(at the cost of infinite energy needed to make $|\vec{H}| = |\vec{E}|$). For the isolated charge, $\vec{\mathcal{E}} \sim g\hat{q}/r^2$ as before, but $\vec{E}$, which is responsible for forces, is proportional to r, while the energy density is like $1/r$, giving divergence at $r=\infty$ but not at $r=0$. I can see little hope of successfully quantizing such a theory, but perhaps that should not be allowed to prevent further exploration.

The quantization of the theory governed by Eq. (14) looks superficially less formidable, since the Lagrangian is conventionally quadratic in the large-field limit, and hence at small distances where ultraviolet divergences occur. However, nonpolynomial Lagrangians are notoriously awkward to deal with and the problems may be insurmountable. The attempt will be made using techniques developed by Wilson and myself[4] in a different context. Whether infrared problems are improved or worsened by the proposed modification remains to be seen.

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[1]H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973); S. Weinberg, Phys. Rev. Lett. 31, 494 (1973), and Phys. Rev. D 8, 4482 (1973); D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973).

[2]G. 't Hooft, Nucl. Phys. B33, 173 (1971); B. W. Lee and J. Zinn-Justin, Phys. Rev. D 5, 3121, 3137 (1972), and D 7, 1049 (1973).

[3]H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); G. 't Hooft, unpublished.

[4]See D. E. Wilson, "Nonanalytic Interactions in Scalar Quantum Field Theory," Ph.D. thesis, 1972, The Ohio State University (unpublished).

Determining Meson Radiative Widths from Primakoff-Effect Measurements

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We suggest that the measurement of vector-meson radiation decays $V\to P\gamma$ in the Primakoff-effect experiments on nuclei should be reanalyzed including isovector hadronic exchange. Its inclusion invalidates the assumption, made in data analyses, of A independence of the strength of the strong-production amplitude and could well remove the disagreement between theory and experiment for $\Gamma(\rho\to\pi\gamma)$ and $\Gamma(K^{*0}\to K^0\gamma)$.

The outstanding problem in the radiative decays of the generic kind V (vector)$\to P$ (pseudoscalar) $+\gamma$ and $P\to V+\gamma$ has been to understand the measured rates (1) $\Gamma(\rho\to\pi\gamma)=35\pm10$ keV (Refs. 1 and 2) compared to the nonet-symmetry (or naive quark-model) expectation of $\simeq 90$ keV, and (2) $\Gamma(K^{*0}\to K^0\gamma)=75\pm30$ keV (Ref. 3) compared to the nonet-symmetry value of $\simeq 210$ keV. Considerable theoretical effort[4-6] has been made in attempts to understand these anomalously low rates in broken-symmetry schemes. One may say in summary that it is not difficult to fit $\Gamma(K^{*0}\to K^0\gamma)$ in a broken-symmetry scheme but it is not possible to understand the low value of $\Gamma(\rho\to\pi\gamma)$ simultaneously with the measurement[7] of $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)$, which proves to be a strong constraint.[5] The best one can do in the schemes of Edwards and Kamal[3] is to obtain $\Gamma(\rho\to\pi\gamma)\simeq 70$ keV.

The purpose of this Letter is to propose a mechanism which, when incorporated in the data analysis, could raise $\Gamma(\rho\to\pi\gamma)$ [and $\Gamma(K^{*0}\to K^0\gamma)$] to higher values consistent with the quark-model expectations.

Both $\Gamma(\rho\to\pi\gamma)$ and $\Gamma(K^{*0}\to K^0\gamma)$ have been measured in Primakoff-effect[8] experiments on various nuclei at Brookhaven National Laboratory with a pion beam of momentum 22.7 GeV/c and a $\bar{K}^0$ beam of momentum 8 to 16 GeV/c, respectively. At these momenta the coherent Coulomb production in $P+(A,Z)\to V(A,Z)$ interferes with the coherent strong production. The experiment measures $d\sigma/dt'$ for the coherent V production. The Coulomb-production amplitude[9-11] is built up of the coherent contribution from the Z protons. In the data analyses[1-3] the strong-production amplitude has been assumed to be generated by ω exchange. This isoscalar natural-parity (1^-) exchange gives an amplitude with the same Lorentz structure as that from the Coulomb production, and A times the elementary amplitude $P+n$ (or p) $\rightarrow V+n$ (or p). The strong-production amplitude with ω exchange has a form[2,9,10]

$$F_{\text{nucl}} = A(\vec{\mathrm{h}}\cdot\vec{\mathrm{q}}_\perp/q_\perp)F_{\text{strong}}. \qquad (1)$$

F_{strong} can be found in Refs. 2, 9, and 10; $-t'=q_\perp{}^2$ and $\vec{\mathrm{h}}$ is proportional to $C_0^{1/2}(\vec{\epsilon}\times\vec{\mathrm{k}})$, where $\vec{\epsilon}$ is the V polarization vector, $\vec{\mathrm{k}}$ is the incident momentum in the laboratory system, and $C_0^{1/2}$ measures the strength of the elementary production amplitude on a nucleon. The normalization of $\vec{\mathrm{h}}$, which is of no consequence to us, is so chosen that in the absence of any nuclear or Coulomb absorption one has[2] $d\sigma/dt'=C_0A^2q_\perp{}^2$.

In the data analyses,[1,3] $d\sigma/dt'\propto|F_{\text{Coul}}+e^{i\varphi}F_{\text{nucl}}|^2$ was fitted by varying C_0 and the phase φ to get the best fit to the data. *An essential criterion for the goodness of the fit was that C_0 should not depend on the nucleus.*[1] Gobbi *et al.*,[1] however, do have a solution for $\Gamma(\rho\rightarrow\pi\gamma)$ with the rate varying from a low of 57 ± 6 keV for Ag to a high of 77 ± 5 keV for U with $\varphi=90°$ and C_0 decreasing monotonically from 3.4 ± 0.2 mb/GeV^4 for the lightest element (Cu) to 1.8 ± 0.3 mb/GeV^4 for the heaviest (U).

If ω exchange were the only natural-parity exchange then the above criterion of constancy of C_0 with A would be appropriate. However, a variation of C_0 with A would be expected if an isovector natural-parity exchange were contributing. A candidate is A_2 exchange with $I^G=1^-$ and $J^P=2^+$. It produces an amplitude with the same Lorentz structure as the ω exchange, but because it is an isovector the amplitude is proportional to $Z-N$ rather than A, N being the neutron number. For lighter nuclei this effect will be small but it will grow in importance with A. If the A_2-exchange amplitude interferes with the ω-exchange amplitude one should expect an effect on C_0 of the form

$$C_0(N,Z)=C_0(N=Z)|1+\delta(Z-N)/A|^2, \qquad (2)$$

where δ measures the amount of interference. For $\delta\simeq1$ the departure from unity can be quite large. The correction factor is 0.84 for Cu, 0.76 for Ag, 0.62 for Pb, and 0.60 for U. This variation of C_0 is in the same direction and of the same size as that needed by Gobbi *et al.*[1] to make their solution for $\Gamma(\rho\rightarrow\pi\gamma)$ acceptable.

The physics is somewhat more involved. The importance of the A_2 exchange relative to the ω exchange depends on their relative phases and sizes. For exotic reactions ($K^+p\rightarrow K^{*+}p$, $\pi^+p\rightarrow\rho^+p$) one expects approximate ω-A_2 exchange degeneracy to give largely a real amplitude and a null relative phase between the ω and A_2 amplitudes for production on a nucleus. For nonexotic reactions ($\bar{K}^0p\rightarrow\bar{K}^{*0}p$, $\pi^-p\rightarrow\rho^-p$) the amplitudes are rotating and have significant real and imaginary parts (see, for example, Fig. 12 of Ref. 13 for amplitudes calculated with absorption effects included). The actual phase varies from reaction to reaction.

The sizes are determined mainly by the coupling strengths. The ω amplitude is proportional to $g_{\omega\rho\pi}g_{\omega NN}$ where $g_{\omega NN}$ is the vector-coupling constant. Similarly, the A_2 contribution is proportional to $4m_Ng_{A_2\rho\pi}g_{A_2NN}$ ($4m_N$ coming from conventions). The meson coupling $g_{\omega\rho\pi}$ calculated[12] from $\Gamma(\omega\rightarrow\pi\gamma)$ together with vector-meson dominance is $g_{\omega\rho\pi}\simeq16$ GeV^{-1}. $g_{A_2\rho\pi}$ is calculated to be $\simeq10$ GeV^{-2} from $\Gamma(A_2\rightarrow\rho\pi)$.[12] The coupling $g_{\omega NN}$ is accurately measured by the C-odd contribution to the NN total cross section, while g_{A_2NN} is well determined by the value of $d\sigma(\pi^-p\rightarrow\eta n)/dt$ in the forward direction, where only the nonflip amplitude contributes. From the detailed analysis by Kane and Seidl[13] we find $g_{\omega NN}\simeq12$ and $g_{A_2NN}\simeq7$. Thus the ratio of the *sizes* of the two contributions is of order

$$A_2/\omega\simeq280/192\simeq1.5 \qquad (3)$$

for the basic reaction on a nucleon.

In practice, absorption corrections will alter the individual phases and magnitudes and an appropriate general form for $d\sigma/dt'$ in $P+(A,Z)\rightarrow V+(A,Z)$ is

$$d\sigma/dt'\propto|F_{\text{Coul}}+e^{i\varphi}AF_\omega+e^{i\delta}(N-Z)F_{A_2}|^2. \qquad (4)$$

Clearly the advantages one gains by going to the heavier nuclei are offset by the theoretical uncertainties. Simple analysis can be done for $N\approx Z$ nuclei only.

The remarks on the importance of the A_2 exchange for heavier nuclei will also apply to the measurement[3] of $\Gamma(K^{*0}\rightarrow K^0\gamma)$.

In summary, the data analyses for $\Gamma(\rho\rightarrow\pi\gamma)$ (Refs. 1 and 2) and $\Gamma(K^{*0}\rightarrow K^0\gamma)$ (Ref. 3) are suspect for heavier nuclei insofar as one cannot assume the constancy of the strength of the strong production amplitude with A. Reliable analysis can be done only for the light nuclei with $N\approx Z$. Indeed, for Cu, Gobbi *et al.* have $\Gamma(\rho\rightarrow\pi\gamma)=66\pm8$ keV for $\varphi=90°$. At Fermilab energies (150 to 200 GeV/c) one will get a cleaner separation of the Coulomb and strong production peaks with little interference and our remarks will not apply. However, the disadvantage of going to higher energies is that the Coulomb peak shifts closer to the zero of t' and the data analysis will have accompanying uncertainties.

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[1]B. Gobbi *et al.*, Phys. Rev. Lett. 33, 1450 (1974), and 37, 1439 (1976).

[2]L. Strawczynski, Ph.D. thesis, Rochester University, 1974 (unpublished).

[3]W. C. Carithers *et al.*, Phys. Rev. Lett. 35, 349 (1975).

[4]B. J. Edwards and A. N. Kamal, Phys. Rev. Lett. 36, 241 (1976), and 39, 66 (1977), and Ann. Phys. (N.Y.) 102, 252 (1976), and Phys. Rev. D 15, 2019 (1977).

[5]A. N. Kamal, Phys. Rev. D 18, 3512 (1978).

[6]P. J. O'Donnell, Phys. Rev. Lett. 36, 177 (1976), and Can. J. Phys. 55, 1301 (1977); D. H. Boal, R. H. Graham, and J. W. Moffatt, Phys. Rev. Lett. 36, 714 (1976); N. Isgur, Phys. Rev. Lett. 36, 1262 (1976); A. Bohm and R. B. Teese, Phys. Rev. Lett. 38, 629 (1977), and Phys. Rev. D 18 , 330 (1978).

[7]C. J. Zanfino *et al.*, Phys. Rev. Lett. 38, 930 (1977).

[8]H. Primakoff, Phys. Rev. 81, 899 (1951); A. Halprin, C. M. Anderson, and H. Primakoff, Phys. Rev. 152, 1295 (1966); V. Glaser and R. A. Ferrell, Phys. Rev. 121, 886 (1961); C. Chiuderi and G. Morpurgo, Nuovo Cimento 19, 497 (1961); G. Morpurgo, Nuovo Cimento 31, 569 (1964); S. M. Berman and S. D. Drell, Phys. Rev. 133, 791 (1964); L. Stodolsky, Phys. Rev. 144, 1145 (1966).

[9]C. Bemporad *et al.*, Nucl. Phys. B51, 1 (1973).

[10]G. Fäldt, Nucl. Phys. B43, 591 (1972).

[11]G. Fäldt, D. Julius, H. Pilkuhn, and A. Müllensiefen, Nucl. Phys. B41, 125 (1972).

[12]Particle Data Group, Phys. Lett. 75B, 1 (1978).

[13]G. L. Kane and A. Seidl, Rev. Mod. Phys. 48, 309 (1976).

Confinement and the Critical Dimensionality of Space-Time

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Using Monte Carlo techniques, we study pure SU(2) gauge fields in four and five space-time dimensions and a compact SO(2) gauge field in four dimensions. Ultraviolet divergences are regulated with Wilson's lattice prescription. Both SU(2) in five dimensions and SO(2) in four dimensions show clear phase transitions between the confining regime at strong coupling and a spin-wave phase at weak coupling. No phase change is seen for the four-dimensional SU(2) theory.

The standard theory of hadronic interactions is based on quarks interacting with non-Abelian gauge fields. The viability of this picture depends on the conjectured phenomenon of confinement, wherein the only physically observable particles are invariant under the gauge group. Thus far, the only demonstration of this property is in the strong-coupling limit and with a space-time lattice regulating ultraviolet divergences.[1] Approximate renormalization-group arguments[2] suggest that four space-time dimensions represent a criical case where confinement persists for all couplings when the gauge group is non-Abelian. In contrast, Abelian groups should exhibit a phase transition to a nonconfining weak-coupling phase containing massless gauge bosons. Thus arises the conjecture that in our four-dimensional (4D) world, the lattice formulation of electrodynamics can avoid confinement of electrons, while the continuum limit of the strong-interaction gauge theory can exhibit asymptotic freedom, a vanishing coupling at short distances.

Recent Monte Carlo results have given mixed support for these arguments. For the four-dimensional gauge-invariant Ising model, the observed transition is first order, contrary to the approximate renormalization-group prediction of a second-order transition analogous to that in the conventional two-dimensional Ising model.[3] However, for Z_n with $n \geqslant 5$ and SO(2) symmetries, the predicted similarities between the four-dimensional gauge models and the two-dimensional