Magnetic Moment of Weak Bosons Produced in pp and $p\bar{p}$ Collisions

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We suggest that the reactions $pp \to W^{\pm}\gamma X$ and $p\bar{p} \to W^{\pm}\gamma X$ are good candidates for measuring the magnetic moment parameter κ in $\mu_W = (e/2M_W)(1+\kappa)$. The angular distribution of the W bosons in $p\bar{p} \to W^{\pm}\gamma X$ is particularly sensitive to this parameter. For the gauge-theory value of $\kappa = 1$, we have found a peculiar zero in $d\sigma(d\bar{u} \to W^-\gamma)/d\cos\theta$ at $\cos\theta = -\frac{1}{3}$, the location of this zero depending on the quark charge through $\cos\theta = -(1+2Q_d)$. A similar zero occurs in $d\sigma(u\bar{d} \to W^+\gamma)/d\cos\theta$. We can offer no explanation for this behavior.

Expectations are high that the weak intermediate vector bosons $W^{\pm}$ and Z^0 will be discovered in the next generation of accelerators[1] (the CERN $p\bar{p}$ project, ISABELLE at Brookhaven, and the $p\bar{p}$ project at Fermilab). The theoretical predictions appear to be on firm ground: The highly successful Weinberg-Salam SU(2)⊗U(1) theory predicts the boson masses to lie below 100 GeV/c^2, and this theory coupled with the Drell-Yan picture of quark-antiquark annihilation in hadron-hadron collisions predicts[2] cross sections around 10^{-33} cm^2, which is well matched by the luminosities of the projects mentioned above.

Assuming that such a discovery will indeed take place, we ask ourselves what properties of weak bosons are likely to be measured. The mass and spin can probably be measured by looking at decay distributions,[2,3] which also serve as a signal for the production process. Another property in which we are interested is the magnetic moment μ_W of the $W^{\pm}$ bosons or, more generally, the coupling between the weak and electromagnetic fields $W_\mu^{\pm}$ and A_μ. (The interaction of $W_\mu^{\pm}$ fields with the Z_μ field is even more interesting from the gauge-theory point of view but, of course, it is harder to detect, requiring W^+W^- and $W^{\pm}Z^0$ pair production.[4])

In principle the electromagnetic coupling of weak bosons could be measured in the reaction[5] $e^+e^- \to W^+W^-$ or in photoproduction[6] $\gamma p \to W^{\pm}X$. However, it will be a long time before we achieve the necessary center-of-mass energies in these reactions. We have, therefore, concentrated on pp and $p\bar{p}$ collisions, and, since an electromagnetic field must clearly be involved, we are naturally led to the processes

$$p + p \text{ or } p + \bar{p} \to W^{\pm} + \gamma + X, \tag{1}$$

which we believe are very good candidates for measuring μ_W. We have used the commonly accepted quark-parton model to describe those processes which involve hard collisions to produce a massive W boson accompanied by a hard photon. In the course of our investigation we found a peculair angular distribution which we cannot yet explain.

In the usual manner we calculate first the basic process with quarks before embedding them in physical particles. In this case the process is

$$q_i + \bar{q}_j \to W^{\pm} + \gamma. \tag{2}$$

The Feynman diagrams and our notation are shown in Fig. 1. Of course, we are after diagram (c). To see how reaction (2) depends on μ_W, we use the standard generalization of the $W_\alpha(p+k)$-

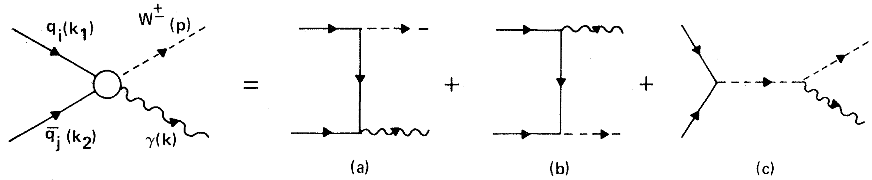

FIG. 1. Feynman diagrams for the process $q_i(k_1)\bar{q}_j(k_2) \to W^{\pm}(p)\gamma(k)$.

$\gamma_\mu(k)$-$W_\beta(p)$ vertex[7]:

$$V_{\alpha\mu\beta} = -ie[g_{\alpha\beta}(2p+k)_\mu - g_{\alpha\mu}(p+k+\kappa k)_\beta - g_{\beta\mu}(p-\kappa k)_\alpha]. \quad (3)$$

The magnetic moment is given by[7]

$$\mu_W = (e/2M_W)(1+\kappa).$$

In the standard model $\kappa = 1$ and, therefore, $\mu_W = e/M_W$.

We have calculated the differential cross section and find

$$\frac{d\sigma}{dt}(q_i\bar{q}_j \to W^-\gamma) = \frac{\alpha}{s^2}\frac{M_W^2 G_F}{\sqrt{2}} g_{ij}^2 \left\{ \left(Q_i + \frac{1}{1+t/u}\right)^2 \frac{t^2+u^2+2sM_W^2}{tu} + (\kappa-1)\left(Q_i + \frac{1}{1+t/u}\right)\frac{t-u}{t+u} + \frac{(\kappa-1)^2}{2(t+u)^2}\left[tu + (t^2+u^2)\frac{s}{4M_W^2}\right]\right\}, \quad (4)$$

where $s = (k_1+k_2)^2$, $t = (p-k_1)^2$, and $u = (p-k_2)^2$, with $s+t+u = M_W^2$, $g_{ij} = \cos\theta_C$ for $q_i\bar{q}_j = d\bar{u}$ and $s\bar{c}$, and $g_{ij} = \sin\theta_C$ for $q_i\bar{q}_j = s\bar{u}$ and $d\bar{c}$. $Q_i|e|$ is the charge of the quark q_i; we have set $Q_j = Q_i + 1$. All quark masses have been dropped. The cross section for $q_j q_i \to W^+\gamma$ is also given by Eq. (4) with t taken to be the four-momentum transferred between the antiquark and W^+. For definiteness, we will concentrate on $W^-\gamma$ production. We have checked Eq. (4) against the corresponding expression for $\gamma q_i \to W^\pm q_j$ in Ref. 6.

The peculiar behavior referred to earlier can be seen in Fig. 2, where we plot the angular distribution of the W^- obtained from Eq. (4):

$$d\sigma(d\bar{u}\to W^-\gamma)/d\cos\theta = \tfrac{1}{2}(s-M_W^2)\,d\sigma(d\bar{u}\to W^-\gamma)/dt, \quad (5)$$

where θ is the angle between W^- and d (γ and $\bar{u}$) in the $W^-\gamma$ center-of-mass frame: $t = -\frac{1}{2}(s-M_W^2)(1-\cos\theta)$. The smooth behavior for $\kappa = -1$ or 0 is radically changed when we set $\kappa = 1$, its gauge-theory value. An unexpected zero appears now at $\cos\theta = -\frac{1}{3}$. Since the same curves apply for $s\bar{c}\to W^-\gamma$ and, except for an overall factor of $\tan^2\theta_C$, to $s\bar{u}\to W^-\gamma$ and to $d\bar{c}\to W^-\gamma$ also, we conclude that *in all cases the standard theory predicts the vanishing of $d\sigma/dt$ at a particular angle.* This zero in $d\sigma/dt$ can be traced to the coefficient $[Q_i + (1+t/u)^{-1}]^2$ in Eq. (4), which vanishes for $t^*/u^* = -(1+1/Q_i)$ implying

$$\cos\theta^* = -(1+2Q_i). \quad (6)$$

Since $Q_i = -\frac{1}{3}$, we get $\cos\theta^* = -\frac{1}{3}$. We have no physical explanation for this zero; we know of no other case where a quark cross section vanishes at a particular value of $\cos\theta$ depending on the charge of the quark.[8] Furthermore, we have shown that no other set of κ, Q_i, etc., gives a zero in the physical region.

We point out that the condition $\cos\theta^* = -(1+2Q_i)$ does *not* involve M_W^2/s, which is the only other dimensionless quantity that could possibly appear in Eq. (6). This has interesting implications for the physical processes in Eq. (1), which we now consider.

The measurable quantity that comes closest to the q-$\bar{q}$ differential cross section is $d\sigma/d\cos\theta_{\text{c.m.}}$, the angular distribution of W bosons or photons in the $W\gamma$ center-of-mass frame, where $\theta_{\text{c.m.}}$ is the angle, say between the W and the direction of one of the colliding beams, which we take to be a proton beam. Let us consider $p\bar{p}$ collisions; then

$$\frac{d\sigma}{d\cos\theta_{\text{c.m.}}}(p\bar{p}\to W^-\gamma X) = \tfrac{1}{3}\sum_{i=d,s}\iint dx_A\,dx_B\,P_i^{p}(x_A)P_{\bar{u}}^{\bar{p}}(x_B)\left(\frac{s-M_W^2}{2}\right)\frac{d\sigma}{dt}(q_i\bar{q}_u\to W^-\gamma) + \tfrac{1}{3}\sum_{i=d,s}\iint dx_A\,dx_B\,P_{\bar{u}}^{p}(x_A)P_i^{\bar{p}}(x_B)\left(\frac{s-M_W^2}{2}\right)\frac{d\sigma}{dt}(q_i\bar{q}_u\to W^-\gamma)_{u\leftrightarrow t}. \quad (7)$$

In the first term $\theta = \theta_{\text{c.m.}}$, while in the second term $\theta = \pi - \theta_{\text{c.m.}}$. In both terms $s = x_A x_B S$, where S is the square of the $p\bar{p}$ c.m. energy, $S = (p_p + p_{\bar{p}})^2$. To be specific we use $\sqrt{S} = 540$ GeV, which is the projected energy for the CERN $p\bar{p}$ machine.

Were the p and $\bar{p}$ to consist purely of q's and $\bar{q}$'s, respectively, $d\sigma/d\cos\theta_{\text{c.m.}}$ would vanish identically at $\cos\theta_{\text{c.m.}} = -\frac{1}{3}$ if $\kappa = 1$. This is because the second term in Eq. (7), the so-called sea contribution, would be absent, and in the first term (valence contribution) the zero in $d\sigma(d\bar{u} \to W^-\gamma)/dt$ is not affected by the integration over x_A and x_B. [Remember, the condition $\cos\theta^* = -(1+2Q_d)$ does not involve s over which we are integrating in Eq. (7).]

We find that the sea contribution does not change this picture very much. Though finite, $(d\sigma/d\cos\theta_{\mathrm{c.m.}})_{\kappa=1}$ is only 0.76×10^{-39} cm^2 at $\cos\theta_{\mathrm{c.m.}} = -\frac{1}{3}$, compared to 0.32×10^{-36} cm^2 at $\cos\theta_{\mathrm{c.m.}} = +\frac{1}{3}$, for example. In Fig. 3 we plot Eq. (7) for $\kappa = -1$, 0, 1. The quark distributions P_i, etc., are parametrized in the same way as in Ref. 4. To make sure that the photon is observed, we have made a cut of $E_\gamma > (E_\gamma)_{\min} = 30$ GeV, where E_γ is the energy of the photon is the laboratory frame (which we always take to be the pp or $p\bar{p}$ c.m. frame).

We see that $d\sigma/d\cos\theta_{\mathrm{c.m.}}$ is highly sensitive to the value of κ. This sensitivity is increased if we increase $(E_\gamma)_{\min}$, but then, of course, the rates get smaller. The converse occurs if we decrease $(E_\gamma)_{\min}$. 30 GeV appears to be a reasonable compromise between sensitivity to κ and acceptable rates, though other values are certainly possible. Note that we are *not* doing radiative corrections to single-W production.

A measurement of $d\sigma/d\cos\theta_{\mathrm{c.m.}}$ requires detailed measurement of the momenta and directions of both the $W^\pm$ and the photon to reconstruct their center-of-mass motion event by event. As an example of a much cruder experiment, we show in Fig. 4 the behavior of $d\sigma/d\cos\theta_{\mathrm{lab}}$, where θ_{lab} is the direction of the W^- with respect to one of the colliding beams, which we again take to be a proton beam, in the laboratory frame. This quantity requires a complicated phase-space in-

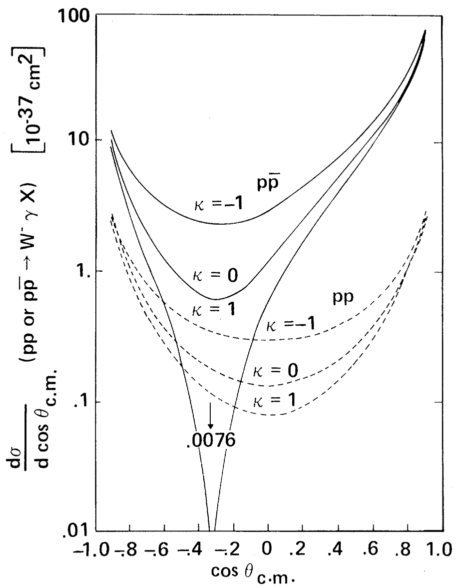

FIG. 3. The differential cross section for $pp \to W^-\gamma X$ and $p\bar{p} \to W^-\gamma X$, with a photon energy cut $E_\gamma > 30$ GeV. $\theta_{\mathrm{c.m.}}$ is the angle between the W^- and the proton direction in the $W^-\gamma$ c.m. frame. $\sqrt{S} = 540$ GeV and $M_W = 85$ GeV/c^2.

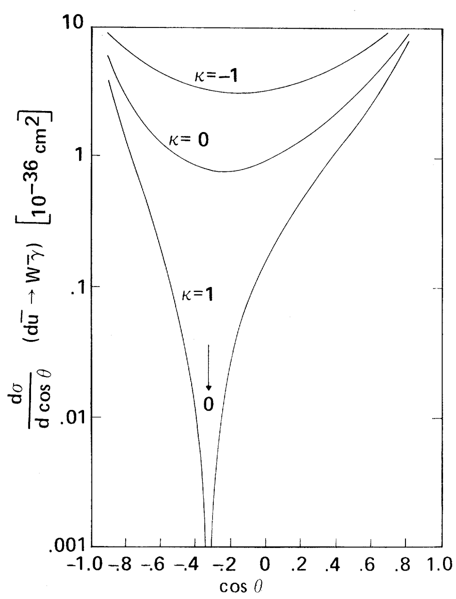

FIG. 2. The differential cross section for $d\bar{u} \to W^-\gamma$. θ is the angle between W^- and d, or between γ and $\bar{u}$, in the c.m. frame. $\sqrt{s} = 200$ GeV and $M_W = 85$ GeV/c^2.

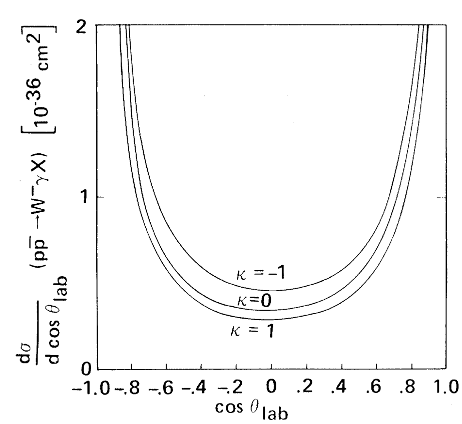

FIG. 4. The differential cross section for $p\bar{p} \to W^-\gamma X$ with a photon energy cut $E_\gamma > 30$ GeV. θ_{lab} is the angle between the W^- and the proton in the laboratory. $\sqrt{S} = 540$ GeV and $M_W = 85$ GeV/c^2.

tegration and we shall omit the details. The same $E_\gamma > 30$ GeV cut has been applied in Fig. 4.

We see that $d\sigma/d\cos\theta_{\rm lab}$ is mildly sensitive to κ, with a 30–40% difference around $\theta_{\rm lab} = 90°$ where the detectors will be. The reason for this behavior is the following: As one integrates over all variables except $\theta_{\rm lab}$, one sweeps over a fairly large range of θ, and the dip at $\cos\theta^* = -\frac{1}{3}$ gets filled in, though not completely. Obviously, one can do better by measuring a doubly differential cross section. The same remarks made earlier concerning the value of $(E_\gamma)_{\rm min}$ apply here also.

Both $d\sigma/d\cos\theta_{\rm c.m.}$ and $d\sigma/d\cos\theta_{\rm lab}$ are large near the forward-backward directions where they do not discriminate between different values of κ. This means that if one performs the third and last integral to obtain $\sigma_{\rm total}$, the results will be rather insensitive to κ. Indeed, we find[9] only a 7% difference between $\sigma_{\rm total}(\kappa = -1)$ and $\sigma_{\rm total}(\kappa = +1)$.

Finally, we consider $W^+\gamma$ production and pp vs $p\bar{p}$ collisions:

(a) Everything we said about the W^- applies unchanged to the W^+ where now the various angles θ, $\theta_{\rm c.m.}$, or $\theta_{\rm lab}$ are taken between the W^+ and the antiquark or antiproton; e.g., in $p\bar{p} \to W^+\gamma X$ for $\kappa = 1$, there is a dip when the $W^+(\gamma)$ makes an angle of about 110° (70°) with the direction of the $\bar{p}$.

(b) pp collisions differ in two ways from $p\bar{p}$ collisions. The absence of valence antiquarks implies lower rates in general, and we find that the differential cross sections in pp collisions are an order of magnitude smaller than in $p\bar{p}$ collisions over most of the phase-space region. This, however, may be compensated by the larger luminosities of pp machines. A more serious setback for pp collisions perhaps is the decrease in sensitivity to κ. For example, in the equation corresponding to Eq. (7), both terms will contribute with equal weights and if $d\sigma(d\bar{u} \to W^-\gamma)/dt$ vanishes in the first term then it does not vanish[10] in the second (note that the $q\bar{q}$ cross section is symmetric under $x_A \leftrightarrow x_B$ because it depends only on $s = x_A x_B S$ and $\cos\theta$). Hence, each one of the terms will substantially fill in the dip left by the other. For comparison we have included in Fig. 3 curves for pp collisions at the same energy $\sqrt{S}$. Our remarks are clearly substantiated by these curves. We should add, however, that it may be possible to find a totally or doubly differential cross section in pp collisions which reflects the κ dependence of the $q\bar{q}$ process, and we hope to do this in the future.[11]

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[8] For $\bar{\nu}_e e \to W^-\gamma$, $Q_i = -1$ and $d\sigma/d\cos\theta$ vanishes in the forward direction.

[9] To obtain $\sigma_{\rm total}$ a finite quark mass m_q must be used in the propagators: $1/t \to 1/(t - m_q^2)$ and $1/u \to 1/(u - m_q^2)$. We have set $m_q = 300$ MeV$/c^2$.

[10] It could vanish if $d\sigma/d\cos\theta$ (see Fig. 2) had another zero at $\cos\theta^* = \frac{1}{3}$, or if $\theta^* = \pi - \theta^* = 90°$, which would be the case if $Q_d = -\frac{1}{2}$.

[11] The idea is to take advantage of the fact that the antiquarks in the sea are found at low x_A or x_B, and study the correlation between the energy and direction of the W bosons or the associated photons.