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Possible Explanation of Nonlinear Conductivity in Thin-Film Metal Wires

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We introduce the idea of an incoherence length L_2 at which inelastic collision broadening equals electronic energy-level separation. The two-dimensional conductance crosses over at L_2 from a $\ln L$ dependence suggested recently by Abrahams *et al.* to Ohmic behavior. These ideas and plausible relaxation and heating models are used to explain the nonlinear conductivity observations of Dolan and Osheroff.

Dolan and Osheroff, in the following Letter,[1] note a logarithmic dependence of conductivity on current in thin-film wires of which a typical one is about a micron wide by one or two millimeters long, with conductivity of about 5000 Ω per square, for currents in the neighborhood of 10^{-9} A and temperature 10–1000 m mK. They observe that this behavior saturates at low fields giving a linear I-V curve with a logarithmic dependence of conductivity on T at zero field as well.

In a recent Letter, Abrahams *et al.*[2] proposed a scaling theory of localization based on ideas of Thouless.[3] The essential idea is that the conductance $G = ge^2/\hbar$ of small samples of metal scales with sample size L. For one dimension the conductance scales rapidly to zero as $L \to \infty$ in all cases,[4] and for three dimensions, it approaches σL for large enough samples and large enough σ. But in two dimensions our theory predicted that

$$g(L) = g_0 - (\alpha/\pi^2)\ln L \quad \text{when } g \gg \pi^{-2}, \tag{1}$$

and for small g it predicted exponential localization. A preliminary perturbation treatment gave $\alpha = 1$ for two noncommunicating gases of electrons with spins up and down, or $\alpha = \frac{1}{2}$ if the elastic spin-flip scattering length is short compared to the other lengths in the problem.

The meaning of conductance at a finite scale is described by Thouless[3] in a number of ways. One way is to define it in terms of a time t_D to diffuse a finite distance L, which gives one

$$t_D(L) = L^2/D(L),$$

and one may measure t_D by allowing a wave packet localized in a length somewhat less than L to spread. The conductivity is then determined in

terms of L from the Einstein relation $\sigma = e^2 D dn/dE$. Thouless proves the equivalence to this of a second definition:

$$G = e^2\hbar^{-1}\langle \Delta E_\alpha\rangle/\langle \delta E_\alpha\rangle ,$$

where $\langle \Delta E_\alpha\rangle$ is a measure of the shift in energy of an average level E_α upon changing the boundary-condition phase by π, and $\langle \delta E_\alpha\rangle$ is the mean separation of adjacent energy levels E_α, i.e., dE/dNL^d.

Thouless has discussed, in another paper[5] aimed at the one-dimensional case, the process of conduction at finite T. In his picture, a wave packet diffuses for a time equal to the inelastic scattering time τ_{inel} and then loses phase coherence, so that the appropriate scale length which will give the observed conductivity at temperature T is given by

$$L_1^2 = D\tau_{\text{inel}} = \tfrac{1}{2} l_{\text{el}} l_{\text{inel}} , \tag{2a}$$

where l_{el} and l_{inel} are the elastic and inelastic mean free paths.

We believe that the correct length scale, if it is shorter, is the one given by the following simple argument: The energy levels are broadened by $\Delta = \hbar/\tau_{\text{inel}}$ and the granularity of the levels which causes the behavior (1) is no longer evident when

$$\langle \delta E_\alpha\rangle = \hbar/\tau_{\text{inel}} \quad \text{or} \quad L_2^{\,d} = (\tau_{\text{inel}}/\hbar) dE/dN. \tag{2b}$$

For $d = 2$ L_1 and L_2 are proportional but not generally equal. For any dimension they turn out to be always equal near $g = 1$ (the conventional localization criterion). One, L_1, may be described by $\langle \Delta E_\alpha\rangle = \hbar/\tau_{\text{inel}}$, the other, L_2, by $\langle \delta E_\alpha\rangle = \hbar/\tau_{\text{inel}}$. We feel that the basic condition for elastic diffusive behavior involving only the elastic scattering processes is that *both* δE and $\Delta E < \hbar/\tau_{\text{inel}}$. The fundamental scaling process on which (1) is based breaks down at either length. We will see that our shorter length is favored by experiment. The ratio of the lengths for $d = 2$ can be expressed as $(L_1/L_2)^2 = \hbar/\sigma e^2$.

Thouless[5] suggested several relaxation mechanisms for τ_{inel}. The phonon ones obey

$$\hbar/\tau_{\text{inel}} \propto (T/\Theta_D)^{d_{\text{ph}}} f(q l_{\text{el}}) E_F , \tag{3}$$

where Θ_D is the Debye temperature, E_F the Fermi energy, and q, the phonon wave vector, is given by

$$q = k_B T/\hbar s .$$

$f(x)$ is constant for $q l_{\text{el}} \ll 1$, and $f = x$ when $q l_{\text{el}} \gg 1$. d_{ph} is the phonon dimensionality, which may be 2 when the phonons propagate in the metal film (as they will at high temperature) or 3 when they are in the substrate (as is likely at low T).[6] Thus we may expect $L^2 \propto T^{-p}$ with $p = 2$, 3, or 4 depending on film and temperature. At low enough T, $p = 4$, so that we predict

$$\Delta\sigma = (2e^2/\pi^2\hbar)\alpha \ln T , \tag{4}$$

and Osheroff and Dolan's experiments furnish a value for α of $\frac{1}{4}$ to $\frac{1}{3}$ (see below for a p-independent determination of α).

Frankly, we find this result a little disturbing. The coefficient α is, according to the arguments of Ref. 2, universal for noninteracting electrons, and the value calculated by perturbation theory in the weak-scattering limit[7] should be correct. We therefore expect $\alpha = \frac{1}{2}$. We note only that there is no argument requiring universality for interacting electrons in real materials; it will be interesting to study the material dependence experimentally. It is just possible that experimental error would account for some of the discrepancy.

For finite electric fields, one can propose several mechanisms, such as field-induced tunneling, which might be important under some circumstances. A simple one which seems to be the operative one is σE^2 heating of the electrons. With such long inelastic scattering times as (3), the electrons are, at 10 mK, almost out of thermal contact with the phonons, and they will heat up under a field until (3) can soak up the energy (note that electron-electron collisions are ineffective for this purpose). This means that

$$T(E) = \Delta T = (\sigma E^2/C_{sp})\tau_{\text{inel}} , \tag{5}$$

or, it may be shown,

$$\pi k_B T(E) = Ee(l_{\text{el}} l_{\text{inel}})^{1/2} . \tag{6}$$

Thus the ratio of coefficients of $\ln V$ (or $\ln I$ in the weak-coupling regime) to that of $\ln T$ is $1/(1+p/2)$. The data are consistent (see following Letter) with $p \gtrsim 3$ at low E and T, and with $p \simeq 2$ at high T and E. We ascribe the crossover to a shift of phonon behavior from metal to substrate.

A way to present the data which is a check on the Ohmic-heating model as well as being independent of relaxation mechanism is to determine the slopes of σ vs $\ln E$ at low T and σ vs $\ln T$ at low E. The difference of their inverses is the inverse of the universal slope versus $\ln L$. This is easily derived as follows. From (1) and the defi-

nition of L_1 or L_2 (which are proportional in two dimensions) we have

$$g - g_0 = -(\alpha/\pi^2)\ln L$$

$$g - g_0 = -S_L \ln L = -\tfrac{1}{2} S_L \ln \tau_{\text{inel}},$$

or

$$\ln \tau_{\text{inel}} = (g - g_0)/(-\tfrac{1}{2} S_L).$$

Dolan and Osheroff measure $g - g_0 = S_T \ln T$ for small E and $g - g_0 = S_E \ln E$ for small T_0. From Eq. (5) for the heating model and noting that $C_{\text{sp}} = \gamma T$, we have

$$\ln T(E) = \ln E + \tfrac{1}{2} \ln \tau_{\text{inel}}(E),$$

so that

$$\frac{1}{g - g_0} \tfrac{1}{2} \ln \tau_{\text{inel}}(E) = -\frac{1}{S_L} = \left(\frac{1}{S_T} - \frac{1}{S_E}\right).$$

Indeed, where we have more than one regime and for all samples, we find a universal number within 20–30%.

The success of the heating model indicates that the electron-electron inelastic scattering process is inoperative. This is, of course, true as far as the Ohmic-heating model is concerned, since electron-electron processes cannot carry energy away, but it also suggests that the electron-electron processes themselves localize as the electronic degrees of freedom localize (or else that in these samples they are very weak, which would be a bit surprising).

Finally, we note that from (6) we can get a quantitative measure of L_1 and hence L_2, by comparing σE^2 with T^2 for the same σ. This indicates that L_1 is several microns and L_2 is somewhat smaller under typical conditions for these films. The absence of a one-dimensional crossover for the 1-μm film suggests that experimentally L_2 is favored.

It should not be too difficult, with use of thinner wires and not much smaller currents, to achieve one dimensionality where we believe the corresponding behavior of the resistance is a rise at low T proportional to T^{-p}, and at low field proportional to $E^{-p/(1+p/2)}$ or thereabouts—a much more precipitous behavior, but less easily seen at high E and T. Finally, with slightly higher-resistance films (10^4 Ω per square) it should be easy to observe a crossover to exponential localization giving behavior presumably like the usual Mott $T^{-1/3}$ or $T^{-1/2}$ laws, with correspondingly strong field dependence. The physics of this regime is very different, in that conventional quantum transport breaks down totally as soon as the behavior becomes exponential and phonon-assisted tunneling takes over. We also can expect negative-resistance problems in this regime, leading to channeling and noise as has actually been observed in preliminary experiments.

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