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Euclidean Group as a Dynamical Symmetry of Surface Fluctuations: The Planar Interface and Critical Behavior

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The statistical mechanics of the interface between two discrete thermodynamic phases is studied in terms of a field f which describes the deviation of the interface from planar. Exploiting the dynamical Euclidean invariance of the Hamiltonian of the field f , we construct ϵ expansions for Ising-like critical behavior in $1+\epsilon$ dimensions, with a critical temperature of order ϵ . Scaling functions for the interface profile and width in a pinning potential are obtained.

Recent interest in the interface between two discrete thermodynamic phases has focused on two main characteristics. One is the divergence of the interfacial width with the correlation length¹ as the temperature T is raised to the critical temperature T_c ; this behavior can be described for example by ϵ expansions in $4-\epsilon$ dimensions² starting from the conventional Ginzburg-Landau-Wilson Hamiltonian. The second is the collective wandering of the interface as a whole,³ which arises from the inability of surface tension to maintain a planar interface. In particular, the lowest-order calculations⁴ in the capillary-wave approach imply that in the absence of a stabilizing potential, such as gravity, the mean square width of the interface diverges for all $T < T_c$ in the thermodynamic limit for dimension of space

$d \leq 3$. This statement must be modified for lattice Ising models where for $d = 3$ the finite lattice spacing succeeds in limiting the wandering of the interface to a finite region for all temperatures less than a roughening temperature⁵ T_R ($< T_c$). Above T_R the wandering is unbounded, as in the continuum models using capillary waves.

The capillary-wave approach is basically a low-temperature description of the interface. It incorporates into the statistical mechanics apparently only the low-energy configurations—the large-distance deviations from planar of an essentially sharp interface, with surface tension as the restoring potential. In this paper we describe how the renormalization group enables one to control the short-distance behavior in the interface in such a model. Hence we can construct, from this model of a sharp interface, a renormalized interface, whose local width diverges as the correlation length when T approaches T_c . In order to control such calculations in a perturbation theory in T it is necessary that T_c be small in some sense. In fact, we show that perturbative calculations are limited to an ϵ expansion in $1 + \epsilon$ dimensions,⁶ with a T_c of order ϵ . The fact that the capillary waves transform as a nonlinear realization of the Euclidean group of d dimensions⁷ plays an important role in these calculations, and conversely these calculations establish that the capillary waves can be interpreted as the Goldstone modes whose fluctuations lower to zero the critical temperature as $d \rightarrow 1^+$. They therefore play a role for systems with discrete internal symmetry which is equivalent to that played by spin waves in a continuous internal symmetry.⁸

Our starting point is the reduced Hamiltonian for a field f which describes the deviation from planar of an essentially sharp interface (see for example Bull, Lovett, and Stillinger⁴),

$$\mathcal{H}(f) = T^{-1} \int d^{d-1}x \left\{ [1 + (\nabla f)^2]^{1/2} + \frac{1}{2} m^2 f^2 \right\}. \quad (1)$$

The first term is the surface area of the interface; the coefficient $1/T$ represents $\sigma/k_B T$ where σ is the interfacial energy per unit area at zero temperature. The mass term in \mathcal{H} represents a pinning potential such as gravity, or step-function magnetic field $H(z) = H\epsilon(z)$. Starting from the Ginzburg-Landau-Wilson description of the interface,² it is straightforward to see that m^2 represents $HM\xi^{-1}$ for such a magnetic field, where M is spontaneous magnetization and ξ is the correlation length. The loop expansion for the partition function $Z = \int Df e^{-\mathcal{H}}$ and correlation functions is an expansion in powers of T ; it is obtained by expansion of the square root to give a free propagator $T(q^2 + m^2)^{-1}$ and vertices $-\frac{1}{8}T^{-1}[(\nabla f)^2]^2$, etc.

The ultraviolet and infrared properties of the model are controlled by the dimension of T . Since \mathcal{H} is dimensionless, $T = \kappa^{-(d-1)}$ where κ is an inverse length, or momentum. We therefore expect that for $d > 1$ the loop integrals which accompany powers of T in perturbation theory do not change the low-momentum behavior of the free propagator and vertices. This property ensures that the description of the large-distance interface wandering by the evaluation of $\langle f^2 \rangle$ at lowest order in perturbation theory is valid. Correspondingly \mathcal{H} is naively nonrenormalizable for $d > 1$, i.e., its high-momentum behavior is not controllable in straightforward perturbation in T . Now, it is intuitively clear that if we are to have an interface which is "fuzzy" over a length scale of the correlation length ξ , then this will be obtained only by incorporating fluctuations of a sharp interface over *all* momenta $q > \xi^{-1}$. The description of critical behavior in the model (1) is therefore equivalent to controlling the high-momentum behavior of the model.

This problem can be solved perturbatively in $1 + \epsilon$ dimensions using renormalization-group techniques. Our approach follows closely the calculations of Brézin and co-workers⁹ for Heisenberg models in $2 + \epsilon$ dimensions. Using dimensional regularization,⁹ we have calculated the two- and four-point vertex functions up to and including two loops.¹⁰ The renormalization required to remove poles in $(d - 1)$ are a coupling-constant renormalization

$$T \equiv \kappa^{-(d-1)} t_0 = \kappa^{-(d-1)} \left\{ t + t^2(d-1)^{-1} + t^3 \left[(d-1)^{-2} + \frac{1}{4}(d-1)^{-1} \right] + O(t^4) \right\},$$

and a mass renormalization $m^2/T = m_R^2 \kappa^{d-1}/t$. Here t is the dimensionless renormalized coupling constant; it is defined to absorb a factor

$$I = - \int d^{d-1}q (2\pi)^{-d+1} q^2 (q^2 + 1)^{-1},$$

which can be extracted from each loop integral. No wave-function renormalization is required. We have shown that this is true to all orders in perturbation theory as a consequence of the Ward identities

resulting from the nonlinear Euclidean-group transformations on f in (1). This can also be understood because f represents a length, the deviation from planar, and hence develops no anomalous dimension.

The resulting renormalization-group equation for all vertex functions Γ_R is

$$\left(\kappa \frac{\partial}{\partial \kappa} + \beta(t) \frac{\partial}{\partial t} + \gamma_1(t) m_R^2 \frac{\partial}{\partial m_R^2} \right) \Gamma_R = 0, \quad (2)$$

where $\beta(t) = (d-1)t - t^2 - \frac{1}{2}t^3 + \dots$ and $\gamma_1(t) = -(d-1) + (\beta/t)$. The fixed points of $\dot{t} = \beta(t)$ are (i) $t=0$, which is infrared stable [$\beta'(0) > 0$] and controls the mean-field low-momentum behavior of \mathcal{H} , and (ii) $t_c = (d-1) - \frac{1}{2}(d-1)^2 + O((d-1)^3)$, which is ultraviolet stable [$\beta'(t_c) = -(d-1) - \frac{1}{2}(d-1)^2 + O((d-1)^3)$], and is the effective coupling for the high-momentum behavior. The interpretation of t_c as the critical temperature is supported by the following arguments.

(i) It is convenient to introduce a physical length scale, the correlation length $\xi \equiv \kappa^{-1} \zeta(t)$, which is independent of the pinning potential m_R , and invariant under a renormalization-group transformation, i.e., $[\kappa \partial/\partial \kappa + \beta(t) \partial/\partial t] \xi = 0$. The solution of this equation is

$$\xi = \kappa^{-1} t^{1/(d-1)} \exp \int_0^t dt' [1/\beta(t') - t^{-1}(d-1)^{-1}].$$

For small t , we see $\xi = \kappa^{-1} t^{1/(d-1)} [1 + O(t)]$; this correlation length can be identified with the phase coherence length introduced for Heisenberg and Ising systems.¹¹ As t is increased to t_c , ξ diverges according to $\xi \propto (t_c - t)^{-\nu}$ where $\nu = -1/\beta'(t_c) = (d-1)^{-1} - \frac{1}{2} + O(d-1)$. This agrees with the lowest-order results from lattice models.⁶

(ii) The interfacial free energy per unit area contains a contribution $1/T$ from the Hamiltonian (1). As $t \rightarrow t_c$, the bare temperature $T \rightarrow \infty$ and hence the interfacial energy vanishes. The way in which it vanishes is obtained by solving $dt_0/dt = (d-1)t_0/\beta(t)$ for t_0 , leading to

$$1/T = \kappa^{d-1} \exp \left[- (d-1) \int^t dt' / \beta(t') \right] \propto \xi^{-(d-1)}.$$

Thus the interfacial energy vanishes according to the expected scaling law (see, e.g., Widom¹ and also Ref. 12).

(iii) The solution of the renormalization-group equation (2) for the mean square width of the interface $\langle f^2 \rangle$ has the scaling form $\langle f^2 \rangle = \xi^2 P_2(y)$ where $y = m_R^2 \xi^{d+1} \kappa^{d-1} t^{-1}$ is the scaling variable associated with the pinning potential m_R^2 . We see that this is compatible with the identification $m^2 \sim HM \xi^{-1}$, i.e., HM scales like ξ^{-d} as expected. The loop expansion gives the function $P_2(y)$ as a power series in $y^{(d-1)/2}$. The explicit form of the first two terms is $P_2(y) = y^{-(3-d)/2} [1 - y^{(d-1)/2} + O(y^{d-1})]$; we have discarded numerical coefficients (of order 1) which are only physically significant when $O(y^{d-1})$ is calculated. The behavior of $P_2(y)$ for y large, i.e., large pinning potential, can be obtained from a simple model similar to that used by Weeks⁵ to study roughening. If we take cells of base area $\sim \xi^{d-1}$, the interface height over a single cell may be modeled by a single variable f , with a probability distribution proportional to $\exp(-m^2 \xi^{d-1} f^2/T)$; we neglect intercell interactions. This model for large y gives $\langle f^2 \rangle = T m^{-2} \xi^{-(d-1)} \propto \xi^2 y^{-1}$. Thus we see that for $y \ll 1$ the divergence of $P_2(y)$ describes the divergence of interface width due to wandering for $d < 3$, for $y \sim 1$, $\langle f^2 \rangle \sim \xi^2$, and finally $\langle f^2 \rangle$ tends to zero for large pinning potential ($y \rightarrow \infty$). The $\ln y$ term expected in three dimensions⁴ is also indicated.

(iv) Loop expansions can be similarly obtained for the expectation values $\langle f^{2n} \rangle = \xi^{2n} P_{2n}(y)$. These functions can be regarded as the moments $P_{2n} = \int_{-\infty}^{\infty} P(z) z^{2n} dz$ of a probability distribution $P(z)$ for the position of the center of the interface. Given the explicit forms of the first two orders in $\langle f^{2n} \rangle$, one can perform an inverse Mellin transform to obtain the probability distribution $P(z)$. To first order, a Gaussian distribution is obtained, a result reported by Huang and Webb¹³ for the model of Buff, Lovett, and Stillinger.⁴ In second order, a new type of graph appears for $n \geq 2$. If we denote by R the ratio of the integral associated with this new type to the $n=1$ type, corrections to the Gaussian distribution can be written explicitly. The scaling form of $P(z)$ produced by this kind of renormalization-group treatment is

$$P(z) \propto \exp \left\{ -\frac{1}{2y} (z/\xi)^2 \left[1 - (R-1) y^{(d-1)/2} + \frac{1}{6} R y (z/\xi)^2 + \dots \right] \right\}. \quad (3)$$

In principle, systematic corrections to higher order can also be obtained. The function $P(z)$ can also be associated with a mean interface profile $\varphi(z)$ according to $d\varphi/dz = P(z)$. For small z , φ is the error function, and we may anticipate good fits to data on optical reflectivity¹⁴ with this distribution.

In conclusion we see that the renormalization group enables one to control the high-wave-vector capillary waves. These waves can be identified as the Goldstone modes whose fluctuations reduce the critical temperature of Ising-like models to zero as the dimension of space is lowered to 1. Perturbative calculations yield ϵ expansions in $1+\epsilon$ dimensions. Although these are numerically rather poor for the exponent ν , and the role of nonperturbative effects such as overhangs¹⁵ requires elucidation, these calculations do appear to support a useful interface phenomenology.

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Two-Body Hadronic Decays of D Mesons

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This Letter accounts for the observed (i) enhancement of $\Gamma(D^0 \rightarrow K^- K^+)/\Gamma(D^0 \rightarrow K^- \pi^+)$ and (ii) suppression of $\Gamma(D^0 \rightarrow \pi^- \pi^+)/\Gamma(D^0 \rightarrow K^- \pi^+)$ relative to $\tan^2 \theta_C$ in terms of (i) the ratio f_K/f_π of leptonic decay constants and (ii) different effective Cabibbo angles in non-charm and charm sectors. Strong new constraints on mixing angles in the six-quark model are implied.

In the four-quark model of the weak interaction,¹ relations between two-body channels in weak decays of charmed hadrons have been derived from SU(3) symmetry.² In particular, the ratios of Cabibbo-suppressed to allowed decay modes for D^0 decay were predicted to be

$$\frac{\Gamma(D^0 \rightarrow K^- K^+)}{\Gamma(D^0 \rightarrow K^- \pi^+)} = \frac{\Gamma(D^0 \rightarrow \pi^- \pi^+)}{\Gamma(D^0 \rightarrow K^- \pi^+)} = \tan^2 \theta_C \approx 0.05.$$

Taking into account phase-space corrections based on physical π and K masses, the SU(3) predictions become

$$\begin{aligned} \Gamma(D^0 \rightarrow K^- K^+)/\Gamma(D^0 \rightarrow K^- \pi^+) &= 4.7\%, \\ \Gamma(D^0 \rightarrow \pi^- \pi^+)/\Gamma(D^0 \rightarrow K^- \pi^+) &= 5.5\%. \end{aligned}$$

Recent measurements at SPEAR are markedly different from these original theoretical expectations. The reported experimental ratios are³

$$\begin{aligned} \Gamma(D^0 \rightarrow K^- K^+)/\Gamma(D^0 \rightarrow K^- \pi^+) &= (11.3 \pm 3.0)\%, \\ \Gamma(D^0 \rightarrow \pi^- \pi^+)/\Gamma(D^0 \rightarrow K^- \pi^+) &= (3.3 \pm 1.5)\%. \end{aligned} \quad (1)$$

In this Letter, we show that the experimental re-

sults can be understood in a model based on charmed-quark decay with hard-gluon corrections. Essential ingredients of the explanation are (i) known symmetry breaking in the ratio of K to π leptonic decay constants and (ii) weak-current angles in the six-quark model⁴ allowed by analyses of the K^0 - \bar{K}^0 mass matrix.^{5, 6}

Weak decays of mesons that have new flavor are nominally presumed to proceed through the decay of the heavy quark, with the light-quark constituent acting as a spectator. For two-body decays, estimates suggest that diagrams in which a quark-antiquark pair is created from the vacuum are negligible. With use of this model, charm-decay rates into two-body hadronic channels were first examined in asymptotically free gauge theory by Ellis, Gaillard, and Nanopoulos.⁷ Further calculations of these modes were made by Fakirov and Stech⁸ and then by Cabibbo and Maiani.⁹ In the present work we extend the predictions to Cabibbo-suppressed two-body decays, with the mixing angles of the six-quark model.

The effective nonleptonic Lagrangian for $c \rightarrow \bar{u} \bar{b}$ including the short-distance renormalization effects of hard gluons is^{7, 10}

$$\mathcal{L}_{\text{eff}} = \frac{G}{\sqrt{2}} U_{ub} U_{c\alpha}^* \left[\frac{1}{2} (c_+ + c_-) (\bar{u}\beta) (\bar{\alpha}c) + \frac{1}{2} (c_+ - c_-) (\bar{\alpha}\beta) (\bar{u}c) \right], \quad (2)$$

where $(q_1 \bar{q}_2)$ denotes a color-symmetric $V-A$ current and U is the Kobayashi-Maskawa⁴ mixing matrix of the six-quark model (hereafter called KM matrix). The renormalization constants are $c_- = [\alpha(m_c^2)/$