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[1]See, for instance, R. Field, in *Proceedings of the Nineteenth International Conference on High Energy Physics, Tokyo, Japan, 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo, 1979); B. Sakita, *ibid.*; Y. Nambu, *ibid.*

[2]K. Wilson, Phys. Rev. D 10, 2445 (1974); R. Balian, J. M. Drouffe, and C. Itzykson, Phys. Rev. D 10, 3376 (1974), and 11, 2098, 2104 (1975).

[3]J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975); J. Kogut, D. K. Sinclair, and L. Susskind, Nucl. Phys. B114, 199 (1976).

[4]S. Mandelstam, Phys. Rep. 23C, 307 (1976); G.'t Hooft, Nucl. Phys. B138, 307 (1978).

[5]Such a wave function was also suggested by K. Wilson, Cornell University Report No. 405, June 1978 (to be published).

[6]Y. Nambu, Phys. Lett. 80B, 375 (1979); J. L. Gervais and A. Neveau, Phys. Lett. 80B, 255 (1979); E. Corrigan and B. Hasslacher, Phys. Lett. 81B, 181 1979).

Higher-Order Quantum Chromodynamic Corrections in e^+e^- Annihilation

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Nonleading quantum chromodynamic corrections to e^+e^- annihilation into hadrons are computed. Comparison with experiment is briefly discussed.

In quantum chromodynamics (QCD), processes which probe the structure of hadrons at short distances may be investigated with use of perturbation theory and the renormalization group.[1] The photon vacuum polarization tensor,

$$\Pi_{\mu\nu}(q)=i(q^2 g_{\mu\nu}-q_\mu q_\nu)\Pi(-q^2)$$

for q^2 large and spacelike, is one such short-distance probe. $\Pi(-q^2)$ and $R=\sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$ can be related through dispersion relations or smearing methods.[2] In regions between new quark thresholds, where the cross section is reasonably smooth, one may hope to obtain R directly from the discontinuity of $\Pi(-q^2)$.

The leading QCD corrections to $\Pi(-q^2)$ are well known,[3] and arise from the renormalization-group improvement of the graphs of Fig. 1(a). In this paper we report a calculation of $\Pi(-q^2)$ through order g^4, arising from the graphs of Fig. 1(b). This calculation is necessary in order to determine if higher-order corrections are small, and in order that one may compare the strong coupling constant determined from measurement of R with that measured in other processes, such as deep-inelastic scattering. To address the first issue we will employ two renormalization schemes, the minimal scheme (MS) of 't Hooft and a modified scheme ($\overline{\text{MS}}$) due to Bardeen *et al.*[4] This latter scheme has been shown to lead to a more satisfactory perturbation series than MS in deep-inelastic and photon-photon scattering, and the same will be seen to be true here.

The problem of determining the strong-coupling constant can be understood in terms of the

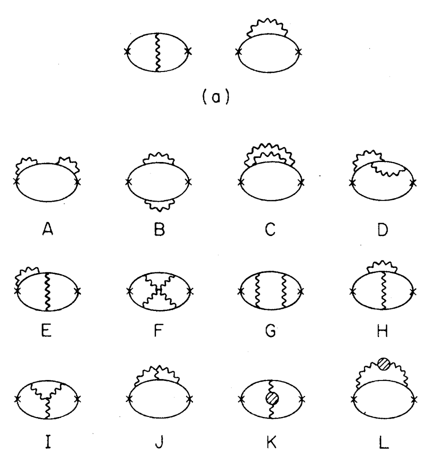

FIG. 1. (a) Graphs whose discontinuity gives R to order g^2. (b) Graphs whose discontinuity gives R to order g^4.

mass, Λ, which is frequently used to parametrize the running coupling. The running coupling constant, $\alpha_s(-q^2)$, may be written[4]

$$\alpha_s(-q^2)=\alpha_s^0(-q^2)-(\beta_1/4\pi\beta_0)[\alpha_s^0(-q^2)]\ln\ln(-q^2/\Lambda^2)+O((\alpha_s^0)^3), \quad (1)$$

where

$$\alpha_s^0(-q^2)=4\pi/\beta_0\ln(-q^2/\Lambda^2) \quad (2)$$

$$\beta_0=(11/3)C_A-\tfrac{2}{3}N_f \quad (3a)$$

$$\beta_1=(34/3)C_A^2-(10/3)C_AN_f-2C_FN_f \quad (3b)$$

(Ref. 5). In these expressions, C_A and C_F are the quadratic Casimir operators for the adjoint and fermion representations, respectively, and N_f is the number of quark flavors [for SU(N), $C_A=N$ and $C_F=(N^2-1)/2N$]. As several authors have noted, in order to make a meaningful determination of Λ, it is necessary both to use the expression above for α_s and to compute all corrections to the process of interest through order $(\alpha_s^0)^2$.[4,6] The necessary calculations have already been performed for deep-inelastic and photon-photon scattering[4]; our calculation is necessary to put R on the same footing.

The calculation was performed using the dimensional regularization procedure of 't Hooft and Veltman,[7] in which Feynman amplitudes are continued to $4-\epsilon$ dimensions and ultraviolet divergences appear as poles as $\epsilon\to 0$. We work in Feynman gauge and set all quark masses equal to zero. Integrals involving self-energy insertions were performed by use of spectral representations. The remaining integrals were performed by introducing Feynman parameters and performing the momentum integrals. Subdivergences in the resulting parameter integrals were treated by adding and subtracting from the integrand simpler functions with the same singularity structure, along the lines of Cvitanović and Kinoshita.[8] This procedure yielded a finite integral which was evaluated numerically,[9] along with divergent integrals which were performed analytically.

For each diagram we obtained only the coefficient of $q_\mu q_\nu$, which can be identified with $q_\mu q_\nu - q^2 g_{\mu\nu}$ in gauge-invariant sets of diagrams. The results are presented in Table I, where the coefficients of $1/\epsilon^3$, $1/\epsilon^2$, and $1/\epsilon$ are given. The numerical errors in each diagram are less than 0.4%, though, as a result of large cancellations, the error in the sum is 2%. Calling the sum of the graphs of Fig. 1(b) $\Pi_{\rm un}^{(6)}$ we find

$$\Pi_{\rm un}^{(6)}=\frac{\alpha}{\pi}\left(\frac{g^2}{4\pi^2}\right)^2 C_F\left[-\frac{\beta_0}{12\epsilon^2}+\frac{B}{\epsilon}+O(1)\right]\Gamma\left(1+\frac{3\epsilon}{2}\right)(4\pi)^{3\epsilon/2}\left(-\frac{q^2}{\mu^2}\right)^{-3\epsilon/2}, \quad (4)$$

Table I. Pole terms from the graphs of Fig. 1(b), where a common factor

$$\left[\frac{\alpha}{\pi}\left(\frac{g^2}{4\pi^2}\right)^2\Gamma\left(1+\frac{3\epsilon}{2}\right)(4\pi)^{3\epsilon/2}\left(-\frac{q^2}{\mu^2}\right)^{-3\epsilon/2}\right]$$

has been taken out.

	Weight	$1/\epsilon^3$	$1/\epsilon^2$	$1/\epsilon$
A	$2C_F^2$	$-1/18$	$-4/27$	-0.1839
B	C_F^2	$-1/18$	$-4/27$	-0.1978
C	$2C_F^2$	$-1/36$	$-35/432$	-0.1226
D	$2C_F(C_F-C_A/2)$	$1/18$	$29/216$	0.1642
E	$4C_F^2$	$1/12$	$47/144$	0.3016
F	$C_F(C_F-C_A/2)$	0	$1/9$	-0.0625
G	C_F^2	$-1/9$	$-115/216$	-0.5757
H	$2C_F(C_F-C_A/2)$	$-1/18$	$-59/216$	-0.0322
I	$2C_FC_A$	$-1/12$	$-61/144$	-0.4240
J	$2C_FC_A$	$1/12$	$43/144$	0.4714
$K+L$	C_FC_A	0	$\frac{2N_f-5C_A}{36C_A}$	$0.00679N_f/C_A-0.0447$

where

$$B = 0.0212\, C_F - 0.0506\, C_A + 0.00579 N_f . \tag{5}$$

The mass, μ, is arbitrary, and is introduced to give the coupling constant correct dimensions in $4-\epsilon$ dimensions. To take account of the QCD counterterms, we add to $\Pi_{\rm un}^{(6)}$

$$\Pi_{\rm CT}^{(6)} = \frac{\alpha}{\pi}\left(\frac{g^2}{4\pi^2}\right) Z\, C_F\left[-\frac{1}{4\epsilon} + D + O(\epsilon)\right]\Gamma(1+\epsilon)(4\pi)^{\epsilon}\left(-\frac{q^2}{\mu^2}\right)^{-\epsilon}, \tag{6}$$

where

$$D = 0.0564, \tag{7a}$$

$$Z = -(\beta_0/2\epsilon)(g^2/4\pi^2)(1+M\epsilon). \tag{7b}$$

The term in brackets in Eq. (6) is the unsubtracted two-loop contribution. Z is the sum of all one-loop counterterms (the two-loop counterterms cancel). The prescription dependence of the calculation enters through the finite part of Z, which is arbitrary. In the minimal scheme, counterterms are introduced in each order so as to cancel only the pole parts of the divergent quantities. The $\overline{\rm MS}$ scheme is defined by absorbing all factors of $\ln 4\pi - \gamma$, where γ is Euler's constant, into the renormalized coupling constant (this amounts to rescaling Λ). For MS, $M=0$, while for $\overline{\rm MS}$, $M = (\ln 4\pi - \gamma)/2$. Before subtraction, we have, expanding $\Pi_0^{(6)} = \Pi_{\rm un}^{(6)} + \Pi_{\rm CT}^{(6)}$ in powers of ϵ,

$$\Pi_0^{(6)} = \left(\frac{\alpha}{\pi}\right)\left(\frac{g^2}{4\pi^2}\right)^2 C_F\left\{\frac{\beta_0}{24\epsilon^2} + \frac{8B+\beta_0(M-4D)}{8\epsilon} - \frac{\beta_0}{32}\ln^2\left(-\frac{q^2}{\mu^2}\right) + \ln\left(-\frac{q^2}{\mu^2}\right)\left[\frac{\beta_0}{16}(\ln 4\pi - \gamma - 2M + 8D) - \frac{3B}{2}\right] + O(\epsilon)\right\}. \tag{8}$$

The $\ln^2(-q^2/\mu^2)$ represents the explicit beginning of the renormalization-group improvement of Fig. 1(a). For $C_A=0$, $C_F=1$, and $N_f=0$, this expression reproduces a result due to Rosner for quantum electrodynamics (QED).[10]

In both the MS and the $\overline{\rm MS}$ schemes, the μ dependence, and hence the scaling properties of the renormalized $\Pi(-q^2)$, are completely determined by $C_1(g^2)$, the coefficient of the simple pole of Π_0. In either scheme,

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right]\Pi(-q^2) = \frac{\partial}{\partial g^2}[g^2 C_1(g^2)]. \tag{9}$$

From our calculation and well-known QED results,[11] we have

$$C_1(g^2) = -\tfrac{2}{3} - \tfrac{1}{4}C_F\left(\frac{g^2}{4\pi^2}\right) + C_F\left[B + \frac{\beta_0}{8}(M-4D)\right]\left(\frac{g^2}{4\pi^2}\right)^2 + O(g^6). \tag{10}$$

Equation (9) is readily solved in terms of the running coupling constant. If we assume that we may obtain R by taking the discontinuity of Π,[12] we obtain, specializing to SU(3),

$$R = \sum Q_i^2\left(1 + \frac{\alpha_s(s)}{\pi} + \begin{cases}(7.35 - 0.442 N_f)\left(\dfrac{\alpha_s(s)}{\pi}\right)^2 & \text{MS}\\[2mm] (1.98 - 0.115 N_f)\left(\dfrac{\alpha_s(s)}{\pi}\right)^2 & \overline{\text{MS}}\end{cases}\right). \tag{11}$$

Thus just as in deep-inelastic and photon-photon scattering the perturbation theory appears more satisfactory in the $\overline{\rm MS}$ than in the MS scheme.[13]

In order to confront theory with the experimental value of R, several effects must be taken into account. To illustrate their relative importance, we work in the $\overline{\rm MS}$ scheme, taking $\sqrt{s} = 6$ GeV, $\Lambda = 0.5$ GeV. This choice of Λ is motivated by recent analyses, including higher-order QCD corrections, of deep-inelastic scattering data.[4] Then, in order of decreasing importance, one must consider the following:

(1) The lowest-order result. With the presently accepted four quarks the contribution is $R_1 = 10/3$.

(2) The first QCD correction. Using $\alpha_s^{\,0}$ one

gets a contribution to R, $R_2=0.32$.

(3) QED radiative corrections.[14] In the experimental analysis of R, account is taken only of radiation from the initial electrons and the electron-loop contribution to vacuum polarization. There fore, to the theoretical prediction of R we must add vacuum polarization contributions from muons, taus, and hadrons, along with radiation from the quark lines. The last effect is negligible, but the vacuum polarization terms give $R_3=0.13$.

(4) Mass corrections. The first QCD correction is computed in the zero-mass limit, and while this approximation is satisfactory for u, d, and s quarks, it is not valid for the charmed quark. Following the treatment of Poggio, Quinn, and Weinberg,[2] The effect of taking the charmed-quark mass to be 1.5 GeV is $R_4=.088$.

(5) Higher-order QCD corrections. In $\overline{\text{MS}}$, although the correction due to the graphs of Fig. 1(b) is 0.047, inclusion of the effect of the two-loop β function cancels the effect so that $R_5=-0.029$.

The smallness of R_5 relative to the other contributions to R is the most important feature of our calculation; higher-order QCD corrections are small and make no qualitative change in the results obtained from the first-order analysis.[15] Adding the above corrections gives $R=3.84$ at $s=36\ \text{GeV}^2$, to be compared with the experimental value[16] $R=4.17\pm 0.09\pm 0.42$, where the first error is statistical and the second systematic. Clearly the data do not rule out the existence of an additional charge-$\frac{1}{3}$ quark or spinless boson, but the large systematic error prohibits a definite conclusion. The use of smearing techniques and dispersion relations is currently under study, but we do not expect that our conclusions will be qualitatively modified.

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[1]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).

[2]E. C. Poggio, H. R. Quinn, and S. Weinberg, Phys. Rev. D 13, 1958 (1976); A. De Rújula and H. Georgi, Phys. Rev. D 13, 1296 (1976); R. Shankar, Phys. Rev. D 15, 755 (1978); S. L. Adler, Phys. Rev. D 10, 3714 (1974).

[3]T. Appelquist and H. Georgi, Phys. Rev. D 8, 4000 (1973); A. Zee, Phys. Rev. D 8, 4038 (1973).

[4]G. 't Hooft, Nucl. Phys. B 62, 444 (1973); W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. D 18, 3998 (1978); E. G. Floratos, D. A. Ross, and C. T. Sachrajda, Nucl. Phys. B129, 66 (1977), and B139, 545(E)(1978); A. J. Buras and W. A. Bardeen, Fermilab Report No. PUB-78191, 1978 (unpublished).

[5]W. E. Caswell, Phys. Rev. Lett. 33, 244 (1974); D. R. T. Jones, Nucl. Phys. B75, 531 (1974).

[6]M. Bace, Phys. Lett. 78B, 132 (1978).

[7]G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972).

[8]P. Cvitanović and T. Kinoshita, Phys. Rev. D 10, 3978 (1974).

[9]The numerical integration programs used were SHEP, described by A. Dufner in Proceedings of the Colloquium on Computational Methods in Theoretical Physics, Marseille, 1970 (unpublished), and VEGAS [G. P. Lepage, J. of Comp. Phys. 27, 192 (1978)]; most trace evaluations were performed with the use of REDUCE [A. C. Hearn, Stanford University Report No. ITP-247 (unpublished)].

[10]J. L. Rosner, Ann. Phys. 44, 11 (1967); W. Brandt, Ph.D. thesis, University of Washington (unpublished).

[11]R. Jost and J. M. Luttinger, Helv. Phys. Acta 23, 201 (1950).

[12]R. G. Moorehouse, M. R. Pennington, and G. G. Ross, Nucl. Phys. B124, 285 (1977).

[13]W. Celmaster and R. J. Gonsalves, University of California, San Diego, Report No. 10P10-210 (to be published).

[14]F. J. Ynduráin, Nucl. Phys. B136, 533 (1978).

[15]Cases in which higher-order corrections are quite large are given in W. Celmaster, Stanford Linear Accelerator Center Report No. SLAC-PUB-2151, 1978 (unpublished); R. Barbieri, E. D'Emilio, G. Curci, and E. Remiddi CERN Report No. CERN-TH-2622, 1979 (unpublished).

[16]R. Schwitters and J. Siegrist, in preparation.

[17]K. G. Chetyrkin *et al.*, Institute for Nuclear Research, Moscow, Report No. 126, 1979 (to be published).