Shape Optimization of Tokamak Plasmas to Localized Magnetohydrodynamic Modes

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(Received 23 March 1979)

We employ a numerical technique to optimize the shape of tokamak plasmas to achieve maximum stable volume-averaged β with respect to ballooning modes. We have examined dee shapes with moderately peaked current profile, poloidal $\beta = 1$, and $R/a = 2.76$. The optimal shape is a strongly modified dee with a large indentation on the inside edge of the plasma. Maximum beta increases with elongation and exceeds 14% when $b/a = 3$.

It is generally believed that magnetohydrodynamic (MHD) ballooning modes may determine the maximum β at which tokamaks can operate. Many authors have considered the effect of current profile and plasma shape upon the maximum β stable to ballooning modes.[1-4] However, before now an automatic search for an optimum plasma shape for stability to ballooning modes has not been attempted. The advent of Class-VI computers and efficient, appropriate ballooning criteria make such a search practical and useful for comparing configurations with different external parameters such as elongation. Here we describe first results from just such a code.

We begin by calculating a free-boundary equilibrium problem in a fixed rectangular box of elongation b/a with no current-carrying conductors inside the box. The maximum β stable to ideal MHD pressure-driven modes is determined numerically for this equilibrium. By changing the boundary conditions in the direction in which maximum stable β is increasing, we are able to optimize the shape for a fixed box and fixed current profile. This approach is similar to an earlier shape-optimization study[5] for stability to axisymmetric modes. Previous shaping studies for ballooning mode stability[1-4,6,7] have examined a restricted set of shapes.

The equilibrium is obtained from the familiar Grad-Shafranov equation,

$$\Delta^*\psi = -(4\pi/c)RJ_\varphi = -4\pi R^2 p' - ff', \quad (1)$$

where $2\pi\psi$ is the poloidal flux, R is the distance from the major axis, J_φ is the toroidal current density, and p is the plasma pressure. The prime denotes differentiation with respect to ψ, and $f(\psi) = RB_t$, where B_t is the toroidal magnetic field.

We must specify a current profile, $p'(\psi)$ and $ff'(\psi)$. For this study, we choose $ff' = 0$, corresponding to poloidal $\beta = 1$, and

$$p'(\psi) = \frac{p'(\psi_a)}{e-1}\left\{\exp\left[1 - \left(\frac{\psi - \psi_a}{\psi_l - \psi_a}\right)^4\right] - 1\right\}, \quad (2)$$

where ψ_a and ψ_l are the values of ψ at the magnetic axis and at the limiter, respectively. This current profile is moderately peaked in radius and was chosen because in an earlier shape study[8] it yielded equilibria which were near-marginal stability for kink modes and which could be easily wall stabilized to axisymmetric modes. A broader profile would yield a higher β but would be less stable to external MHD modes.

Each equilibrium is calculated as a free-boundary problem with use of the geometry shown in Fig. 1. The inner rectangle is the limiter which determines the outermost plasma flux surface. The outer rectangle is a mathematical boundary for the equilibrium problem. The poloidal flux,

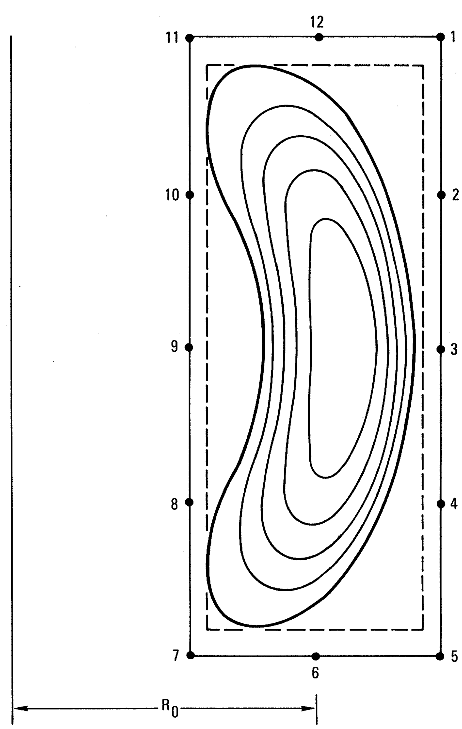

FIG. 1. Geometry used to solve equilibrium problem for $b/a = 2.5$. H/W equilibrium boundary is 300 cm/120 cm; $R_0 \approx 145$ cm. Locations where ψ is specified are labeled like the face of a clock.

$2\pi\psi$, is specified on the outer boundary at each of the twelve points indicated on Fig. 1. Linear interpolation is then used to specify ψ on the rest of the boundary. The twelve points are intended to be a crude model for twelve field-shaping coils which determine the plasma shape. Since we impose up-down symmetry there are only six independent parameters.

Once an equilibrium is calculated, the stability to localized interchange and ballooning modes is evaluated. Both of these modes are driven by pressure gradients in the plasma and are stabilized by increases in the magnetic field energy when the magnetic field lines are bent. By properly shaping the flux surfaces, the magnetic-field-line bending may be maximized and the stability to internal pressure-driven modes improved.

The stability to ballooning modes is determined from the second-order ordinary differential Euler equation obtained in the localized δW analysis of Dobrott *et al.*,[9] with the eigenfunction representation of Connor *et al.*[10] We integrate this equation numerically on 25 flux surfaces across the plasma and obtain an eigenvalue for each surface. The vacuum toroidal magnetic field, $B_{t,\text{vac}}$, is then adjusted to make the least stable surface marginally stable. The corresponding value of the volume-averaged β,

$$\beta = \frac{8\pi}{B_{t,\text{vac}}^2} \frac{\int_{\text{vol}} p\, dV}{\int_{\text{vol}} dV},$$

is then the value of β for which the equilibrium is marginally stable to ballooning modes.

The Mercier criterion for ideal interchanges is contained within the ballooning-mode criterion.[4] Near the magnetic axis the ballooning mode is difficult to evaluate and we instead evaluate the analytic equation for interchange stability derived by Yavlinskii.[11]

The limiting β for the equilibrium is the minimum of the two values determined from the two localized analyses.

The optimization method we use is somewhat similar to that used by Rebhan and Salat[5] to determine the optimum shape for stability of skin-current equilibria to axisymmetric modes. We begin by calculating an equilibrium using a given set of ψ boundary values and determining the maximum stable β for this shape. Then one of the boundary ψ values is altered in fixed increments of $d\psi$, with a new equilibrium and a new maximum β calculated at each step, until a relative maximum in β is found. This set of boundary conditions is used as the starting point for varying the next ψ boundary value and so on. One optimization cycle is completed by sequentially varying each independent boundary value until a new relative maximum β is reached. We continue to do additional complete cycles until β(present cycle) $\leq 1.01\beta$(preceding cycle).

When the optimization technique is applied to equilibria constrained by a rectangular limiter with height-to-width ratio, b/a, the optimum shape exhibits a strong indentation on the inside edge of the plasma. This shape is similar to the one found by Mercier.[4] The indentation can produce nonmonotone q profiles and even approach separatrix formation in the plasma. The maximum stable β for these equilibria exceeds 18%. However, the elongation at the magnetic axis of the plasma becomes twice that of the limiter and the numerical stability analysis becomes very difficult. We have chosen to restrict the results presented in this paper to equilibria with monotone q profiles which are easier to analyze and probably more stable to internal kinks and resistive modes. With this restriction, the optimum shape is typically of the form shown in Fig. 1 for $b/a = 2.5$. The current profile is given by Eq. (2) and the aspect ratio is 2.76. The volume-averaged β for this case was 13.1%; the elongation at the magnetic axis was 3.9:1, and q on axis was 1.20. The safety profile is shown in Fig. 2.

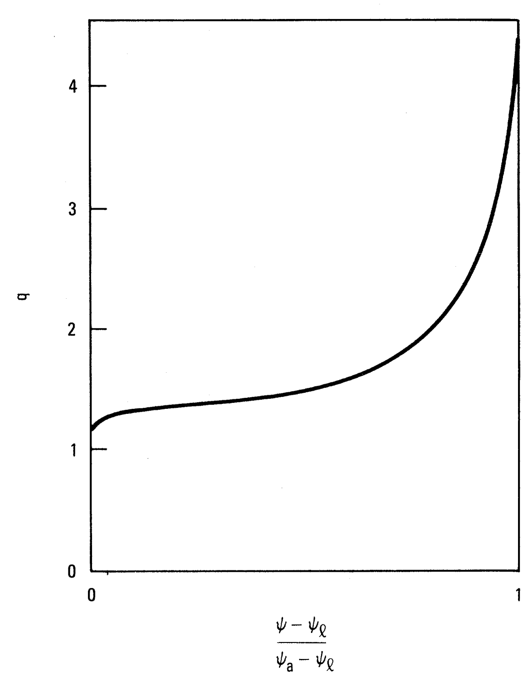

FIG. 2. $q(\psi)$ profile for equilibrium shown in Fig. 1.

The indentation produces an extremely large triangularity at the magnetic axis and hence the Mercier stability is improved. In addition, the indentation reduces the connection length in the bad-curvature region on the outside of the plasma and enhances the good curvature on the inside of the torus, both of which help to stabilize the ballooning mode.

This approach to increasing β may be compared with recent ideas on improving ballooning mode stability[4,12] by use of high β_p to produce equilibria with short connection lengths. In our case the short connection length is achieved at low β_p by use of external shaping coils, rather than by use of current/pressure profiles to generate high-β_p equilibria.

The maximum β increases with b/a (subject to the restriction mentioned above) as shown in Fig. 3. It is interesting to note that β is smaller for $b/a = 3.5$ than for $b/a = 3$. The $b/a = 3.5$ case does not touch the top or bottom limiter and is a more standard dee shape. This indicates that the field coil array was not close enough to the plasma to produce the high-β equilibria. The cross sections of the equilibria of Fig. 3 are shown in Fig. 4. Notice that the plasma in the $b/a = 1.5$ case did not utilize the full volume available to it, but rather created a cross section with elongation equal to 2.4.

To determine the increase in β achieved over that of a standard dee, we consider the case where $\psi_8 = \psi_9 = \psi_{10}$, thus making indentation impossible. The results for these equilibria are also shown in Fig. 3. We see that for $b/a = 2.5$ and 3.0, the indentation increases β by roughly a factor of 2 over β for the more standard dee shape.

By utilizing the shape-optimization technique outlined above, we have generated an optimum plasma shape with extremely high volume-averaged β (for this current profile and aspect ratio) which is stable to localized interchange and ballooning modes. This equilibrium is characterized by an indentation on the inside edge of the plasma which increases the triangularity, reduces the connection length on the outside of the plasma, and enhances the good curvature on the inside edge of the plasma.

The combined optimization of current profile and plasma shape to both internal ballooning modes and external $n = 1$ kinks is being done to extend this shaping study. Some cases have already been studied in which the current profile was varied and the same general shape was obtained. Optimization with another model field-coil configuration or a specific realistic configuration may produce different results. Nonetheless, the striking enhancement of β_{crit} even within these constraints is clear evidence of the possible gains with optimal choice of cross-sectional shapes.

Questions which have not been addressed here include (1) the stability of these equilibria to external MHD modes, which is now being investigated using the two-dimensional ideal MHD stability code ERATO,[13] and (2) the degree of control required to maintain these equilibria, which may be considered using the General Atomic Company $1\frac{1}{2}$-dimensional transport code.[14] Finally, the issue of topology has not been adequately ex-

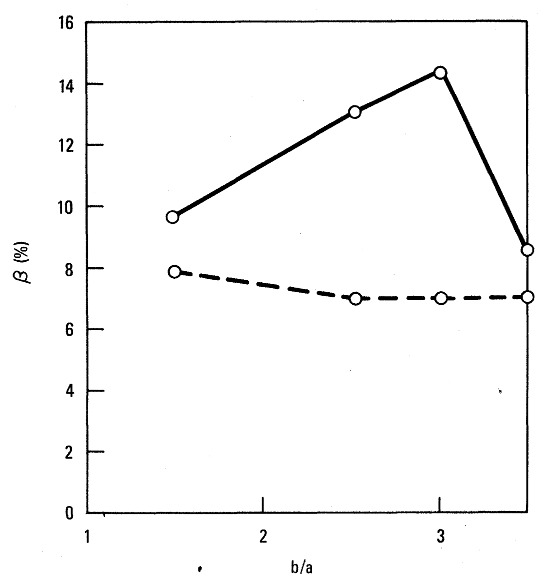

FIG. 3. Plot of β vs b/a of the limiter. Dashed line, sequence are nonindented dee shapes.

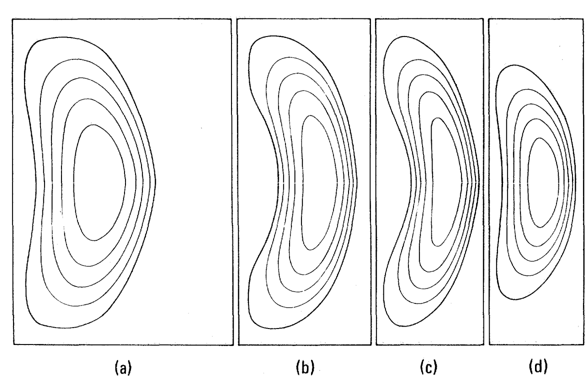

FIG. 4. Optimal cross section for b/a equal to (a) 1.5, (b) 2.5, (c) 3.0, and (d) 3.5.

plored. The numerically more difficult optimization of doublet shapes is now under investigation.

It is a pleasure to thank Gareth Guest, Don Dobrott, Fred McClain, and Wayne Pfeiffer for stimulating discussions, and Bob Davidson and Boni Dy for programming assistance. This work has been supported by the U. S. Department of Energy under Contract No. EY-76-C-03-0167, Project Agreement No. 38.

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Confinement of Fusion-Produced Tritium in the Princeton Large Torus

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Tritium is produced in deuterium discharges in the Princeton Large Torus by the reaction D(d,p)T. These tritons undergo reactions D$(t,n)^4$He creating 14-MeV neutrons which have been detected by two independent techniques at a level of 1% of the 2.5-MeV neutrons from the reaction D$(d,n)^3$He. The magnitude of the 14-MeV neutron emission is consistent with finite banana width, neoclassical predictions for the confinement of the energetic tritons.

A requirement for operation of an ignited fusion reactor is that the charged fusion reaction products be well confined since their energy is required to overcome the plasma energy losses. In this Letter, we report experimental evidence that such fusion reaction products are confined in a tokamak plasma [PLT (Princeton Large Torus)] consistent with finite–banana-width, neoclassical predictions.

In deuterium discharges, 1.01-MeV tritons are produced by the reaction D(d,p)T with nearly equal probability as the 2.45-MeV (d-d) neutrons produced by the reaction D$(d,n)^3$He. A fraction of the tritons may then react further to produce 14-MeV (d-t) neutrons by the reaction D$(t,n)^4$He. The ratio of d-t to d-d neutrons produced in a given discharge is strongly dependent on the confinement of the tritons as they slow down from 1.01 MeV through the maximum of the D$(t,n)^4$He cross section near 170 keV. About 10^{-2} of the *confined* tritons undergo reactions D$(t,n)^4$He. Unconfined tritons still encounter deuterium in the vacuum vessel wall,[1] but due to the shorter triton slowing-down time in the metal, only about 10^{-6} undergo reactions D$(t,n)^4$He. The d-t/d-d neutron flux ratio of about 1/100 observed on PLT is thus an indication that the tritons have been confined during most of their slowing-down time.

In these experiments, PLT (500 kA, 3.2 Tesla, 40 cm minor radius, and 130 cm major radius)[2] was heated by hydrogen or deuterium neutral-beam injection (37 keV, 1.4 MW)[3] into a deuterium target plasma resulting in plasma conditions of $n_e(0)=4\times10^{13}$ cm^{-3}, $T_e(0)=2$ keV, $T_i(0)\simeq2$ keV. The Thomson scattering[4] deduced electron temperature profile during neutral-beam injection indicated a highly peaked $T_e(r)$ profile similar to $[1-(r/a)^2]$.[4] The $Z_{\rm eff}$ was about 3, and spectroscopic observations indicated that the main impurity was carbon. As a result of the neutral-beam injection, the neutron emission rate rose from about 10^8 to 10^{11} per second for H^0 injection and to 10^{12} per second for D^0 injection.

The 14-MeV neutron flux was measured with two independent systems. The first was a 13-cm ×13-cm-diam NE213 liquid scintillator in which

