observed in the coincidence spectra has to be associated with an integral fission probability of at least half the value of the fission probability of the underlying continuum. This value was deduced by taking a ratio of areas above and below the dashed lines in Fig. 4(a) for the singles and coincidence spectra. It should be kept in mind, however, that such a choice of background for the coincidence spectrum would mean that the ratio of the fission probabilities between the resonance structure and background would vary between 1.3 and 0 for the energy range 7.5–11.5 MeV, and be 0 for the remaining resonance region between 11.5 and 17.5 MeV, which is clearly unreasonable. We therefore feel that this simple choice of background for the coincidence spectrum represents an upper limit and, consequently, the quoted ratios of fission probabilities should be taken as lower limits, with the probable values being considerably above these limits. The division between resonance and compound-nucleus contributions is therefore likely to be lower than drawn in Fig. 4(a), and so would lead to a higher value than $\frac{1}{2}$ quoted above for the ratio between fission probabilities of the resonance and compound-nucleus components.

The fission probability varies strongly over the energy region of the resonance structure observed in the singles spectra. For excitation energies above about 13 MeV this energy dependence is mainly due to the occurrence of second- and third-chance fission. Additional structure might be caused by the contribution of several multipolarities[3,4] to the resonant structure and the splitting of the quadrupole resonance[8] into its $K=0$, 1, and 2 components. It is possible that different fission probabilities have to be associated with these various contributions. To pursue further this interesting question it will be necessary to compare directly the fission probabilities for a reaction condition in which the resonance is strongly excited with a reaction condition where it is only weakly excited, over an extended energy range.

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# Determination of the Asymptotic *D*- to *S*-State Ratio of the Deuteron Wave Function

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Cross sections and tensor analyzing powers in *dp* elastic scattering, measured at 35 and 45 MeV, are used to determine the asymptotic *D*- to *S*-state ratio of the deuteron wave function. The results agree with the value determined at lower energies and provide a considerably more precise averaged value: $\rho_D = 0.0263 \pm 0.0013$.

Recent reviews of the deuteron[1,2] have emphasized the importance of the determination of its *D*-state probability, $P_D$, and/or its asymptotic *D*- to *S*-state ratio, $\rho_D$. More precise knowledge of these basic deuteron properties would establish significant constraints on the nucleon-nucleon intermediate- and long-range tensor force and, thus, on the various potential models of the $NN$ interaction. Such constraints would also be important in calculations of the triton binding energy and in the determination of the saturation properties of nuclear matter, both of which are very sensitive to variations of $P_D$.[2] Even though $P_D$ is known only within a factor of 2,[1] the prospects for a significant improvement in its determination are poor because of the model dependence among the various techniques used to deduce $P_D$ from experimental data.[2] In contrast, it has been shown recently that the asymptotic ratio $\rho_D$ can be obtained in a clear model-independent fashion from measurements of the tensor analyzing-power component $T_{22}(\theta)$ in $dp$ scattering[3]; and the method has been used to determine the value $\rho_D = 0.027 \pm 0.005$ from $dp$ data at $E_p = 10$ MeV.[4] We report here on $dp$ measurements of the cross section $\sigma(\theta)$ and $T_{22}(\theta)$ at $E_d = 35$ and 45.2 MeV, from which we extract independently determined values of $\rho_D$. The consistency among our two values and that of Ref. 4 is extremely good, and the resulting averaged value of $\rho_D$ has a substantially reduced error.

The original prescription[3] for the determination of $\rho_D$ was to fit the experimental angular distribution $T_{22}(\theta)/\sin^2\theta$ with a Legendre polynomial expansion in $z = \cos\theta$, and to extrapolate this function to the neutron exchange pole at

$$z_p = -\left[\tfrac{5}{4} + \tfrac{9}{4}(B/E_d)\right], \qquad (1)$$

with $B$ the deuteron binding energy and $E_d$ the deuteron laboratory kinetic energy. The extrapolated value at $z_p$ is then directly proportional to $\rho_D$. Soon thereafter, Grüebler *et al.*[5] found that their 20-MeV measurements of $T_{22}(\theta)/\sin^2\theta$ exhibited a rather complex angular dependence, and the higher-order polynomials required to fit these data led to unstable extrapolated values at the pole position. Subsequently, Amado *et al.*[4] showed that a more suitable quantity for extrapolation is given by

$$f(z) = k^2\sigma(z)T_{22}(z)(z - z_p)^2/(1 - z^2), \qquad (2)$$

where $k$ is the $dp$ c.m. wave number. The construction of $f(z)$ is quite clear. The quantity $\sigma(z)T_{22}(z)$ is the difference between two spin-dependent cross sections, which has a second-order pole at $z = z_p$ that is removed from $f(z)$ by the factor $(z - z_p)^2$. The zeros of $T_{22}(z)$ at $z = \pm 1$ are canceled by the factor $(1 - z^2)$. Extrapolation of $f(z)$ to the pole then gives directly the value

$$\rho_D = -0.0542 f(z_p). \qquad (3)$$

Independent determinations of $\rho_D$ at higher energies $E_d$ are essential for the following reasons:

(i) The value of $\rho_D$, thus $f(z_p)$, must be independent of $E_d$. There is always the question in pole extrapolations of the "background" contribution from other singularities.[6] Thus, changes in the relative positions of $z_p$ and other singularities with respect to the physical region, produced by variation of $E_d$, can provide an important experimental check on the background contribution. As seen in (1), the pole moves closer to the physical region as $E_d$ increases.

(ii) The proportionality constant between $\rho_D$ and $f(z_p)$ in (3) was obtained[4] from expressions for $nd$ scattering since, in leading order, $f(z)$ is Coulomb independent. Any remaining exchange-pole Coulomb distortion effects will be reduced at the higher energies.

Measurements were made of both $\sigma(\theta)$ and $T_{22}(\theta)$ in $dp$ scattering at $E_d = 35$ MeV, while $T_{22}(\theta)$ alone was measured at $E_d = 45.2$ MeV. Cross-section values at the higher energy were obtained from quadratic least-squares interpolations of data in the literature.[7] The absolute normalization of our 35-MeV $\sigma(\theta)$ distribution was made in the same way, and an additional check was made through a comparison of optical-model calculations with measured forward-angle cross sections in $d$ + Xe elastic scattering. Absolute $\sigma(\theta)$ normalization errors of 2.3% and 3.0% were assigned at 35 and 45 MeV, respectively. The absolute normalization errors in $T_{22}(\theta)$ were 2.9% and 3.4%, respectively.[8] The $f(z)$ values are plotted in Fig. 1, where the relative errors shown are essentially statistical.

In order to extrapolate the measured distribution $f(z)$ and to determine the quantity $f(z_p)$, a least-squares Legendre polynomial fit was made. The fit procedure used the method of multiple linear regression as described by Bevington.[9] The data points $(z_i, f_i)$ were fitted by use of a Legendre polynomial series of order $L$: $f(z) = \sum_{l=0}^{L} a_l P_l(z)$. The fit program was modified in order to obtain the covariances as well as the variances of the fit parameters $a_l$. A correct error assignment to the extrapolated value $f(z_p)$ can be made only if the covariances (correlation between parameters) as well as the variances are included.

A crucial point of the analysis is the maximum order $L$ of the Legendre polynomials to be used

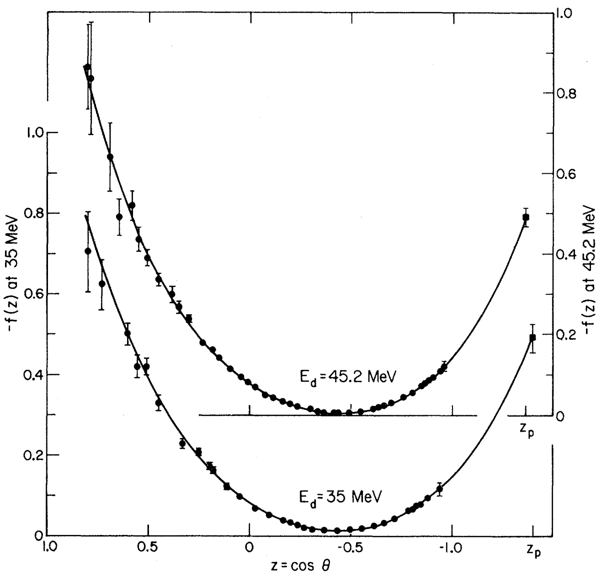


FIG. 1. The quantity $-f(z)$, Eq. (2), in $dp$ elastic scattering is plotted vs $z=\cos\theta_{\text{c.m.}}$ at $E_d=35$ and 45.2 MeV. The relative errors are shown except where they are smaller than the points. The curves are fourth-order Legendre polynomial series fits to the data. Extrapolated values, $-f(z_p)$, are also shown.

in the fitting function, because the error in the extrapolated value increases with $L$. We found that $L=4$ was necessary and sufficient for satisfactory fits at both 35 and 45 MeV. Details of the fit information are contained in Table I. From the $\chi^2$ per degree of freedom, $\chi^2/\nu$, $L=4$ is clearly required. Ciulli, Pompaniu, and Sabba-Stefanescu[10] have established a clear criterion for the determination of the number of terms to be used in the expansion of the data function in order to perform a stable polynomial extrapolation. Following this procedure, the data points were varied slightly within their error corridor, and the number of terms was determined "for which the corresponding extrapolations diverged explosively from each other."[10] This clearly confirmed $L=4$ to be the critical term at which to stop the expansion. The $L=4$ fits and the extrapolated values $f(z_p)$ are also shown in Fig. 1.

With use of (3), our two independently determined values of the asymptotic ratio $\rho_D$ are also given in Table I. The listed errors are those combined from the error in $f(z_p)$ and the absolute normalization errors in $\sigma(\theta)$ and $T_{22}(\theta)$ given above. The weighted average of these two values

TABLE I. Results from fits with $f(z)=\sum_{l=0}^{L} a_l P_l(z)$.

| $E_d$ | $L$ | $\chi^2/\nu$ | $a_L$ | $f(z_p)$ | $\rho_D$ |
|---|---|---|---|---|---|
| 35 | 3 | 1.36 | 0.010(5)[a] | −0.355(13) | |
| | 4 | 0.90 | 0.025(7) | −0.480(36) | 0.0260(21) |
| | 5 | 0.90 | −0.009(9) | −0.572(104) | |
| 45.2 | 3 | 2.01 | 0.006(4) | −0.326(9) | |
| | 4 | 0.72 | 0.041(5) | −0.490(24) | 0.0265(18) |
| | 5 | 0.71 | −0.007(7) | −0.548(62) | |

[a] $0.010(5)=0.010\pm0.005$.

and that of Ref. 4 is

$$\rho_D=0.0263\pm0.0013. \quad (4)$$

Theoretical estimates of $\rho_D$ are also available. Values calculated from various phenomenological-potential models range from 0.025 to 0.0285 (Ref. 1). However, the value (4) now weighs against the old phenomenological Hamada-Johnston–potential value of 0.0285. A model-independent dispersion-relation calculation, which used only one-pion exchange and the $np$ effective-range parameters, gave the value 0.029 with no estimate of its uncertainty.[11]

A different, less direct, method has been used by Knutson and Haeberli[12] to obtain information about $\rho_D$ from measurements of the tensor analyzing powers in the $^{208}$Pb$(d,p)^{209}$Pb reaction at 9 MeV. In their distorted-wave Born approximation (DWBA) calculations, these analyzing powers scale with the deuteron parameter $D_2\simeq\rho_D/\alpha^2$, where $\alpha^2=MB/\hbar$, with $M$ the nucleon mass. Their deduced value of $D_2$ gives $\rho_D=0.0232\pm0.0017$. While uncertainties in the nuclear distorting potentials are included in their error estimate, there may remain some systematic discrepancy from the approximate way[13] in which the $D$ state was included in the DWBA calculations.

In view of the experimental accuracy stated in (4), other sources of error must be considered. The first concerns the effect of background contributions from singularities other than the neutron exchange pole. We cite the remarkable stability of the $\rho_D$ values, derived at energies between 10 and 45 MeV, as substantial experimental evidence against background contributions at the level of more than a few percent. That is, the strongest background contribution to the scattering amplitude in the physical region of momentum transfer comes from the triangular-graph (two nucleons in the intermediate state) branch cut which starts at the branch point $z_a = 1 + 18B/E_d$.[6,14] Then, for $E_d = 10$ and 45 MeV, the corresponding singularity positions $(z_a, z_p)$ are (5.00, − 1.75) and (1.89, − 1.36), respectively. Thus, the relative contributions of the exchange-pole and branch-cut amplitudes will certainly change over this energy interval. The stability of $\rho_D$ then argues well against any significant background contribution from the branch-cut singularity. It would seem, at first, that this conclusion is at odds with the finding of an intrinsic error of 10% in *nd* cross-section extrapolations in momentum transfer, due essentially to just this branch-cut contribution.[6] There, however, the extrapolated quantity was $F(z) = \sigma(z)(z - z_p)^2$. We remark again that the factor $\sigma(z)T_{22}(z)$ in (2) is the *difference* between two spin-dependent cross sections. If the background contributions to them are less spin dependent than the pole contributions, some cancellation could result. This would make $f(z_p)$ more free of these background effects, in agreement with our experimental findings. This argument, however, requires quantitative theoretical verification.

A second source of error, that of the exchange-pole Coulomb distortion, has been evaluated by Amado and Locher.[15] The Coulomb distortion leads to an exchange-channel branch cut which starts at the exchange pole.[14] The contribution from this cut to the calculated *dp* cross section (near $z = -1$) is 3.5% at $E_d = 45$ MeV, 4.5% at 35 MeV, and 8% at 20 MeV. Since the leading contribution to $T_{22}(z)$ comes from *S-D* interference,[3] a considerably smaller correction to $f(z)$ is estimated. Hence, for our extrapolated value accuracies of 5% to 7%, this Coulomb correction at 35 and 45 MeV can safely be neglected.

Certainly the experimental error on $\rho_D$ can be further reduced simply by making repeated independent determinations of its value. This should be done at different energies $E_d$, since the stability of $\rho_D$ as a function of energy provides an important test of the reliability of the pole-extrapolation method. However, in view of the level of accuracy already achieved, a more sophisticated analysis, which includes conformal mapping and the removal of next-nearest singularities, should be made.[3]

The value (4) for $\rho_D$ can now be used in any particular potential model to specify the $D$-state probability $P_D$. That is, with the intermediate- and long-range parts of the $D$-state wave function constrained by the deuteron quadrupole moment and by $\rho_D$, the calculated value of $P_D$ will be similarly constrained. Of course, there will still be variations because of the model dependence of the short-range part of the wave function.

In terms of the modern meson-exchange potentials, our $\rho_D$ valve establishes a constraint on the strength of the long-range part of the tensor force.[16] For example, this component is too strong in the Paris potential,[17] which gives the value $\rho_D = 0.0293$. Meson theory, now including $\rho$-meson exchange, suggests the value $\rho_D \leq 0.026$,[16] in good agreement with our experimental value.

In summary, we have determined the deuteron $D$- to $S$-state asymptotic ratio to an accuracy of ± 5%, and this value provides an important constraint on the *NN* intermediate- and long-range tensor force.

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# Direct-Reaction Calculation of Intermediate-Energy Proton Radiative Capture on $^{11}$B

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Photon spectra of the reaction $^{11}\mathrm{B}(p,\gamma)^{12}\mathrm{C}$ at $E_p = 40$, 60, and 80 MeV are calculated assuming a direct-capture mechanism. Observed capture spectra below 25-MeV excitation energy are reproduced well. Only about half of the observed cross section at the 19.2-MeV peak is found to come from the $4^-$ stretched configurations. Contributions of "charge-exchange currents" are shown to be important.

Kovash *et al.*[1] have recently measured photon yields for the $(p,\gamma)$ reaction on several light nuclear targets. The spectra obtained were for incident proton energies $E_p$ from 40 to 100 MeV. In Fig. 1, we show the observed excitation function for $^{11}\mathrm{B}(p,\gamma)^{12}\mathrm{C}$ at a scattering angle of 60°. At the lower proton energies, peaks are clearly evident in the low-excitation-energy end of the spectrum. These are easily identifiable as corresponding to the ground- and first excited-state transitions. However, an unexpected feature also shows up at the higher-excitation-energy end of the spectrum where we see a distinct peak clearly emergent on top of the rising continuum at an excitation energy of about 19.2 MeV in $^{12}$C. For 80-MeV protons, both the low-lying and high-lying peaks are harder to identify as the resolution width of the photon detector, which is roughly proportional to $E_\gamma$, is now comparable with nuclear level spacings. Since this is the first observation of such a high-lying state in radiative capture measurement, it is of some interest to see if it can be accounted for with presently available models of the radiative capture process.

It was speculated[1,2] that the stretched $4^-$ states[3] in $^{12}$C could be responsible for the observed 19.2-MeV peak, but no calculations were done to verify this point. Here we apply a model,[4] which gives a good quantitative description of the inverse photonuclear process at intermediate energies, to calculate the reaction $^{11}\mathrm{B}(p,\gamma)^{12}\mathrm{C}$. We are interested to see if this model can reproduce the observed features of the photon spectrum, such as the transition strength to the ground and first excited state, the rising trend of the photon spectrum with increasing nuclear excitation energy, and the change in the photon spectrum as the incident proton energy is varied. In particular, we want to investigate whether it is possible to have any localization of strength in the region of high excitation energy. The effect of the mesonic exchange currents will also be examined.

 