Generation of Branch Imbalance by the Interaction between Supercurrent and Thermal Gradient

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A thermal gradient in a current-carrying superconductor gives rise to a branch imbalance and hence to a potential difference between quasiparticles and condensate. We present a theory of this effect valid for a dirty and for a clean superconductor which is in good agreement with recent experiments.

Although in the past years there has been an increased activity of research in this field, thermoelectric phenomena in superconductors seem to remain mysterious.[1] Going back in history to 1927, Meissner found evidence that a thermoelectric voltage was absent in a steady-state superconductor. Seventeen years later, Ginzburg pointed out that there still exist thermoelectric phenomena which can be observed. For instance, there has to be a contact potential between superconductors of different temperatures which is the analog of the mechano-caloric effect (fountain effect) in superfluid ^{4}He. Unfortunately, because of the exceedingly small heat capacity of the degenerate electrons, this effect and many related phenomena are difficult to measure. Their interpretation leaves many questions unanswered. As a striking example, we mention the thermoelectric generation of magnetic flux in a bimetallic superconducting ring where recent results[2] show a discrepancy by orders of magnitude with accepted theories.[3]

In an other type of experiment,[4] a thermally induced difference in the electrochemical potentials of the normal and superfluid component has been observed. Since the pioneering work of Tinkham,[5] such a difference is known to arise from an imbalance in the population of the excitation spectrum branches. However, it is unclear how such a branch imbalance can be generated by a thermal gradient alone.[6]

As the experience has been rather discouraging so far, it seems to us important that recent observations by Clarke, Fjordbøge, and Lindelof[7] can be explained quantitatively as we will show below. These authors report on the generation of branch imbalance by the interaction between a supercurrent and a thermal gradient and they found that the magnitude of this effect is bilinear in both of the driving forces.

For convenience, we consider first a dirty superconductor ($T_c\tau_{\rm imp}\ll 1$), in which case, kinetic equations for the nonequilibrium part δf_E of the quasiparticle distribution function have been presented in a paper by the authors.[8] We have shown there that the branch imbalance Q can be computed by means of the relation[9]

$$Q^* = 2N(0)\int dE\, N_1(E)\,\delta f_E\,, \tag{1}$$

where N_1 is the reduced density of states.

For further discussion, it is advantageous to split the distribution function into two parts $\delta f_E^{(T)}$ and $\delta f_E^{(L)}$, where the transverse (T) and longitudinal (L) parts are even and odd functions of E, respectively. The local variation in temperature $T(\vec r) = T + \delta T(\vec r)$ creates a longitudinal nonequilibrium distribution

$$\delta f_E^{(L)} = -n_T'(E)(E/T)\,\delta T(\vec r)\,, \tag{2}$$

where n_T' is the energy derivative of the Fermi function. It has been demonstrated, that in the presence of a superfluid velocity $2m\vec v_s = -\nabla\theta - 2e\vec A$, the longitudinal and transverse modes are coupled.[10] Thus, $\delta f_E^{(T)}$ is determined as the solution of the Boltzmann equation

$$-K(\delta f^{T}) + 2|\Delta|N_2(E)\,\delta f_E^{(T)} - 4mD\vec v_s N_2(E)R_2(E)\nabla\delta f_E^{(L)} = 0\,, \tag{3}$$

where $D = v_F^2\tau_{\rm imp}/3$ is the diffusion coefficient and where we have assumed that $\delta f^{(T)}$ is constant in space and time. In this equation, K is the phonon collision integral. The second term describes the conversion of superfluid into normal fluid and vice versa (branch-imbalance relaxation) whereas the last term represents a generation mechanism proportional to the product $(\vec v_s\nabla T)$ of superfluid velocity and temperature gradient. We remember that in the presence of phonon scattering (collision rate $1/\tau_E$), there exists always a pair-breaking energy $\Gamma = 1/2\tau_E$. In this case, the spectral quantities are given by ($|\Delta| \to \Delta$)

$$\left.\begin{matrix} N_1 \\ R_1 \end{matrix}\right\} = \left\{\begin{matrix}\mathrm{Re} \\ \mathrm{Im}\end{matrix}\right\} \frac{E + i\Gamma}{[(E+i\Gamma)^2 - \Delta^2]^{1/2}};$$

$$\left.\begin{matrix} N_2 \\ R_2 \end{matrix}\right\} = \left\{\begin{matrix}\mathrm{Re} \\ \mathrm{Im}\end{matrix}\right\} \frac{i\Delta}{[(E+i\Gamma)^2 - \Delta^2]^{1/2}}. \quad (4)$$

These relations will be of importance in a moment.

We have shown in Section 5 of Ref. 8 how to solve Eq. (3) in the limit $\Delta \to 0$, and these arguments have been used by Eckern and Schön[11] to introduce as an approximation a reduced collision operator

$$-K(\delta f^T) = \tau_E^{-1} N_1(E)[\delta f_E^T + n_T'(E) \times \int dE' N_1(E') \delta f_{E'}{}^T], \quad (5)$$

which preserves the conservation of particle number. Effects of nonequilibrium phonons are neglected. Accordingly, we obtain for the branch imbalance the following expression:

$$Q^*/2N(0) = 4mD(\vec{v}_s \cdot \nabla T) T^{-1} V (1-Z)^{-1}, \quad (6)$$

where V and Z are the integrals

$$Z = -\int dE\, n_T'(E) \frac{N_1^2}{N_1 + 2\Delta\tau_E N_2}, \quad (7)$$

$$V = -\int dE\, n_T'(E) \frac{N_1 N_2 R_2 \tau_E E}{N_1 + 2\Delta\tau_E N_2}.$$

In the limit of small pair breaking $\Gamma \ll \Delta$ and small Δ such that $\Delta(\tau_E \Gamma)^{1/2} \ll T$ the range of integration is restricted to $E = O(\Delta)$ which allows us to put $n_T' = -1/4T$ and to neglect the energy dependence of τ_E. In this case the integrals yield [see also Eq. (44) of Ref. 8]

$$1 - Z = 2\Delta\tau_E J_2 = (\pi\Delta/4T)(2\Gamma\tau_E)^{1/2}, \quad (8)$$

$$V = \frac{\Delta}{8T}\left[\ln\frac{4\Delta}{\Gamma} + 2(2\tau_E\Gamma - 1)^{1/2} \arctan(2\tau_E\Gamma - 1)^{1/2}\right],$$

where we have also allowed for the possibility of additional sources of pair breaking in addition to phonon scattering; hence $\Gamma \geq 1/2\tau_E$.

In the limit $\Gamma = 1/2\tau_E$, the result can be put in the simple form

$$\frac{Q^*}{2N(0)} = \frac{2}{3\pi} \frac{p_F l_{imp}}{T} (\vec{v}_s \cdot \nabla T) \ln(8\Delta\tau_E). \quad (9)$$

It is interesting to note that, although electron-phonon collisions are responsible for branch-imbalance relaxation, the dependence of Q^* on τ_E is extremely weak.

A supercurrent acts as a source of pairbreaking adding to Γ a contribution $\delta\Gamma = 2m^2 D v_s^2$. Therefore, we must require $\delta\Gamma \ll 1/\tau_E$ in order to guarantee that Eq. (9) is valid. This is equivalent to the condition that v_s be smaller than the critical velocity at least by a factor $[\tau_E(T_c - T)]^{-1/2}$.

The corresponding calculations for a clean metal ($T_c \tau_{imp} \gg 1$) are rather involved if one uses the Green's-function technique of Ref. 8. As an alternative, we present a Boltzmann equation in the excitation representation which has been laid out by Aronov and Gurevich.[12]

In the presence of a uniform temperature gradient, and for the stationary situation which we are considering, the Boltzmann equation takes the form

$$-\frac{\partial \hat{E}_{\vec{p}}}{\partial \vec{p}} n_T'(\hat{E}_{\vec{p}}) \hat{E}_{\vec{p}} \frac{\nabla T}{T} + I\{n_{\vec{p}}\} = 0, \quad (10)$$

where I is the collision integral, $E_p = (\epsilon_p^2 + \Delta^2)^{1/2}$ and $\hat{E}_{\vec{p}} = E_p + \vec{p}\cdot\vec{v}_s$ is the excitation energy in the presence of a supercurrent. The branch imbalance can now be found from the relation $Q^* = 2N(0) \times \int d\epsilon_p \langle(\epsilon_p/E_p)\delta n_{\vec{p}}\rangle$, where $\langle\cdots\rangle$ denotes an angular average. Inspection shows that it is advantageous to multiply Eq. (9) by $N_1(E)(\epsilon_p/E_p) \times \delta(E - \hat{E}_{\vec{p}})$ and integrate with respect to ϵ_p. Furthermore, we introduce the notation[13]

$$\varphi(E, \vec{p}) = \int d\epsilon_p (\epsilon_p/E_p) \delta n_{\vec{p}}\, \delta(E - \hat{E}_{\vec{p}}). \quad (11)$$

Neglecting for the moment some complications (to be discussed later) in connection with the impurity collisions (which are otherwise assumed to be isotropic), we obtain from Eq. (10)

$$\tau_E^{-1}\{N_1(E - \vec{p}\cdot\vec{v}_s)\varphi(E,\vec{p}) + n_T'(E)\int dE' \tfrac{1}{2}[\varphi(E',\vec{p}) + \varphi(E',-\vec{p})]\} + \tau_{imp}^{-1}[\varphi(E,\vec{p}) - \langle\varphi(E,\vec{p}')\rangle] = 2(\vec{p}\cdot\nabla T/mT) n_T'(E) E, \quad (12)$$

where we have written down the phonon collision operator in the reduced form corresponding to Eq. (5). An important feature is the dependence of the density of states on the shifted energy $E - \vec{p}\cdot\vec{v}_s$. The solution for φ linear in $\vec{p}\cdot\vec{v}_s$ is obtained without difficulty in the case $\tau_{imp} \ll \tau_E$, and we essentially recover Eq. (6) with the following exception: The divergent integral $V = -\frac{1}{2} \times \int dE\, n_T'(E)(\Delta^2/\epsilon^2)$, leads to the previous value (in the case of $\Gamma/\Delta \ll 1$ and $\Gamma + 1/2\tau_E$) only if we cut off the integration below $E = \Delta + \Gamma/2$. In retrospect, it becomes clear that an expansion in powers of $p_F v_s$ requires that $p_F v_s \ll \Gamma$.

A detailed calculation reveals that, in Eq. (12), the impurity-scattering term $\varphi(E,\vec{p})/\tau_{imp}$ carries an extra factor

$$L(E,\vec{p}) = \left\langle \frac{E_p}{\epsilon_p}\frac{E_{p'}}{\epsilon_{p'}}\left[1 - \frac{\Delta^2}{E_p E_{p'}}\right]\right\rangle_{p'}, \tag{13}$$

where $E_p = E - \vec{p}\cdot\vec{v}_s$, etc. One can convince oneself that this factor is relevant only if $p_F v_s \gtrsim \Delta(\tau_{imp}/\tau_E)^{1/2}$. In particular, if $p_F v_s \gg T(\tau_{imp}/\tau_E)^{1/2}$, then phonon scattering becomes unimportant, and we obtain the result

$$\frac{Q^*}{2N(0)} = \frac{4}{15}\frac{p_F l_{imp}}{T}(\vec{v}_s \nabla T)\left[1 + \frac{\Delta}{2T}\ln\frac{a\Delta}{p_F v_s}\right], \tag{14}$$

where a constant of order unity.

In conclusion we may say that in a rather diverse variety of limiting cases, we have obtained almost the same expression for the branch imbalance $Q^* \sim (p_F l_{imp}/T)(\vec{v}_s\cdot\nabla T)$ which is generated by the interaction between supercurrent and heat flow. Furthermore, Q^* depends very little on temperature if $\Delta \lesssim T$. On the other hand, from Eq. (7) it is clear that Q^* vanishes exponentially at low temperatures. In the dirty limit, we may eliminate advantageously $\vec{v}_s$ in favor of $\vec{j}_s$ in Eq. (9) and obtain $Q^* = (4/\pi^2 e\Delta^2)(\vec{j}_s\cdot\nabla T)\ln(8\Delta\tau_E)$ which is independent of material parameters except for a weak dependence on τ_E. In the experiments of Ref. 7 the difference between the quasiparticle potential and the pair potential $V = Q^*/2eN(0)G_{NS}$ (where G_{NS} is the normalized junction conductance to the normal probe) is measured and it is found that the reduced quantity $VG_{NS}/(\vec{j}_s\cdot\nabla T)$ varies proportional to $(1-t)^{-1}$ where $t = T/T_c$. The logarithm in our result changes this dependence only very little: If we use the material parameters listed in Ref. 7, the exponent is shifted from -1 to about -0.9, a result which actually fits the data even better. Furthermore, using the same material parameters we find in the dirty limit the absolute value of $VG_{NS}(1-t)/(\vec{j}_s\cdot\nabla T) = 1.2\times10^{-16}\ \Omega\ \mathrm{cm}^3$, which is in good agreement with the experimental prefactor which varies from 0.8 to 3.8 for different probes.

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