Understanding $\eta' \to \eta\pi\pi$ and Allied Processes Despite Adler Zeros

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With use of the (hitherto obscure) negative-parity meson matrix N_+ of partial symmetry, an $\eta' \to \eta\pi\pi$ amplitude, explicitly satisfying the Adler conditions, has been constructed within the relativistic quark pair-creation model without any adjustable parameters. The model yields a total η' width of 388 keV and $\eta\pi\pi$, $\rho^0\gamma$, and $\gamma\gamma$ branching ratios of 68.0%, 29.9%, and 1.88%, all in excellent agreement with data. The same model gives the π-π scattering lengths as $m_\pi a_0 = 0.19$, $m_\pi a_2 = -0.08$.

Hadronic processes involving more pions than one are known to be governed by current-algebraic partial conservation of axial vector current (PCAC) constraints, especially Adler zeros, as exhibited, e.g., in Weinberg's paper.[1] These are generally obeyed by data (e.g., pion scattering lengths),[2] but there are apparent anomalies like $\eta' \to \eta\pi\pi$ whose expected rate in terms of the σ term (small in the quark model) is much smaller than observed, despite efforts with symmetry-breaking terms.[3,4] Very recently an important mechanism for the enhancement of the $\eta' \to \eta\pi\pi$ amplitude (governed by δ and ϵ exchanges) in the physical region without violating the PCAC constraints has been suggested by Deshpande and Truong[5] via derivative $PP'S$ (P, P' = pseudoscalar, S = scalar) couplings of the form $\partial_\mu P \partial_\mu P'S$.

In view of the obvious PCAC significance of the *derivative $PP'S$* couplings, as against their usual nonderivative forms[6] which have an immediate quark-model counterpart in the so-called quark-recoil effect,[7] it is extremely desirable to have a corresponding quark-model structure[8,9] of the former, preferably in terms of the basic coupling constant g_ρ, so as to facilitate a simultaneous understanding of $\eta' \to \eta\pi\pi$ which is "large," and allied PCAC-constrained processes such as π-π scattering which is "small," within a unified framework without the need for separate parametrizations[5] for individual $PP'S$ or other couplings. It turns out that the twin features of the relativistic quark pair-creation (QPC) model *and* partial symmetry[10,11] have between them the necessary theoretical ingredients for such a unified description, so that the combined framework is rich and broad enough for predicting a fairly wide range of hadronic processes with a single universal coupling constant $g_\rho = (12\pi)^{1/2}$ characterizing relativistic QPC,[12] and the (equally universal) spring constant $\Omega \approx 1$ GeV^2 characterizing the harmonic oscillator model which gives a linear rise of M^2 with excitation.[13] In particular an explicit parameter-free construction is possible (albeit in a heuristic fashion) for $\partial_\mu P \partial_\mu P'S$ couplings, provided one employs the (hitherto obscure) *negative-parity* meson matrix[11] N_+ instead of the more familiar positive-parity matrix M_+ which dominates at low momenta.

The resulting Lagrangian in momentum space for the transition $A(P) \to B(P') + C(S)$, with four momenta $K_{A\mu}, K_{B\mu}, K_{C\mu}$, is

$$\mathcal{L}_{ABC} = [f](\tfrac{3}{2})^{1/2} g_\rho \tfrac{1}{2} m_\rho^{-2} K_A \cdot K_B [1 + \tfrac{2}{9}\lambda(m_A^2, m_B^2, -K_C^2) m_C^{-2}] F \exp(-\tfrac{2}{3} m_\rho^2), \tag{1}$$

where $[f]$ is an SU(6) factor (specified later), λ the usual invariant function of masses, and F the QPC form factor [see Eq. (6) below]. For the transition $C(S) \to B(P') + \bar{A}(P)$, we have $K_{A\mu} \to -K_{A\mu}$ and $\frac{2}{9} \to -\frac{4}{9}$.[14] Equation (1) may be compared with the more familiar Lagrangian for $\pi_A \to \rho_C + \pi_B$, under the same QPC and partial symmetry assumptions, viz.,[9]

$$\mathcal{L}_{\pi_A \rho \pi_B} = g_\rho (K_{A\mu} + K_{B\mu}) \rho_\mu{}^a T_a{}^\pi F \exp(-\tfrac{2}{3} m_\rho^2). \tag{2}$$

The PCAC-constrained structures of (1) and (2) are brought out explicitly by the factors $K_A \cdot K_B$ and $K_{A\mu} + K_{B\mu}$, respectively. When applied to π-π scattering governed by ρ and ϵ exchanges in this model, the ϵ term makes a small contribution because of the quadratic appearance of the $K_A \cdot K_B$ factor (small in this case), and the (ρ dominant) result is

$$m_\pi a_0 = +0.19, \quad m_\pi a_2 = -0.08, \tag{3}$$

thus bringing out the typical smallness of these scattering parameters in conformity with theory[1] as well as present estimates ($m_\pi a_0 \approx 0.26 \pm 0.1$).[2] On the other hand, the factor $K_A \cdot K_B$ in (1) has the effect of a big enhancement[5] in $\eta' \to \eta\pi\pi$, and the result in this case is

$$\Gamma(\eta' \to \eta\pi\pi) = 265 \text{ keV}. \tag{4}$$

The relativistic QPC model gives for the two principal η' radiative modes $\rho^0\gamma$ and $\gamma\gamma$ the values 116 and 7.3 keV,[15] so that the total η' width (assuming saturation by these three modes) is predicted as

$$\Gamma_{\text{tot}} = 388 \text{ keV } (0.28 \pm 0.10 \text{ MeV}), \tag{5}$$

the latest measured value[16] being shown in parentheses. Indeed, the *branching* ratios, where the data (in parentheses) are more reliable, are 68.0% [(67.6 ± 1.7)%], 29.9% [(30.4 ± 1.7)%], and 1.88% [(1.92 ± 0.27)%] for $\eta\pi\pi$, $\rho^0\gamma$, and $\gamma\gamma$, respectively, thus bringing out the more quantitative nature of the agreement. The same QPC constant g_ρ also explains most radiative meson decays,[16] as well as strong decays[17] (e.g., $\Gamma_{\rho\pi\pi} = 110$, $\Gamma_{K^*K\pi} = 47.5$, $\Gamma_{\varphi K^+K^-} = 1.94$, etc.).

Before discussing these results further, we give a brief outline of the essential ingredients of relativistic QPC *and* partial symmetry which lead to these and allied results. QPC was originally conceived in a nonrelativistic spirit[12] and has had several successful applications in the charmed sector.[18] For relativistic applications, on the other hand, one needs a minimal set of relativization prescriptions, such as those described in one of the applications of QPC to hadron electromagnetic masses through a fixed ($J = -2$) pole mechanism,[19] in excellent accord with data.[2] The general techniques of evaluation of relativistic QPC matrix elements are described in a recent review.[9] However, to widen the scope of physical applications it has been found necessary to sharpen[15] some of these prescriptions without affecting in any manner the results of electromagnetic mass calculations.[19] These concern (i) the choice of the so-called radiation quantum, (ii) a new QPC description of s-wave hadron couplings through the meson matrix N_+, and (iii) a simpler (albeit intuitive) form of relativization of the body form factor which agrees with the earlier prescription[19] for the equal-mass case, viz., the replacement[14] ($\Omega = 1$ GeV^2, $\vec{K}_A \to K_{A\mu}$, etc.)

$$\exp[-\tfrac{1}{3}(\vec{K}_A{}^2 + \vec{K}_B{}^2 + \vec{K}_C{}^2)] \to \exp[-\tfrac{1}{3}(K_A{}^2 + K_B{}^2 + K_C{}^2)] \equiv F \tag{6}$$

which defines the function F, Eq. (6), for the process $A(P) \to B(P') + C(S)$ where $K_{A,B}{}^2 = -m_{A,B}{}^2$ but $K_C{}^2 \neq -m_c{}^2$. This prescription needs to be used with the utmost caution since its range of validity (it certainly has a small nonrelativistic domain of validity to start with) must be severely limited by unitarity. A conservative estimate for applicational purposes has been taken as $|K_\mu{}^2| \lesssim 1$ GeV^2 which is not only adequate for the present purposes, but has recently been employed for charge-exchange reactions through t channel and A_2 (Reggeized) exchanges with unexpected success.[20]

The question of choice[15] of the radiation quantum (mostly a V meson) does not much concern us here, and so the main part of this discussion is devoted to the arguments for the QPC construction of s-wave $PP'S$ couplings, Eq. (1). Similar constructions are applicable to more general situations which may be abstracted as follows. $PP'S$ is an example of PPV couplings with an L-excited V meson but with $J = L - 1 = 0$, which is *lower* than the maximum ($L + 1 = 2$) allowed in this case. Whenever such is the case, the coupling occurs in a *lower* partial wave ($l = 0$) than the maximum that is allowed ($l = 2$) with the highest J value ($L + 1$). A similar situation occurs when one of the P mesons instead of the V meson is L excited, e.g., in $B\omega\pi$ coupling which is again s-wave dominated (with a small d-wave mixture). For all such cases of L-excited PPV couplings, dominated by a *lower* partial wave than the allowed maximum, we propose their construction *via the* N_+ *matrix*[11] rather than the more usual M_+[10] which dominates in the low-momentum limit and constitutes the principal quark-model mechanism[8,9] for generating hadron couplings. The V-meson parts of M_+, and certain relevant (ρ, π) parts of N_+, are[9]

$$M_+ = \tfrac{1}{2} g_\rho m_\rho{}^{-1}(-i\vec{\sigma} \cdot \vec{V} \times \vec{K} + m_V V_0), \tag{7}$$

$$N_+ = \tfrac{1}{2} g_\rho m_\rho{}^{-1}[(m_\rho + K_{\rho 0})\rho_i{}^a \sigma_i + (m_\pi + K_{\pi 0})\pi^a]\tau_a. \tag{8}$$

M_+ correctly generates all the *highest*-wave couplings ($J=L+1$ states), and does not of course have any problems of Adler zeros. One must have a relativistic meson normalization factor, as in Van Royen and Weisskoff[8] (VW), for all meson couplings. For example, for $\langle\pi|\rho|\pi\rangle$ via the $m_\rho\rho_0$ term of (7), one has $K_{A0}+K_{B0}$, $K_{A,B}$ being the two pion momenta, so that $(K_{A0}+K_{B0})\,V_0$ boosts to $-(K_{A\mu}+K_{B\mu})V_\mu$. Proceeding as in Mitra and Scod[9] (MS), one obtains (2) for the Lagrangian for the (virtual) process $\pi_A\to\rho+\pi_B$, with F as in (6) with appropriate four-momenta. The magnetic term in (7), on the other hand, leads to relativistically invariant couplings without extra indices (e.g., $\omega\rho\pi$, $A_2\rho\pi$) and these already have the correct four-momentum structures consistent with the highest-partial-wave requirements. For such cases, the VW factor proposed in MS[9] was $(-4K_A\cdot K_B)^{1/2}$ as the boosted form of $(4K_{A0}K_{B0})^{1/2}$ characterizing the standard energy normalizations of meson states. The latter was employed recently[14] for $V\to P\gamma$ and $P\to\gamma\gamma$ decays, with very good agreement with data.[2]

However, for s-wave couplings like (1) for which the nonrelativistic QPC structure of the matrix element gives little or no momentum dependence, the entire $\partial_\mu P\partial_\mu P'$ structure must now come, so to say, from a VW-like factor, so that the factor $(-4K_A\cdot K_B)^{1/2}$ (and hence M_+) is no longer adequate. To that end we seek to employ N_+ of Eq. (8), but since a literal use of N_+ (which has negative parity) to generate couplings in analogy with M_+ would give the wrong momentum dependence everywhere one must first "dress it up" with a negative-parity "spurion" to make up for the parity mismatch. The simplest multiplying factor consistent with the original QPC spirit,[12] is $\alpha\vec{\sigma}\cdot\vec{p}_{\rm opp}$, where $\vec{p}_{\rm opp}=\vec{p}-\vec{k}$ is the momentum of the $^3P_0$ loop[12] opposite the meson A undergoing the (real or virtual) transition $A\to B+C$. The factor α must be determined by comparison with a standard reference coupling, say $\pi\rho\pi$, which can be computed from both (7) and (8). Before doing this, however, one must take cognizance of a *renormalization factor* $\frac{3}{2}$ which arises as follows. The magnetic and the charge terms of (7) can again be recovered in a unified fashion by $\vec{\sigma}\cdot\vec{p}_{\rm opp}$ and $\vec{V}\cdot\vec{p}_{\rm opp}$, respectively, and in the process certain "recoil" and "convective" terms[9] would be generated. However, the *translation* (tr) in the *internal* variable $\vec{p}=\vec{k}+\vec{p}_{\rm opp}$ that would arise from the QPC integral $\int d^3\vec{p}\,\psi_A\psi_B\psi_C$ with Gaussian functions would lead to $\vec{p}-\vec{k}=\vec{p}_{\rm tr}-\frac{2}{3}\vec{k}$ and hence to an unwarranted factor $\frac{2}{3}$ at the meson coupling level,[21] such as in $\pi\rho\pi$ of Eq. (2).

To calculate α with the *renormalized* spurion $\frac{3}{2}\alpha\vec{\sigma}\cdot\vec{p}_{\rm opp}$, we take the ρ term of (8) with $m_\rho+K_{\rho 0}\approx 2m_\rho$ (allowed for small momentum), and do a relativistic QPC calculation[9] between π_A and π_B states, taking the VW factor as $(-4K_A\cdot K_B)^{1/2}$, just as for magnetic couplings described above (no free indices left for boosting). Comparison with (2) then gives

$$|\alpha|^{-1}=(2m_\rho^2-4m_\pi^2)^{1/2}\approx\sqrt{2}\,m_\rho. \tag{9}$$

With this value of α, we now use (8) to generate the s-wave $\pi\eta\delta$ coupling in the QPC model, this time via the *pion* term whose coefficient can be equally well approximated for small momenta as $2K_{\pi 0}$. The VW factor for the (virtual) process $\eta\to\delta+\pi$ now comes from the η energy, viz., $2K_{\eta 0}$, as in the original VW approach,[8] so as to yield the desired boost $4K_{\pi 0}K_{\eta 0}\to -4K_\pi\cdot K\eta$. The relativization of the other factors in the QPC matrix element goes through as in MS,[9] leading finally to (1).

The derivation is unavoidably heuristic, since the very premises of (7) and (8) are nonrelativistic in content, yet the final forms (1) and (2) have the desired theoretical features, and being free from adjustable parameters, their physical interest should stem from their capacity to make unambiguous and testable predictions. Thus the SU(6) factor $[f]$ in (1) equals $\frac{2}{3}\sqrt{2}$, $\frac{2}{3}\sqrt{3}$, $\frac{2}{3}\sqrt{6}$, and 2 for $\eta'\eta\epsilon$, $\eta'\pi\delta$, $\eta\pi\delta$, and $\pi\pi\epsilon$, respectively. These are based on an η-η' mixing angle $\beta=-\cot^{-1}\times 2\sqrt{2}$ suggested by Greco[22] on theoretical grounds, where $\eta'=\eta_1\cos\beta+\eta_8\sin\beta$ and $\eta=\eta_8\cos\beta-\eta_1\sin\beta$. For ϵ an ω-like assignment has been taken, with $m_\epsilon\approx m_\delta(980)$. (This last is not sensitive to $\eta'\to\eta\pi\pi$ which is dominated by δ exchange.) With the relevant relativistic Lagrangians (1) thus specified at the hadronic level, the δ and ϵ exchange amplitudes may be calculated as in Ref. 5, but the details are omitted for brevity. The Adler condition is now explicitly satisfied and the deviation from phase space is small since $1+\frac{2}{9}\lambda m^{-2}$ is practically unity. Since there are no adjustable parameters, the results (4) and (5), as well as the branching ratios, represent a nontrivial test of relativistic QPC,[9] together with the partial symmetry operator N_+ of Eq. (8).

Additional checks on (1) are provided by $\epsilon\to\pi\pi$ and $\delta\to\eta\pi$ decays which are predicted as 325 and 96 MeV, respectively. Again, in conformity with the remarks preceding Eq. (7), if we calculate $B\to\omega\pi$ coupling via the pion term of N_+, we get

$\Gamma_{B\to\omega\pi}=132$ MeV (125 ± 10),[2] and $|M(0)/M(\pm1)| = 0.40$ as against the two quoted values of 0.68 ± 0.12 and 0.47 ± 0.20.[12]

To summarize, we have found a simple mechanism, free of adjustable parameters, within the twin framework of partial symmetry (N_+ matrix)[11] and relativistic QPC, for a simultaneous understanding of "small" quantities like π-π scattering and "large" quantities like $\eta'\to\eta\pi\pi$ width, signifying different degrees of PCAC constraints in the physical region. The success of the mechanism is based solely on the momentum structure of $K_A\cdot K_B$ for s-wave $PP'S$ couplings, which represents a particular case of a more general momentum dependence of this nature whenever at least one of the P or V mesons in a PPV coupling is L excited and the transition can occur in a *lower* wave than the allowed maximum, e.g., in $B\to\omega\pi$ decay. We hope that these results will provide a semiphenomenological guide towards a deeper understanding of the pion and PCAC in a more fundamental approach. A fuller account (with other allied processes) will be published elsewhere.

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