SU(3) Predictions for Charmed-Meson Decays

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The predictions of SU(3) for charmed-meson decay are reanalyzed, allowing for expeoted new weak-mixing angles.

Recent discoveries[1] have suggested an unexpected[2] richness in the spectrum of quark flavors. We now expect at least six flavors and a plenitude of mixing angles, analogous to the Cabibbo angle, describing their weak interactions. It is of course of great interest to determine these angles experimentally.

Previously SU(3) predictions for two-body nonleptonic charmed-meson decays were analyzed[3,4] assuming the original four-quark model of Glashow, Iliopoulos, and Maiani.[5] We have carried out a more general analysis, allowing the ratio $V_{cd}/V_{cs} \equiv -\rho$ of c-d to c-s couplings to differ from the value $\tan\theta_C$ suggested by the four-quark model. The results for D^0, D^+, and F^+ decaying to two pseudoscalars are given in Table I. Throughout this note we ignore the contributions from coupling to the b quarks. We believe these con-

TABLE I. We give the decay rates of D^0, D^+, and F^+, to two pseudoscalars. A, B, C, D, and E are the five independent amplitudes, where A, B, and C are identical to those used in Ref. 3. c_1, c_2, s_1, and s_2 are related to the mixing angles by $c_1 \equiv V_{ud}$, $s_1 \equiv V_{us}$, $c_2 \equiv V_{cs}$, and $-s_2 \equiv V_{cd}$, where the V's are defined the same as in Ref. 6.

	(a) D^0 decay rates
$K^-\pi^+$	$2\vert c_1c_2A\vert^2$
K^-K^+	$2\vert s_1c_2(A+D)-c_1s_2D\vert^2$
$\pi^-\pi^+$	$2\vert s_1c_2D-c_1s_2(A+D)\vert^2$
$K^+\pi^-$	$2\vert s_1s_2A\vert^2$
$\bar{K}^0\pi^0$	$\vert c_1c_2B\vert^2$
$\bar{K}^0\eta^0$	$\frac{1}{3}\vert c_1c_2B\vert^2$
$K^0\bar{K}^0$	$\frac{1}{2}\vert(s_1c_2-c_1s_2)(6E-D)\vert^2$
$\pi^0\pi^0$	$\vert s_1c_2D-c_1s_2(D-B)]^2$
$\eta^0\eta^0$	$4\vert s_1c_2(E-\frac{1}{3}B)-c_1s_2(E+\frac{1}{6}B)\vert^2$
$\pi^0\eta^0$	$\frac{2}{3}\vert s_1c_2[-B+\frac{2}{3}(D-2E)]-c_1s_2\frac{3}{2}(D-2E)\vert^2$
$K^0\pi^0$	$\vert s_1s_2B\vert^2$
$K^0\eta^0$	$\frac{1}{3}\vert s_1s_2B\vert^2$
	(b) D^+ decay rates
$\bar{K}^0\pi^+$	$2\vert c_1c_2(A+B)]^2$
$\bar{K}^0K^+$	$2\vert s_1c_2[A+\frac{1}{2}(3D-6E)]-c_1s_2[A-C+\frac{1}{2}(3D-6E)]^2$
$\pi^0\pi^+$	$\vert c_1s_2(A+B)\vert^2$
$\eta^0\pi^+$	$\frac{1}{3}\vert s_1c_2(-2B+3D-6E)-c_1s_2(3A+B-2C+3D-6E)]^2$
$K^+\pi^0$	$\vert s_1s_2C\vert^2$
$K^+\eta^0$	$\frac{1}{3}\vert s_1s_2C\vert^2$
$K^0\pi^+$	$2\vert s_1s_2(A+B-C)\vert^2$
	(c) F^+ decay rates
$\bar{K}^0K^+$	$2\vert c_1c_2(A+B-C)\vert^2$
$\pi^0\pi^+$	0
$\eta^0\pi^+$	$\frac{4}{3}\vert c_1c_2C\vert^2$
$K^0\pi^+$	$2\vert s_1c_2[A-C+\frac{1}{2}(3D-6E)]-c_1s_2[A+\frac{1}{2}(3D-6E)]\vert^2$
$K^+\pi^0$	$\vert s_1c_2[A-C+\frac{1}{2}(3D-6E)]-c_1s_2[-B+\frac{1}{2}(3D-6E)]^2$
$K^0\eta^0$	$\frac{1}{3}\vert s_1c_2[3A+2B-C+\frac{1}{2}(3D-6E)]-c_1s_2[-B+\frac{1}{2}(3D-6E)]\vert^2$
K^0K^+	$2\vert s_1s_2(A+B)\vert^2$

tributions are probably small. In any case they merely redefine the coefficience of D and E in the table.

The most interesting results of this analysis are as follows: (1) The weak effective Hamiltonian now has the structure $\underline{3}+\underline{6}^*+\underline{15}$. In the four-quark case the $\underline{3}$ was absent. There are now five invariant amplitudes describing the decay of charmed mesons into two light pseudoscalars, whereas previously there were three. The modifications introduced by letting $|\rho|\neq\tan\theta_C$ do not of course affect the relative rates of the leading $\Delta c=\Delta s$ decays (assuming $|\rho|\ll 1$, as is indicated[6,7]). Thus the decay $F^+\to\pi^0\pi^+$ is still forbidden by SU(3) and the amplitudes for $D^0\to K^-\pi^+$, $\sqrt{2}(D^0\to\bar{K}^0\pi^0)$, and $D^+\to\bar{K}^0\pi^+$ satisfy triangle inequalities, which can be easily obtained from Tables Ia and Ib.

(2) The previous simple prediction

$$\frac{\Gamma(D^0\to K^-K^+)}{\Gamma(D^0\to K^-\pi^+)}=\frac{\Gamma(D^0\to\pi^-\pi^+)}{\Gamma(D^0\to K^-\pi^+)}=\tan^2\theta_C \tag{1}$$

no longer obtains. A new reduced matrix element from the $\underline{3}$ part of the Hamiltonian appears, complicating matters. The amplitudes a_1, a_2, and a_3 for $D^0\to K^-\pi^+$, K^-K^+, and $\pi^-\pi^+$, respectively, are now given by[8]

$$a_1=\sqrt{2}\,AV_{ud}V_{cs}, \tag{2}$$

$$a_2=\sqrt{2}\,AV_{us}V_{cs}+\sqrt{2}\,D(V_{us}V_{cs}+V_{ud}V_{cd}), \tag{3}$$

$$a_3=\sqrt{2}\,AV_{ud}V_{cd}+\sqrt{2}\,D(V_{us}V_{cs}+V_{ud}V_{cd}), \tag{4}$$

where A and D are reduced matrix elements which *a priori* we expect to be of the same order of magnitude. From these derive the triangle inequalities[8]

$$\left(\frac{\Gamma(D^0\to K^+K^-)}{\Gamma(D^0\to K^-\pi^+)}\right)^{1/2}-\left(\frac{\Gamma(D^0\to\pi^-\pi^+)}{\Gamma(D^0\to K^-\pi^+)}\right)^{1/2}\leq\left|\frac{V_{us}}{V_{ud}}+\rho\right|\leq\left(\frac{\Gamma(D^0\to K^+K^-)}{\Gamma(D^0\to K^-\pi^+)}\right)^{1/2}+\left(\frac{\Gamma(D^0\to\pi^+\pi^-)}{\Gamma(D^0\to K^-\pi^+)}\right)^{1/2}, \tag{5}$$

$$\left(\frac{\Gamma(D^0\to K^+K^-)}{\Gamma(D^0\to K^-\pi^+)}\right)^{1/2}-\left(\frac{\Gamma(D^0\to\pi^-\pi^+)}{\Gamma(D^0\to K^-\pi^+)}\right)^{1/2}\leq\left|1+\frac{D}{A}\right|\left|\frac{V_{us}}{V_{ud}}-\rho\right|\leq\left(\frac{\Gamma(D^0\to K^+K^-)}{\Gamma(D^0\to K^-\pi^+)}\right)^{1/2}+\left(\frac{\Gamma(D^0\to\pi^+\pi^-)}{\Gamma(D^0\to K^-\pi^+)}\right)^{1/2}. \tag{6}$$

These inequalities are not inconsistent with preliminary measurements[9] of the relevant branching ratios, $\Gamma(K^+K^-)/\Gamma(K^-\pi^+)=0.113\pm0.030$ and $\Gamma(\pi^+\pi^-)/\Gamma(K^-\pi^+)=0.033\pm0.015$, and theoretical estimates[6,7] constraining ρ even for modest values of D/A. However, taking literally the central values of Shrock *et al.* in Ref. 7 and central values of the experiment, we require $|D/A|\gtrsim 5$.

(3) The decay $D^0\to K^0\bar{K}^0$, previously forbidden, now occurs at a rate proportional to $\approx|V_{us}/V_{ud}-\rho|^2$.

(4) The ratio

$$\left(\frac{\Gamma(D^+\to\pi^+\pi^0)}{\Gamma(D^+\to\bar{K}^0\pi^+)}\right)=\tfrac{1}{2}|\rho|^2 \tag{7}$$

affords a direct measure of ρ.

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[8]$|V_{us}/V_{ud}|$ is very nearly $\tan\theta_C$, see Ref. 6.

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