Energy Scales for Noise Processes in Metals

P. Dutta, P. Dimon, and P. M. Horn
The James Franck Institute and the Department of Physics, The University of Chicago, Chicago, Illinois 60637
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We ascribe the excess noise in metals to random fluctuations with thermally activated characteristic times. From the known temperature dependence of the magnitude of the noise in Ag we infer a narrow distribution of activation energies that peaks in the vicinity of 1 eV. Frequency power laws close to f^{-1} are deduced and the small variations with temperature are consistent with observations. The predicted spectrum, like the observed one, has an apparent power-law nature throughout the frequency region accessible to experiment.

Excess noise with a $1/f$ frequency spectrum has the property that it is time- (and hence energy-) scale invariant. It is exactly this scale invariance that makes $1/f$ noise both interesting and difficult to explain. Generic $1/f$ noise (i.e., with a frequency spectrum proportional to $1/f^{\alpha}$ with $0.8 \leq \alpha \leq 1.4$) has been observed in a tremendous variety of experimental systems. However, the absence of any knees or bends in the frequency spectrum has obscured the time scales of the processes involved and has hampered attempts to determine the origin of the phenomena. We show in this paper that, in the framework of a random-fluctuation model, the temperature dependence of the frequency exponent $\alpha(T)$ observed in metal films[1] naturally suggests energy scales for the processes that generate the excess noise. As a consequence of these energy scales, we show that most of the features of excess noise in metals, especially the strong anomalous temperature dependence,[1] can be described in a simple way. Finally, we present data on the thickness dependence of the excess noise in silver films which provide an essential check of our assumptions.

Our conclusions follow from three physically plausible premises. We assume that (1) the noise arises from random fluctuations with characteristic times that are thermally activated; (2) the resistance is linearly coupled to the fluctuating quantity with a coupling constant that is nonsingular at long wavelengths; and (3) all samples studied are inhomogeneous, with the degree of inhomogeneity (which can be arbitrarily small) to be determined *a posteriori*. These assumptions are not new to the noise literature. Indeed, it is exactly these assumptions that led McWhorter[2] to suggest that $1/f$ noise in semiconductors arises from the random trapping and detrapping of carriers with a uniform distribution of trap depths. However, in the present case the fluctuating quantity need not be specified and in general is not known. Furthermore, we will show that the "$1/f$" noise observed in metals leads to a distribution of activation energies that is far from uniform.

A process causing random resistance fluctuations with a single characteristic time τ leads to the well-known shotlike spectrum for the voltage noise: $S_v(\omega) \propto \tau/(\omega^2\tau^2+1)$. If τ is thermally activated, i.e., $\tau = \tau_0 \exp(E/k_B T)$, and $\mathfrak{D}(E)$ is the distribution of activation energies E throughout the inhomogeneous sample,[2-4] then

$$S_v(\omega, T) \propto \int \frac{\tau_0 e^{E/k_B T}}{\omega^2 \tau_0^2 e^{E/k_B T} + 1} \mathfrak{D}(E) dE. \quad (1)$$

Here τ_0 is a characteristic "attempt time" which for many activated processes in solids is of the order of an inverse phonon frequency. Equation (1) can be integrated to give

$$S_v(\omega, T) \propto \frac{k_B T}{\omega}\left[\mathfrak{D}(\tilde{E}) + \sum_{n=1}^{\infty} \frac{\mathcal{E}_n}{(2n)!}\left(\frac{\pi k_B T}{2}\right)^2 \frac{d^{2n}}{dE^{2n}}\mathfrak{D}(E)\Big|_{E=\tilde{E}}\right], \quad (2)$$

where $\tilde{E} \equiv -k_B T \ln(\omega\tau_0)$ and $\mathcal{E}_n$ is the nth Euler number. To illustrate the physics involved, let us assume that the width of $\mathfrak{D}(E)$ is much larger than $k_B T$. Then, to an excellent approximation,

$$S_v(\omega, T) \propto \frac{k_B T}{\omega}\mathfrak{D}(\tilde{E}). \quad (3)$$

The limit $\mathfrak{D}(E) = \text{const}$ of Eq. (1) is equivalent to the proposal of van der Ziel[3] and du Pre[4] to generate exact $1/f$ noise. In this limit $S_v \propto k_B T/\omega$ and the frequency exponent is $\alpha = 1$. However, Eberhard and Horn have reported[1] that α in metal films varies with temperature. This immediately suggests that $\mathfrak{D}(E) \neq \text{const}$, which in turn requires that the temperature dependence of the noise magnitude deviate from $S_v \propto T$. This is consistent with experiments.[1] It remains to be shown

that a given $\mathfrak{D}(E)$ can explain both the small deviations of α from 1 and the strong dependence of S_v on T. To do so, we note that Eq. (3) implies a relationship between the frequency and temperature dependences of $S_v(\omega, T)$. In particular the exponent α, now defined locally as $\alpha \equiv -\partial \ln S_v / \partial \ln \omega$, can be written as

$$\alpha(\omega, T) = 1 - \frac{1}{\ln(\omega\tau_0)}\left[\frac{\partial \ln S_v(\omega, T)}{\partial \ln T} - 1\right]. \quad (4)$$

In Fig. 1, the known dependence of noise magnitude on temperature for 800-Å silver films has been used to predict α as a function of T via Eq. (4). We have assumed $\tau_0 = 10^{-14}$ sec, although the results are extremely insensitive to the exact value. We find that not only is the predicted α always within the accepted limits for generic $1/f$ noise, but that the trends in the data[1] are well reproduced. $\mathfrak{D}(E)$ for this system is plotted in Fig. 1(c). The distribution has a half-width of about 0.2 eV and peaks at $E_p \simeq 1$ eV. (Changing τ_0 by two orders of magnitude changes the energy scale by only about 15%.) Note that the noise magnitude peaks at $T_p \simeq 400$ K $\ll E_p/k_B$ because $-\ln(\omega\tau_0)$ is large. For the same reason, α is always close to 1.

As a further check of Eq. (4) we have examined the noise in 250- and 1400-Å Ag films (Fig. 2). In each case the temperature dependence of the noise is slightly different from that for the 800-Å film, and the implications of this for $\alpha(T)$ agree well with the trends actually observed. Further, the stronger temperature dependence observed in the thick film leads to a narrower $\mathfrak{D}(E)$, whereas the available data for the thin film suggest a broader $\mathfrak{D}(E)$ [Fig. 2(c)]. This leads to the physically plausible picture that sample inhomogeneity causes a broadening of the energy distribution. Indeed, the temperature dependence of the noise in Cu whiskers[5] is considerably stronger than in Cu films.

Apart from Ag, the temperature dependence of $S_v(\omega)$ has been measured in Cu, Au, and Ni[1] and in Bi.[6] In each case the magnitude varies strongly with temperature; and in the case of Cu and Bi, peaks are seen within the observed temperature range. Thus, segments of $\mathfrak{D}(E)$ may be calculated for these materials as well. In all cases $\alpha(T)$ shows the same general trends as for Ag, so that there is qualitative agreement with Eq. (4).

In writing Eq. (1) it has been implicitly assumed that the coupling constant and hence the integrated noise power are temperature independent. The ability to reproduce the experimental data with such a model suggests that the actual integrated noise power is at most weakly temperature dependent in the range $T < 500$ K. To estimate the

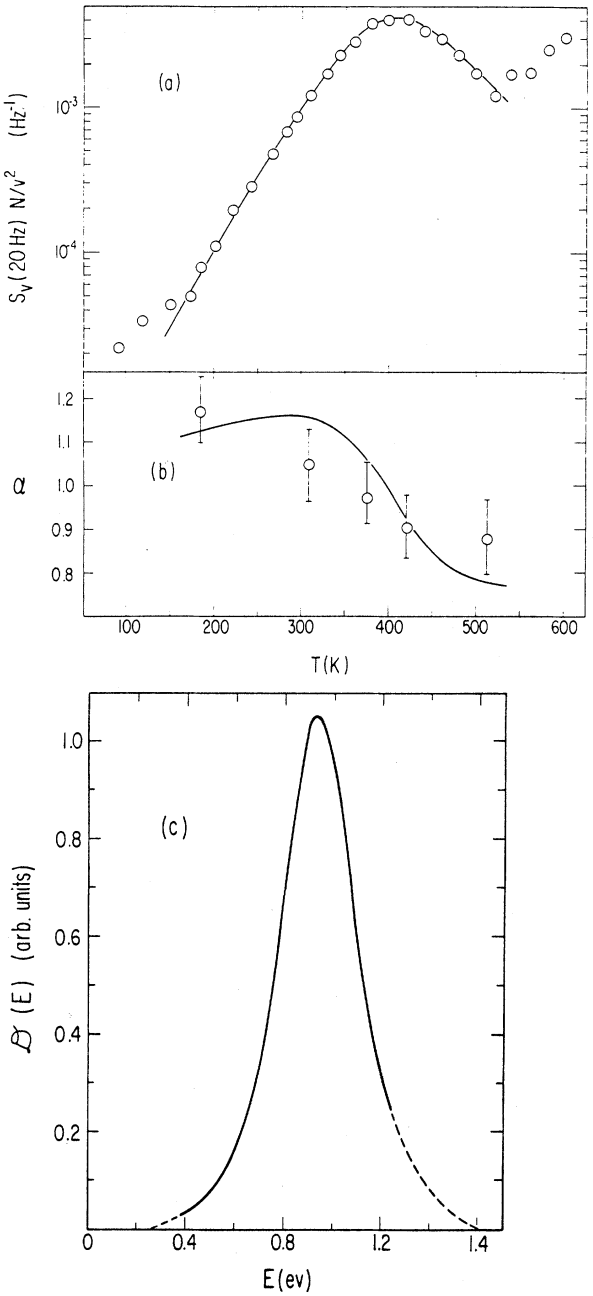

FIG. 1. (a) Noise magnitude at 20 Hz (corrected for dependence on voltage and sample volume) vs temperature for 800-Å Ag film (data from Ref. 1). The solid line is a smooth fit to the noise peak. (b) Frequency exponent vs temperature for 800-Å Ag film (data from Ref. 1). The solid line is the prediction of Eq. (4) using the curve in (a). (c) Energy distribution calculated from the curve in (a). Dashed lines are extrapolations to estimate the area under the curve.

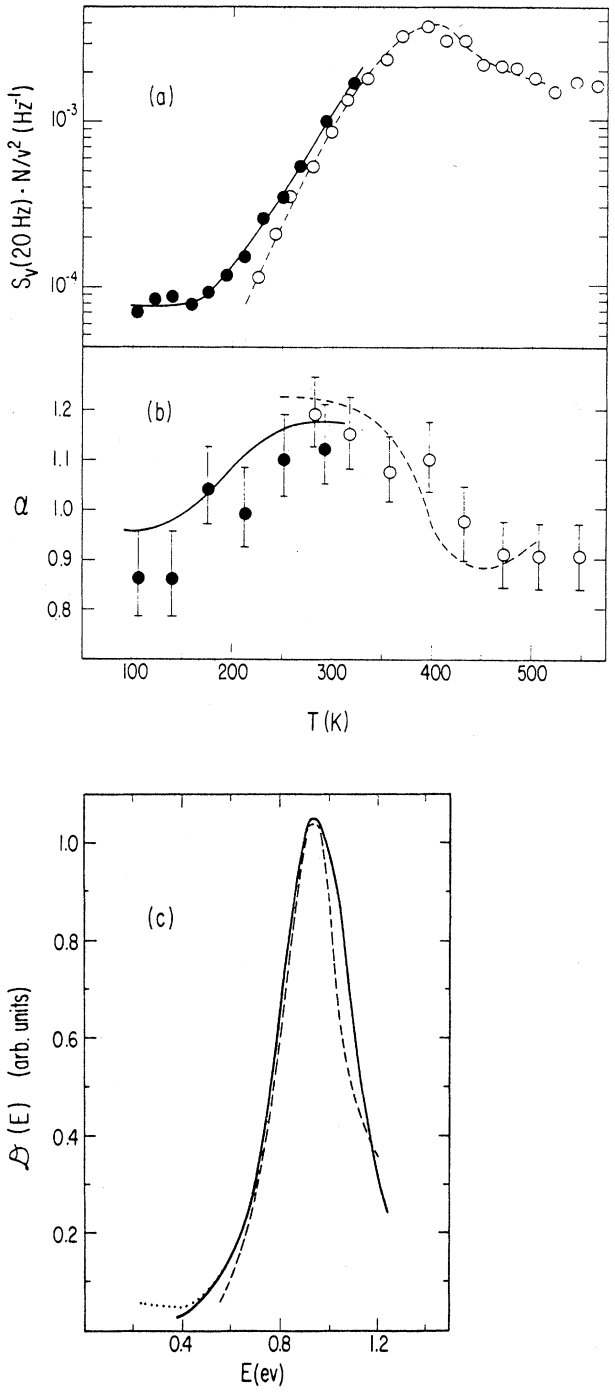

FIG. 2. (a) Normalized noise magnitude at 20 Hz vs temperature for (i) 250-Å Ag film (solid circles) and (ii) 1400-Å Ag film (open circles). Lines are smooth fits to data. (b) α vs T for (i) 250-Å Ag film (solid circles) and (ii) 1400-Å film (open circles). Lines are predictions from Eq. (4) using the corresponding curves in (a). (c) Energy distributions for (i) 800-Å film [solid line—same as Fig. 1(c)], (ii) 250-Å Ag film (dotted line), and (iii) 1400-Å Ag film (dashed line).

magnitude of the integrated noise we extrapolate $\mathcal{D}(E)$ to zero as shown by the dashed lines in Fig. 1(c). We find for Ag $\langle\delta V^2\rangle/V^2 \gtrsim 0.9/N$ and for Cu $\langle\delta V^2\rangle/V^2 \gtrsim 0.3/N$, where N is the number of atoms in the sample. These are approximate lower limits: In the case of Ag, for example, the rise in the noise with temperature above 500 K is not taken into account. The data may indicate either a second peak or the onset of a temperature-dependent coupling constant. However, the numbers do provide an order-of-magnitude consistency check for proposed origins of the noise.

It should be noted that the energy scales of electron volts that have now been associated with excess noise in metals are in a range that is typical of many processes in metallic crystals. Since the temperature T_p at which the noise peaks increases with the peak energy E_p, we have attempted to correlate T_p with known characteristic energies of the materials involved. At present, peaks in the noise magnitude versus temperature have been observed in continuous films of Bi,[6] Ag, and Cu.[1] In Au, no peak has been observed but the noise reaches a plateau at about 550 K.[1] In Fig. 3 we have plotted T_p for these materials versus the cohesive energy E_c. While the data are too limited for us to draw definite conclusions, the correlation of T_p with E_c may mean that the noise is associated with vacancy-interstitial formation and recombination. It is difficult to understand, however, why such a process

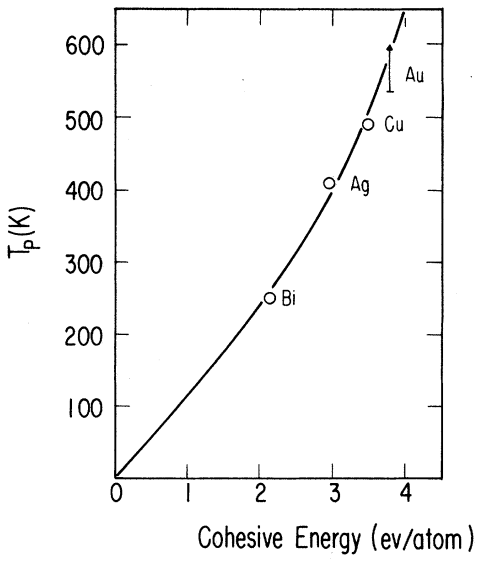

FIG. 3. Temperature T_p at which the noise peaks vs cohesive energy, for various metals. In the case of Au, only an approximate lower limit is known.

would have a constant integrated noise magnitude. At present other proposed origins for the noise in metals would be equally speculative. Also, different mechanisms for generating the noise may predominate in different temperature regions.[7,8]

In conclusion, we have shown that the existing noise data in metals can be transformed to obtain the distribution of activation energies for the process that generates the noise. We should stress that this transformation [Eq. (3)] is in no way a tautology; its validity can be investigated experimentally. There are many noise mechanisms for which Eq. (3) would fail. One obvious example is noise from fluctuations in a diffusing variable as in, for example, the temperature-fluctuation model in its simplest form.[9]

Finally, we note that the observed scale invariance of the noise spectrum is reproduced in our model as an artifact: For $\omega \ll 1/\tau_0$, $\tilde{E}$ is extremely insensitive to changes in ω. For example, the model predicts that, at fixed T, to reproduce the small changes in α seen in our temperature-dependence measurements would require going from our normal experimental frequency down over sixty decades in ω! Thus, low-frequency noise arising from an activated process will almost always appear to be of power-law [$\alpha(\omega) = \text{const}$] form.

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ERRATUM

ENERGY-MOMENTUM IN GENERAL RELATIVITY. Abhay Ashtekar and Anne Magnon-Ashtekar [Phys. Rev. Lett. 43, 181 (1979)].

In the first column on page 181, a phrase is missing. Line 20 should read "Bondi four-vector, while the assumed physical significance of the ADM four-vector has led to a long series of analyses concerning the sign of its time component."

Line 3 in the *Definition* on page 181 should read "if there exists a space-time, $(M, g_{ab}), \ldots$" [not $(\hat{M}, g_{ab})$]. In the first column on page 182, line 3 should read "q_{ab} is the metric induced by g_{ab}" and line 17 should read "The ADM four-momentum $\tilde{P}_a$."

The integral sign on the left-hand side of Eq. (3) should read $(\int_{\tilde{S}_K} - \int_{S_K})$ rather than $(\int_{\tilde{S}_K} \int_{S_K})$ and that of Eq. (5) should read $\int_{\tilde{S}_K}$ in place of $\int_{\tilde{S}_0}$.

In line 8, column 1 of page 183, read $\dot{J}(i^0) - i^0$ in place of $j(i^0) - i^0$.