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Two-Body Hadronic Decays of *D* Mesons

V. Barger and S. Pakvasa

Department of Physics and Astronomy, University of Hawaii at Manoa, Honolulu, Hawaii 96822
and Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

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This Letter accounts for the observed (i) enhancement of $\Gamma(D^0\to K^-K^+)/\Gamma(D^0\to K^-\pi^+)$ and (ii) suppression of $\Gamma(D^0\to\pi^-\pi^+)/\Gamma(D^0\to K^-\pi^+)$ relative to $\tan^2\theta_C$ in terms of (i) the ratio f_K/f_π of leptonic decay constants and (ii) different effective Cabibbo angles in non-charm and charm sectors. Strong new constraints on mixing angles in the six-quark model are implied.

In the four-quark model of the weak interaction,[1] relations between two-body channels in weak decays of charmed hadrons have been derived from SU(3) symmetry.[2] In particular, the ratios of Cabibbo-suppressed to allowed decay modes for D^0 decay were predicted to be

$$\frac{\Gamma(D^0\to K^-K^+)}{\Gamma(D^0\to K^-\pi^+)}=\frac{\Gamma(D^0\to\pi^-\pi^+)}{\Gamma(D^0\to K^-\pi^+)}=\tan^2\theta_C\simeq 0.05.$$

Taking into account phase-space corrections based on physical π and K masses, the SU(3) predictions become

$$\Gamma(D^0\to K^-K^+)/\Gamma(D^0\to K^-\pi^+)=4.7\%,$$
$$\Gamma(D^0\to \pi^-\pi^+)/\Gamma(D^0\to K^-\pi^+)=5.5\%.$$

Recent measurements at SPEAR are markedly different from these original theoretical expectations. The reported experimental ratios are[3]

$$\begin{aligned}&\Gamma(D^0\to K^-K^+)/\Gamma(D^0\to K^-\pi^+)=(11.3\pm 3.0)\%,\\&\Gamma(D^0\to \pi^-\pi^+)/\Gamma(D^0\to K^-\pi^+)=(3.3\pm 1.5)\%.\end{aligned}\tag{1}$$

In this Letter, we show that the experimental results can be understood in a model based on charmed-quark decay with hard-gluon corrections. Essential ingredients of the explanation are (i) known symmetry breaking in the ratio of K to π leptonic decay constants and (ii) weak-current angles in the six-quark model[4] allowed by analyses of the K^0-$\bar{K}^0$ mass matrix.[5,6]

Weak decays of mesons that have new flavor are nominally presumed to proceed through the decay of the heavy quark, with the light-quark constituent acting as a spectator. For two-body decays, estimates suggest that diagrams in which a quark-antiquark pair is created from the vacuum are negligible. With use of this model, charm-decay rates into two-body hadronic channels were first examined in asymptotically free gauge theory by Ellis, Gaillard, and Nanopoulos.[7] Further calculations of these modes were made by Fakirov and Stech[8] and then by Cabibbo and Maiani.[9] In the present work we extend the predictions to Cabibbo-suppressed two-body decays, with the mixing angles of the six-quark model.

The effective nonleptonic Lagrangian for $c\to\alpha u\bar{\beta}$ including the short-distance renormalization effects of hard gluons is[7,10]

$$\mathcal{L}_{\text{eff}}=\frac{G}{\sqrt{2}}U_{u\beta}U_{c\alpha}^*\left[\tfrac{1}{2}(c_++c_-)(\bar{u}\beta)(\bar{\alpha}c)+\tfrac{1}{2}(c_+-c_-)(\bar{\alpha}\beta)(\bar{u}c)\right],\tag{2}$$

where $(q_1\bar{q}_2)$ denotes a color-symmetric $V-A$ current and U is the Kobayashi-Maskawa[4] mixing matrix of the six-quark model (hereafter called KM matrix). The renormalization constants are $c_-=[\alpha(m_c^2)/$

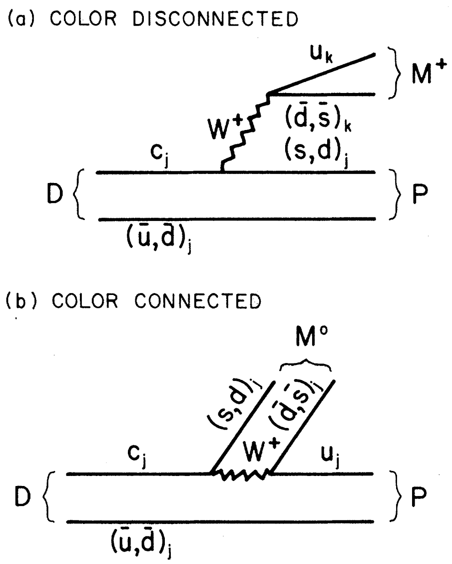

FIG. 1. Diagrams for two-body decays of D mesons; the color indices of the quarks are denoted by subscripts j and k.

$\alpha(m_W^2)]^{\gamma}$ and $c_+ = (c_-)^{-1/2}$, with $\gamma = 12/(33-2F)$ and $\alpha(m^2) = \pi\gamma/\ln(m^2/\Lambda^2)$. In the free-quark limit, $c_- = c_+ = 1$. With $F=4$, $\Lambda = 0.5$ GeV, $m_c = 1.5$ GeV, and $m_W = 84$ GeV, we estimate

$$c_- = 2.09, \quad c_+ = 0.69. \tag{3}$$

With neglect of quark-antiquark pair creation, the matrix element of the Lagrangian between color-singlet hadronic states factorizes in the form

$$\langle MP|\mathfrak{L}_{\text{eff}}|D\rangle = \frac{G}{\sqrt{2}} U_{u\beta}U_{c\alpha}{}^*\chi_+ \langle M|(\bar{u}\beta)|0\rangle\langle P|(\bar{\alpha}c)|D\rangle \tag{4}$$

for $D(c\bar{q}) \to M^+(u\bar{\beta}) + P(\alpha\bar{q})$, and

$$\langle MP|\mathfrak{L}_{\text{eff}}|D\rangle = \frac{G}{\sqrt{2}} U_{u\beta}U_{c\alpha}{}^*\chi_- \langle M|(\bar{\alpha}\beta)|0\rangle\langle P|(\bar{u}c)|D\rangle \tag{5}$$

for $D(c\bar{q}) \to M^0(\alpha\bar{\beta}) + P(u\bar{q})$, where $\chi_\pm = \frac{2}{3}c_+ \pm \frac{1}{3}c_-$. Equations (4) and (5) are obtained from Fierz rearrangements of the terms in Eq. (2) [e.g., $(\bar{\alpha}\beta)(\bar{u}c) = \frac{1}{3}(\bar{u}\beta)(\bar{\alpha}c)$ + color-octet pieces]. The amplitude proportional to χ_+ corresponds to the color-disconnected diagram of Fig. 1(a), and the amplitude proportional to χ_- corresponds to the color-connected diagram of Fig. 1(b). From Eq. (3) we estimate $\chi_+ = 1.16$ and $\chi_- = -0.24$. To test the negative sign of χ_- predicted by renormalization, an interference of χ_+ and χ_- amplitudes must be measured.

For the single-particle matrix elements, the contribution of the axial current is $\langle M|(\bar{u}\beta)|0\rangle = if_M q_\mu$, where $f_M = f_\pi$ for $M = \pi^+$, $f_M = f_\pi/\sqrt{2}$ for $M = \pi^0$, and $f_M = f_K$ for $M = K^+$ or K^0. Here $f_\pi = 0.96 m_\pi$ and $f_K = 1.23 m_\pi$ are the usual pion and kaon leptonic decay constants. The contraction of q_μ with the matrix element of the $D \to P$ transition is proportional to the scalar form factor f_S of $D \to Pl\nu$ decay,[11]

$$q_\mu \langle P|(\bar{\alpha}c)|D\rangle = f_S{}^{D\to P}(M^2)(D^2 - P^2). \tag{6}$$

Here we have denoted the mass of each particle by the particle's symbol. The $q^2 = 0$ value of $f_S(q^2)$ is $f_S(0) = f_+(0)$, where $f_+(q^2)$ is the $D \to Pl\nu$ vector form factor. Assuming that the scalar form factor is dominated by a single meson D_S for $D \to \pi$ (F_S for $D \to \bar{K}$), we have

$$f_S{}^{D\to P}(M^2) = f_+{}^{D\to P}(0)/(1 - M^2/S^2), \tag{7}$$

where S is the mass of the scalar meson with the quantum numbers of $D\bar{P}$. Thus the diagrams of Fig. 1 lead to decay amplitudes of the form

$$A(D \to MP) = \frac{-iG}{\sqrt{2}} U_{u\beta}U_{c\alpha}{}^*\chi_\pm f_M f_+{}^{D\to P}(0)\frac{(D^2 - P^2)}{(1 - M^2/S^2)}.$$

For a given mode all contributing diagrams must be summed. The decay rate is

$$\Gamma(D \to MP) = \lambda^{1/2}(D^2, M^2, P)|A(D \to MP)|^2/16\pi D^3,$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

Relative D rates can be calculated more reliably than absolute rates because ratios of widths are independent of SU(4) breaking. We define effective Cabibbo angles in the noncharm and charm sectors of the six-quark weak current as

$$\tan^2\theta_C = |U_{us}|^2/|U_{ud}|^2, \quad \tan^2\bar{\theta}_C = |U_{cd}|^2/U_{cs}|^2.$$

Then, in the excellent approximation of neglecting π^2/D^2, D-decay rates relative to $D^0 \to K^-\pi^+$

are given in the model by

$$\frac{\Gamma(D^0\to K^-K^+)}{\Gamma(D^0\to K^-\pi^+)}=\frac{\Gamma(D^+\to \bar{K}^0K^+)}{\Gamma(D^0\to K^-\pi^+)}=\tan^2\theta_C\left(\frac{f_K}{f_\pi}\right)^2\frac{(1-4K^2/D^2)^{1/2}}{(1-K^2/D^2)(1-K^2/F_S)^2},$$
$$\frac{\Gamma(D^0\to\pi^-\pi^+)}{\Gamma(D^0\to K^-\pi^+)}=\frac{2\Gamma(D^+\to\pi^0\pi^+)}{\Gamma(D^0\to K^-\pi^+)}=\tan^2\tilde{\theta}_C\frac{r^2}{(1-K^2/D^2)^3}, \tag{8}$$
$$\frac{\Gamma(D^0\to\bar{K}^0\pi^0)}{\Gamma(D^0\to K^-\pi^+)}=\frac{1}{2}\left(\frac{\chi_-/\chi_+}{(1-K^2/D^2)(1-K^2/D_S^2)}\right)^2,\quad \frac{\Gamma(D^+\to\bar{K}^0\pi^+)}{\Gamma(D^0\to K^-\pi^+)}=\left(1+\frac{\chi_-}{\chi_+}\frac{f_K}{f_\pi}\frac{r}{(1-K^2/D^2)(1-K^2/D_S^2)}\right)^2,$$

where $r\equiv f_+^{D\to\pi}/f_+^{D\to K}$ at $q^2=0$. It seems likely that SU(3) breaking in f_+ at $q^2=0$ will be small; so we henceforth take $r=1$. With the measured values of $\tan\theta_C$, f_K, f_π, K, and D, and the assumed mass values $D_S=D+\pi$ and $F_S=F+\pi$, we find the following relative rates in D^0 decay:

$$\Gamma(D^0\to K^-K^+)/\Gamma(D^0\to K^-\pi^+)=8.5\%,$$
$$\frac{\Gamma(D^0\to\pi^-\pi^+)}{\Gamma(D^0\to K^-\pi^+)}=\left(\frac{\tan^2\tilde{\theta}_C}{\tan^2\theta_C}\right)\times 6.3\%,\quad \Gamma(D^0\to\bar{K}^0\pi^0)/\Gamma(D^0\to K^-\pi^+)=2.8\%. \tag{9}$$

The above prediction for $\Gamma(D^0\to K^-K^+)/\Gamma(D^0\to K^-\pi^+)$ is within 1 standard deviation of the measurement, Eq. (1). The predicted rate for $\Gamma(D^0\to\pi^-\pi^+)/\Gamma(D^0\to K^-\pi^+)$ is critically dependent on the effective Cabibbo angle in the charm sector. From calculations of the K_L-K_S mass difference and CP nonconservation within the framework of the siz-quark model, we found the bounds[5,12] $0.17\le|\tan\tilde{\theta}_C|\le 0.24$ (solution I), $0.24\le|\tan\tilde{\theta}_C|\le 0.32$ (solution II), for a t-quark mass greater than 9 GeV. For solution I the CP-nonconserving phase δ of the KM matrix lies in the first quadrant and for solution II δ lies in the second quadrant, with the conventions of Ref. 4. Consistency of Eq. (9) with Eq. (1) rules out solution II; with solution I, we predict the lower bound

$$\Gamma(D^0\to\pi^-\pi^+)/\Gamma(D^0\to K^-\pi^+)\ge 3.6\%.$$

This lower limit is close to the central value of the measurement. For Eq. (9) to be within 1 standard deviation of the $\pi^-\pi^+/K^-\pi^+$ measurement, we must have

$$0.17\le|\tan\tilde{\theta}_C|\le 0.195. \tag{10}$$

In the six-quark model, the corresponding contraint on the mixing angles is

$$0.17<\left|\frac{s_1c_2}{c_1c_2c_3+s_2s_3e^{i\delta}}\right|<0.195, \tag{11}$$

where $c_i=\cos\theta_i$ and $s_i=\sin\theta_i$. Consolidating this with the results of Ref. 4, the preferred ranges of the angles for t-quark masses of 9 GeV, (14 GeV), and [30 GeV], respectively, are

$$s_3=0.50-0.35,\ (0.50-0.40),\ [0.50-0.46];$$
$$s_2=0.71-0.60,\ (0.66-0.58),\ [0.60-0.57]; \tag{12}$$
$$\delta=0.004-0.006,\ (0.0027-0.0035),\ [0.0014-0.0016];$$

where the upper limits correspond to $\tan\tilde{\theta}_C=0.17$ and the lower limits to $\tan\tilde{\theta}_C=0.195$.

It goes without saying that limits on $\tan\tilde{\theta}_C$ can be tested in other D-decay modes, as well as in the relative total rates for $D\to(S=0)$ and $D\to(S=-1)$. The bounds on s_2, s_3, and δ can eventually be tested in B and Υ decays. From Eq. (12) preferred ranges for coupling strength ratios[12] based on t-quark masses of 9 GeV, (14 GeV), and [30 GeV] are

$$|b\to u|/|b\to c|=0.42-0.27,\ (0.56-0.42),\ [0.86-0.77];$$
$$|t\to s|/|t\to b|=0.26-0.29,\ (0.18-0.20),\ [0.11-0.12]; \tag{13}$$
$$|t\to d|/|t\to b|=0.17-0.15,\ (0.16-0.13),\ [0.14-0.13].$$

For the range of angles in Eq. 12, the predicted lifetimes[12] in a heavy-quark decay model with gluon corrections are

$$3.6\times10^{-13}<\tau_D<3.9\times10^{-13}\ \text{sec},\quad 0.4\times10^{-13}<\tau_B(m_B=5.0\ \text{GeV})<1.1\times10^{-13}\ \text{sec} \tag{14}$$

for a t-quark mass in the 9–30-GeV range. The prediction for the D lifetime is in good agreement with the recently measured values,[13]

$$\tau(D^0) = (3.5^{+3.5}_{-1.7}) \times 10^{-13} \text{ sec},$$

$$\tau(D^+) = (2.5^{+3.5}_{-1.5}) \times 10^{-13} \text{ sec}.$$

In the heavy-quark decay model the D^+ and D^0 total decay rates are predicted to be equal. Hence from Eq. (8) we obtain

$$B(D^+ \to \bar{K}^0\pi^+)/B(D^0 \to K^-\pi^+) = 0.49. \quad (15)$$

This agrees well with the present experimental value[14]

$$B(D^+ \to \bar{K}^0\pi^+)/B(D^0 \to K^-\pi^+) = 0.68 \pm 0.33. \quad (16)$$

A more precise experimental determination will provide a crucial test of the hard-gluon renormalization results in Eq. (15).

From Eqs. (8) and (10), we predict

$$\Gamma(D^+ \to \bar{K}^0K^+)/\Gamma(D^+ \to \bar{K}^0\pi^+) = 15.7\%,$$

$$\Gamma(D^+ \to \pi^0\pi^+)/\Gamma(D^+ \to \bar{K}^0\pi^+) = (3.3\text{–}4.3)\%.$$

The dramatic enhancement of the $\bar{K}K$ mode relative to $\bar{K}\pi$ is a consequence of partial cancellation between color-connected and -disconnected diagrams in the $\bar{K}\pi$ amplitude.

We now turn to the absolute rate prediction for $D^0 \to K^-\pi^+$:

$$\Gamma(D^0 \to K^-\pi^+) = G_F^{\,2}D^3(1 - K^2/D^2)^3 \cos^2\theta_C \cos^2\tilde{\theta}_C f_\pi^{\,2}[f_+^{\,D\to K}(0)]^2/32\pi$$
$$= 1.67 \times 10^{11}[f_+^{\,D\to K}(0)]^2 \text{ sec}^{-1}.$$

With the lowest value of $\tau(D^0)$ in Eq. (14), we get $B(D^0 \to K^-\pi^+) = [f_+^{\,D\to K}(0)]^2 \times 6.0\%$. To reproduce the experimental value[14] $B(D^0 \to K^-\pi^+) = (2.2 \pm 0.6)\%$, substantial symmetry breaking is necessary, with $f_+^{\,D\to K}(0) = 0.61 \pm 0.08$. Such large SU(4)-symmetry breaking in the vector form factor is not ruled out.[15] An independent determination of $f_+^{\,D\to K}(0)$ from $D \to Kl\nu$ can test this.

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