $U=0$ interactions induced from the $U=1$ four-fermion interaction through SU(3) breakings presumably dominate over the $U=0$ term of the SU(3) symmetry limit. Therefore, the real enhancement need not be as large as indicated in Fig. 1(b) in that neighborhood.

To summarize, the branching ratios of the two-body nonleptonic D^0 decays as measured by Abrams *et al.* (Ref. 1) can be accommodated in the standard gauge model of sequential $SU(2)_L \otimes U(1)$ if the penguin diagrams are enhanced in the Cabibbo-suppressed charm decays just as I suspect they are in the strange-particle decays. Because of the difficulty in accurate measurement of Cabibbo-suppressed semileptonic decays of the D^0 meson, the nice U-spin property of the nonleptonic decay interactions makes the two-body D^0 decays the most suitable to examine the weak-interaction mixing angles when the accuracy of measurement is improved.

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Generalized Gauge Hierarchies

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We generalize the analysis of coupling-constant renormalization in unified theories to allow for more than one scale of spontaneous symmetry breakdown between unification and SU(3)⊗SU(2)⊗U(1). We find a relation between α, α_s, $\sin^2\theta_W$, and the proton lifetime which is independent of the details of the gauge hierarchy and holds in a large class of unified theories.

In an SU(5) unified theory of strong, weak, and electromagnetic interactions[1] the structure of the gauge hierarchy is unique. If this theory is to describe physics, the SU(5) symmetry must be broken at a very large momentum, $\sim 10^{15}$ GeV, down to SU(3)⊗SU(2)⊗U(1).[2] This symmetry is further broken down to SU(3)⊗U(1) at $M_W \sim 80$ GeV. There is no choice for the first symmetry breakdown because the only proper subgroup of SU(5) which contains SU(3)⊗SU(2)⊗U(1) is SU(3)⊗SU(2)⊗U(1) itself. Consequently, the effect of the gauge hierarchy on the renormalization of $\sin^2\theta_W$ is unique, and any reasonable choice for the color SU(3) coupling α_s implies $\sin^2\theta_W \simeq 0.2$. Experimentally, $\sin^2\theta_W = 0.23 \pm 0.01$.[3] This is closer to 0.2 than it is to $\frac{3}{8}$ (the naive unification value) which is nice, but it seems to be significantly different from the SU(5) prediction. What is going on? One possibility is that the unifying group may be some larger group which contains SU(5). Then, the gauge symmetry may break down to SU(3)⊗SU(2)⊗U(1) in several steps. In this paper, we generalize the analysis of Ref. 2 to this more general situation. We find a useful formula with a simple physical interpretation. We then apply it in some interesting examples.

If a simple gauge group G is broken down to SU(3)⊗SU(2)⊗U(1) in N steps, the gauge hierarchy of the theory can be characterized by $N+1$ masses, μ_x, $x=0$ to N, describing the strengths of the various steps of symmetry breaking. We assume that $\mu_{x-1} \ll \mu_x$. There is a large region between μ_{x-1} and μ_x where the physics can be described by an effective field theory with an S^x gauge symmetry. S^x is the subgroup of G which is left unbroken in steps x through N of the hierarchy.[4] S^x is broken down to S^{x-1} at a mass μ_{x-1}. The gauge bosons associated with S^x but not with S^{x-1} get mass of the order of μ_{x-1}.

The subgroup S^x is a product of simple factors and U(1) factors

$$S^x = \prod_\alpha s_\alpha^{\ x}, \tag{1}$$

where $s_\alpha^{\ x}$ is either a simple non-Abelian subgroup or a U(1). S^1, for example, is U(1)⊗SU(2)⊗SU(3); so we can take $s_1^{\ 1}=\mathrm{U}(1)$, $s_2^{\ 1}=\mathrm{SU}(2)$, and $s_3^{\ 1}=\mathrm{SU}(3)$. S^0 is SU(3)⊗U(1) and $\mu_0 \sim M_W$. Denote the generators of $s_\alpha^{\ x}$ by $T_{\alpha i}^{\ \ x}$. The unification provides a natural normalization of the generators. Choose a convenient representation of G (for example, the adjoint representation) so that

$$\mathrm{tr}(T_{\alpha i}^{\ \ x} T_{\beta j}^{\ \ x}) = \lambda \delta_{\alpha\beta}\delta_{ij}, \tag{2}$$

where λ is any convenient constant.

We will choose λ so that the electric charge Q is

$$Q = T_{23}^{\ \ 1} + (\tfrac{5}{3})^{1/2} T_{11}^{\ \ 1} \tag{3}$$

in terms of the generators of S^1 [$T_{23}^{\ \ 1}$ is the third component of weak SU(2) and $T_{11}^{\ \ 1}$ is the U(1) generator]. The factor $(\frac{5}{3})^{1/2}$ in Eq. (3) is characteristic of groups G with the SU(3)⊗SU(2)⊗U(1) embedded in an SU(5) subgroup [in Ref. 2, $(\frac{5}{3})^{1/2} = -C$].

If S^x contains more than one U(1) factor, we choose all but (at most) one to be orthogonal to the electric charge [$\mathrm{tr}(QT_{\alpha 1}^{\ \ x})=0$]. The orthogon-