Interacting Boson-Fermion Model of Collective States in Odd-A Nuclei

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It is suggested that odd-A nuclei be treated as a system of interacting bosons and fermions. It is shown that a simple choice for the form of the boson-fermion interaction is sufficient to describe the variety of observed spectra.

The purpose of this Letter is to point out that we have developed a new model for describing collective properties of odd-A nuclei, which contains as limiting cases both the particle-vibration (weak-coupling) and Nilsson (strong-coupling) models. It can additionally be used to discuss other situations, including those intermediate between weak and strong coupling. In this model, an odd-A nucleus is treated as a system of interacting bosons and fermions. The bosons represent correlated pairs of particles[1,2] coupled to $L=0$ (called s) and $L=2$ (called d), while the fermions represent the odd, unpaired particles. The total Hamiltonian is written as $H=H_B+H_F+V_{BF}$, where H_B is the boson Hamiltonian of Ref. 1, $H_F=\sum_{jm}\epsilon(j)a_{jm}{}^\dagger a_{jm}$, is the fermion part of H and V_{BF} denotes the boson-fermion interaction. The boson part is fixed by requiring that its eigenvalues describe accurately the adjacent even-even nucleus, which, in the present approach, is calculated simultaneously with its odd-A partner. However, the spectra of odd-A nuclei depend crucially on the boson-fermion interaction, V_{BF}. In this Letter we show that a particularly simple choice of V_{BF} (i) leads, in limiting cases, to spectra analogous to those of the particle-vibration, the Nilsson, and the particle-plus-γ soft-rotor models, and most importantly (ii) is sufficient to describe, to a good approximation, the observed spectra.

In order to write down explicitly this interaction, we begin by showing in Fig. 1(a) all possible couplings linear in the boson and fermion variables. In this figure, we have separated the terms which arise from a two-body fermion-fermion interaction (denoted by a circle) from terms which arise from the antisymmetrization of the fermion lines (exchange terms). We then note that, because the s and d bosons describe pairing and quadrupole correlations,[2] we expect the $J=0$ and $J=2$ terms to be dominant in the diagrams (2), (4), and (6) of Fig. 1(a). We also note that the exchange terms (1) and (3) can be rewritten as a renormalization of the monopole ($J=0$) and quadrupole ($J=2$) coupling terms. We are thus led to suggest that the structure of odd-A nuclei is, to a large extent, dominated by the interplay of three terms: effective monopole and quadrupole interactions, and the exchange diagram (5), as shown in Fig. 1(b). These three terms can be written as

$$V_{BF}=A[(s^\dagger\times s)^{(0)}\times(a_j{}^\dagger\times\tilde{a}_j)^{(0)}+\Gamma\{[(d^\dagger\times s+s^\dagger\times\tilde{d})^{(2)}+\chi(d^\dagger\times\tilde{d})^{(2)}]\times(a_j{}^\dagger\times\tilde{a}_j)^{(2)}\}^{(0)} - \Lambda :[(\tilde{d}\times a_j{}^\dagger)^{(j)}\times(d^\dagger\times\tilde{a}_j)^{(j)}]^{(0)}:, \quad (1)$$

where we have used the same notation of Refs. 1 and 2 with $\tilde{d}_\mu=(-1)^\mu d_{-\mu}$, $\tilde{a}_{jm}=(-1)^{j-m}a_{j,-m}$, and considered, for simplicity, the case in which the odd fermion can occupy only one level j. The parameters A, Γ, Λ denote the strength of the monopole, quadrupole, and exchange terms and they become matrices A_j, $\Gamma_{jj'}$, $\Lambda_{jj'}{}^{j''}$ in the general case in which the odd fermion can occupy several single particle levels j, j', ... Normal ordering (:) has been introduced in the last term in order to remove the contribution arising from the commutator of $\tilde{d}$ and $d^\dagger$. The three parameters A, Γ, Λ (or the three matrices A_j, $\Gamma_{jj'}$, $\Lambda_{jj'}{}^{j''}$ in the general case of several single-particle levels) can, in principle, be calculated from the underlying fermion-fermion dynamics using either a procedure similar to that described by Otsuka, Arima, and Iachello[3] for calculating the parameters appearing in the boson part of H, or using the nuclear field theory of Bortignon *et al.*[4] For the purposes of the present Letter, they will be taken as phenomenological constants to be adjusted to experiment. We now claim that a diagonalization of $H=H_B+H_F+V_{BF}$ with V_{BF} given by (1), produces, in limiting situations, spectra analogous to those of (i) the particle-vibration model, (ii) the Nilsson model, and (iii) the particle-plus-

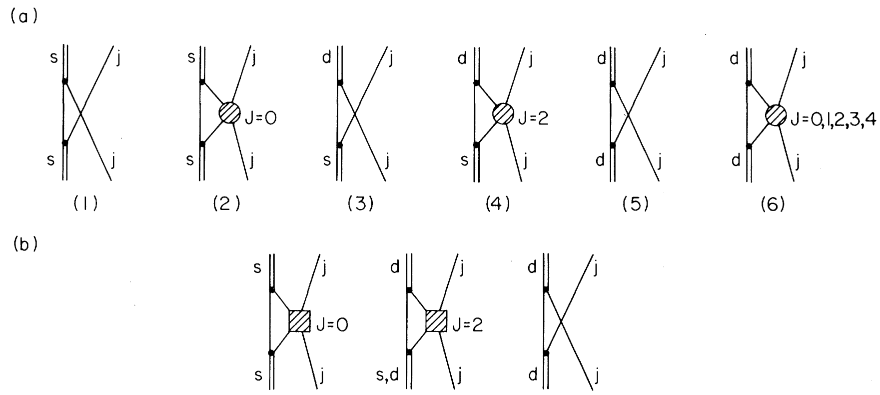

FIG. 1. (a) Diagrams describing the couplings of an odd particle with angular momentum j (single line) to the bosons s and d (double line). Two-body fermion-fermion interactions are denoted by a circle. (b) The three terms which dominate the structure of odd-A nuclei. The square denotes an effective fermion-fermion interaction.

γ soft-rotor model. These limiting situations occur whenever the boson Hamiltonian, H_B, possesses one of its three possible dynamical symmetries[5] SU(5), SU(3), or O(6). Postponing a discussion of the limits (i) and (iii) to a longer paper, we concentrate here only on the limit based on the SU(3) boson symmetry and show that it gives spectra similar to those of the Nilsson model.[6] To this end, we present in Figs. 2(a) and 2(b) the results of two calculations for a particle with $j=\frac{9}{2}$ coupled to an SU(3) boson core with V_{BF} as in (1) and $A=0$. The two calculations correspond to two different choices of the interaction strengths Λ and Γ. In the first calculation, Fig. 2(a), done with $\Lambda=0$ (corresponding to an empty shell) and $\Gamma<0$ (corresponding to a negative deformation), the diagonalization produces a series of rotational bands with bandheads $K_0=\frac{9}{2}, \frac{7}{2}, \frac{5}{2}, \frac{3}{2}$, and $\frac{1}{2}$. These are analogous to the bands which can be constructed in the Nilsson model for a single particle with $j=\frac{9}{2}$ moving in a deformed potential with deformation $\beta<0$. However, the diagonalization produces other excited bands. In order of increasing energy, we find bands with $K=K_0$ and $K=K_0\pm 2$ [Fig. 2(a)]. The bands correspond to the bands built on the SU(3) representation $(2N-4, 2)$ in even-even nuclei[1] and thus could be called β and γ bands. While here they arise automatically, in the Nilsson model they must be either placed *ad hoc* or calculated by use of other methods. In the second calculation, Fig. 2(b), done with $\Lambda>0$ (corresponding to a partially occupied shell) and $\Gamma>0$ (corresponding to positive deformation), the diagonalization still produces a series of rotational bands but with different ordering. The band with $K_0=\frac{5}{2}$ is now the lowest, followed by $K_0=\frac{7}{2}, \frac{3}{2}, \frac{1}{2}$, and $\frac{9}{2}$. Associated with these bands there are, at higher energies, the corresponding β and γ bands as before. The different ordering of the K_0 bands here results from a balance between quadrupole (Γ) and exchange (Λ) couplings. Since, as one can directly demonstrate starting from the shell model,[3,4] the effects of the latter term are proportional to the occupation probability of the shell, this balance is analogous to the balance between deformation, β, and Fermi energy, λ_F, in the Nilsson model; it is the crucial ingredient in producing spectra similar to those of the Nilsson model within the framework of the present approach. The introduction of the monopole term, A, does not modify these conclusions.

The most interesting aspect of the present method, however, is that, in addition to the three limiting cases (i), (ii), and (iii), it is potentially capable of describing intermediate situations. As an example, we show in Fig. 3 the results of a preliminary calculation of the low-lying negative-parity states of the odd Eu isotopes, which originate from a proton in the $1h_{11/2}$ level coupled to the even-even Sm cores. In this calculation, the boson part of H was taken from Scholten, Iachello, and Arima[7] and the parameters A, Γ, and Λ appearing in the boson-fermion interaction were adjusted so as to produce a good overall fit (they are given in the figure caption). Further details, including results for the positive-parity states based on the $g_{7/2}$, $d_{5/2}$ single-particle levels and electromagnetic transition rates will be presented in a longer paper. Here we emphasize that

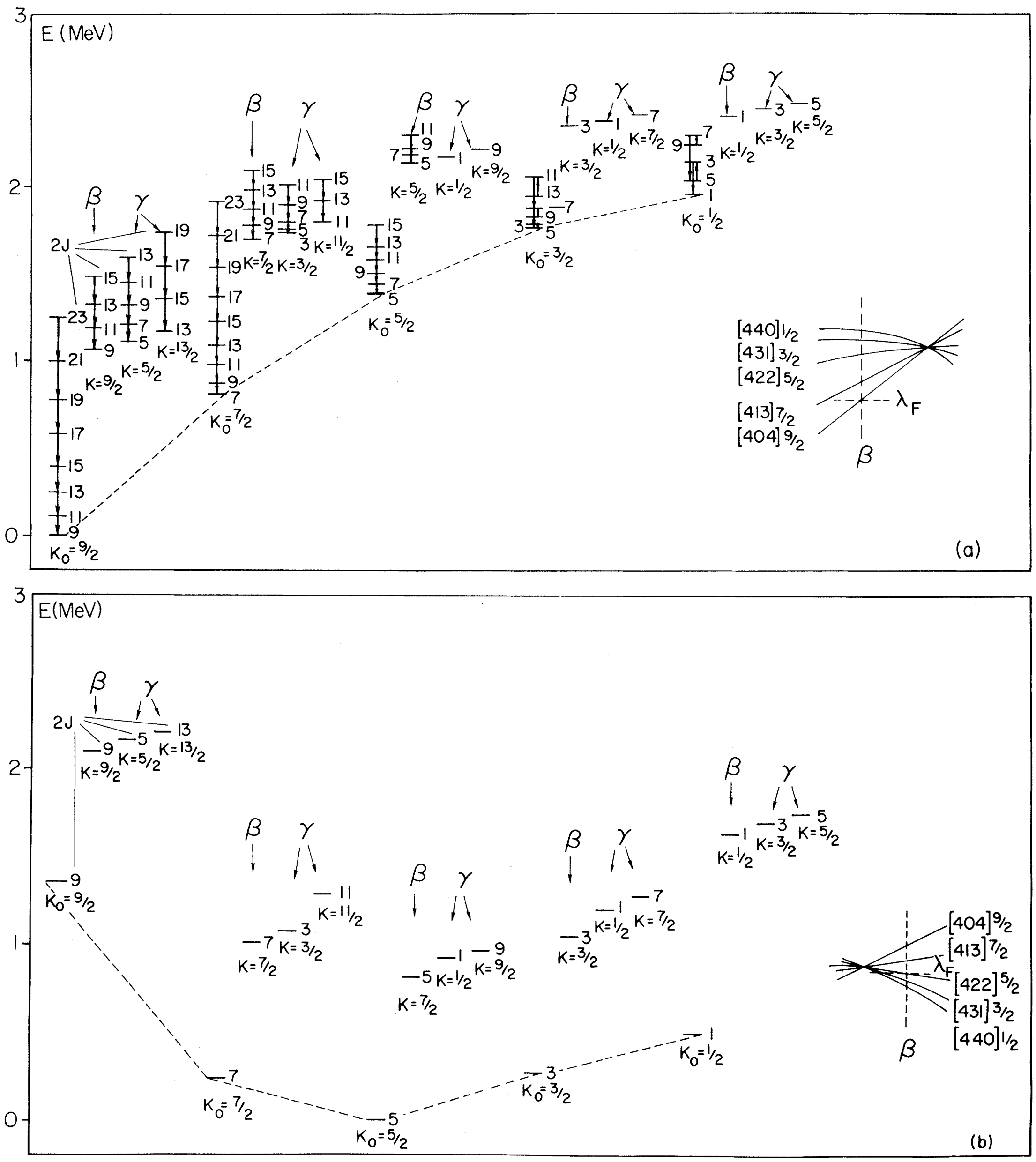

FIG. 2. (a) A typical spectrum in the SU(3) limit of the interacting boson-fermion model. The number of bosons is $N=6$, the odd particle has $j=\frac{9}{2}$ and the energy levels are calculated by diagonalizing the Hamiltonian (Ref. 1) $H=-\kappa(5^{1/2})[Q^{(2)}\times Q^{(2)}]^{(0)}+\Gamma[Q^{(2)}\times(a_j^\dagger\times\tilde{a}_j)^{(2)}]^{(0)}$ with $\kappa=12.5$ keV, $\Gamma=-500$ keV, and $Q^{(2)}=(d^\dagger\times s+s^\dagger\times\tilde{d})^{(2)}$. Only a selected number of levels is shown. The levels have been arranged into bands denoted by the lowest value of the angular momentum, K, contained in the band. This quantum number is only approximately equivalent to the quantum number K in the Nilsson model. In the inset, the corresponding situation in the Nilsson model is shown. (b) Same as above but for $H=-\kappa(5)^{1/2}[Q^{(2)}\times Q^{(2)}]^{(0)}+\Gamma[Q^{(2)}\times(a_j^\dagger\times\tilde{a}_j)^{(2)}]^{(0)}-\Lambda:[(\tilde{d}\times a_j^\dagger)^{(j)}\times(d^\dagger\times\tilde{a}_j)^{(j)}]^{(0)}$: with $\kappa=12.5$ keV, $\Gamma=175$ keV, $\Lambda=1385$ keV. Only the lowest state of each band is shown.

the possibility of following the complex interweaving of levels across a transitional region in odd-A nuclei opens a new perspective into the very rich phenomenology of this class of nuclei.

Finally, we point out that although one of us (F.I.) in collaboration with Arima[8] had already indicated the most general form of the boson-fermion Hamiltonian, we had not previously succeeded in

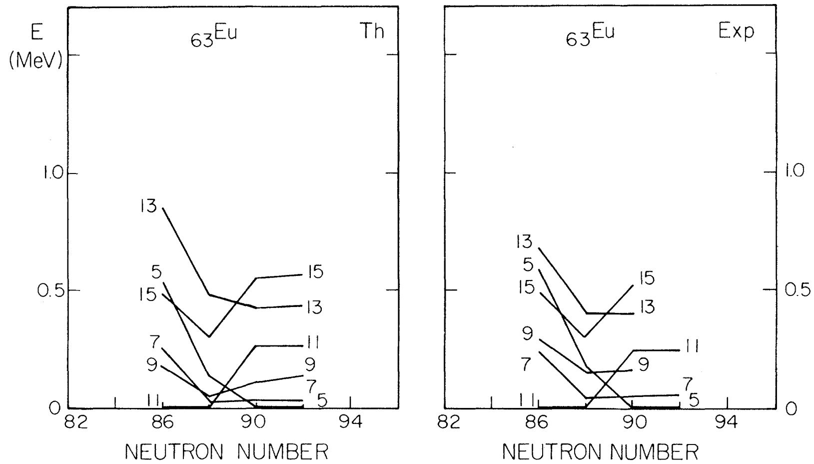

FIG. 3. Low-lying negative-parity states in the odd Eu isotopes. The numbers next to each level denote the values of $2J$. The experimental location of the levels $J^{\pi} = 9/2^-$, $13/2^-$, and $15/2^-$ in ^{155}Eu is not known. The theoretical curves are calculated by use of (1) with $A = 300$ keV, $\Gamma = 1800$ keV, and $\Lambda = 5889$ keV.

showing explicitly that it would produce spectra with the observed properties. The key development has been the recognition of the special role played by the exchange term, Λ, in determining the ordering of the various collective bands. This term had already been introduced by Civitarese, Broglia, and Bes[9] in their particle-vibration description of ^{211}Pb. Its importance in producing a Nilsson-like scheme has also been recently suggested by Bes and Broglia[10] in their nuclear field-theory treatment of the interacting-boson model.

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[1]A. Arima and F. Iachello, Phys. Rev. Lett. 35, 1069 (1975).

[2]A. Arima, T. Otsuka, F. Iachello, and I. Talmi, Phys. Lett. 66B, 205 (1977); T. Otsuka, A. Arima, F. Iachello, and I. Talmi, Phys. Lett. 76B, 139 (1978). In the present paper, for simplicity, we will not distinguish between proton and neutron bosons.

[3]T. Otsuka, A. Arima, and F. Iachello, Nucl. Phys. A309, 1 (1978). T. Otsuka, Ph.D. thesis, University of Tokyo, Japan, 1979 (unpublished).

[4]P. F. Bortignon, P. A. Broglia, D. R. Bes, and R. Liotta, Phys. Rep. C30, 305 (1977).

[5]A. Arima and F. Iachello, Ann. Phys. (N.Y.) 99, 253 (1976), and 111, 201 (1978); A. Arima and F. Iachello, Phys. Rev. Lett. 40, 285 (1978).

[6]S. G. Nilsson, K. Dan Vidensk. Selsk. Mat.-Fys. Medd. 29, No. 16 (1955).

[7]O. Scholten, F. Iachello, and A. Arima, Ann. Phys. (N.Y.) 115, 321 (1978).

[8]A. Arima and F. Iachello, Phys. Rev. C 14, 761 (1976).

[9]O. Civitarese, R. A. Broglia, and D. R. Bes, Phys. Lett. 72B, 45 (1977).

[10]D. R. Bes and R. A. Broglia, to be published.