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Exclusive Processes in Quantum Chromodynamics: The Form Factors of Baryons at Large Momentum Transfer

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The form factors of baryons at large momentum transfer are computed in quantum chromodynamics (QCD) to leading order in $\alpha_s(Q^2)$ and m^2/Q^2. Asymptotically, we predict $Q^4G_M{}^p(Q^2) \rightarrow C[\alpha_s(Q^2)]^{2+4/3\beta}$ and $G_M{}^n/G_M{}^p \rightarrow -\frac{2}{3}$, where $\beta = 11-(\frac{2}{3})n_{\rm flavor}$ and $C>0$. Form factors for processes in which the baryon helicity is changed or in which the initial or final baryon has helicity greater than 1 are suppressed by factors of m/Q. We also give QCD predictions for general exclusive scattering processes at large momentum transfer.

In this Letter, we present a new analysis of exclusive processes involving baryons produced at large transverse momentum. This analysis is an extension of our earlier work on meson form factors in quantum chromodynamics (QCD).[1,2] Here we will describe QCD predictions for the electromagnetic form factors of baryons, for ratios of form factors, and for transition form factors (e.g., $\gamma^*p\rightarrow\Delta$), all at large Q^2. We will also outline the analysis of other large-momentum-transfer exclusive processes in QCD.

The analysis of baryon form factors in QCD is, in essence, identical to that for mesons.[1] Leading terms (in $1/Q^2$) involve only the three-quark component of the baryon's wave function (in light-cone gauge, $A^+=0$). When the leading logarithms in each order of perturbation theory [i.e., $(\alpha_s \ln Q^2)^n$] are summed, the form factor has the form $(-q^2\equiv Q^2)$

$$F_B(Q^2)=\int_0^1[dx_i]\int_0^1[dy_i]\,\varphi^\dagger(x_i,Q)\,T_B(x_i,y_i,Q)\,\varphi(y_i,Q). \tag{1}$$

Here

$$T_B=[C_B\alpha_s(Q^2)/Q^2]^2 f(x_i,y_i),$$

where

$$C_B=(n_{\rm color}+1)/2n_{\rm color}=\tfrac{2}{3},$$

$$\alpha_s=4\pi/\beta\ln(Q^2/\Lambda^2),\quad \beta=11-(2/3)n_{\rm flavor},$$

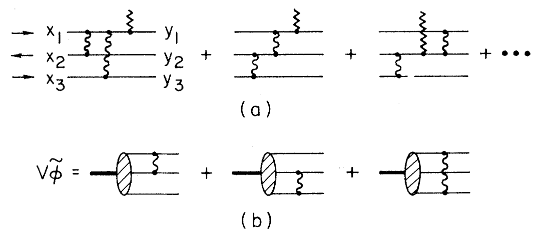

FIG. 1. (a) Diagrams constituting T_B for baryon form factors. The arrows indicate the quark helicity. (b) The one-gluon interaction in Eq. (3).

is the minimally connected amplitude for $\gamma^* 3q \to 3q$ [Fig. 1(a)],[3] and the symbol for symmetric integration over the constituents' longitudinal momenta $[x_i \equiv (k^0+k^3)_i/(p_B{}^0+p_B{}^3);\ \sum_{i=1}^3 x_i = 1]$ is

$$[dx_i] \equiv dx_1\, dx_2\, dx_3\, \delta(1-\textstyle\sum_i x_i).$$

The effective wave function $\varphi(x_i,Q)$ is the three-body qqq Fock-state wave function integrated over transverse momenta $|k_\perp{}^{(i)}|^2 < Q^2 [C_F = (n_c{}^2-1)/2n_c = \frac{4}{3}]$:

$$\varphi(x_i,Q) = \left(\ln\frac{Q^2}{\Lambda^2}\right)^{-(3/2)C_F/\beta} \int_0^Q \prod_{i=1}^{3}\left[\frac{d^2k_\perp{}^{(i)}}{16\pi^3}\right] 16\pi^3 \delta^2(\textstyle\sum_i k_\perp{}^{(i)})\, \psi(x_i, k_\perp{}^{(i)}) \equiv x_1x_2x_3\tilde{\varphi}(x_i,Q). \tag{2}$$

Only baryon states with $L_Z=0$ contribute to the leading power. The factor $(\ln Q^2)^{-(3/2)C_F/\beta}$ is due to vertex and fermion self-energy corrections in T_B which are more conveniently associated with φ rather than T_B. As in the meson case, the leading behavior of φ for large Q^2 is determined ($A^+=0$ gauge) by planar ladder diagrams with the transverse momenta in successive loops strongly ordered, $\lambda^2 \ll (k_\perp{}^1)^2 \ll (k_\perp{}^2)^2 \ll \ldots \ll Q^2$. Three- and four-gluon couplings play no role in this order (other than in standard vertex renormalization) since they destroy the strong ordering. Consequently, defining $\xi = \ln\ln(Q^2/\Lambda^2)$, we can derive an evolution equation for $\tilde{\varphi}(x_i,Q)$ [relating it to $\tilde{\varphi}(x_i,\lambda)$ for some $\lambda < Q$]:

$$x_1x_2x_3\{\partial\tilde{\varphi}(x_i,Q)/\partial\xi + \tfrac{3}{2}(C_F/\beta)\tilde{\varphi}(x_i,Q)\} = \int_0^1 [dy_i]\, V(x_i,y_i)\tilde{\varphi}(y_i,Q), \tag{3}$$

where

$$V(x_i,y_i) = x_1x_2x_3 \frac{2C_B}{\beta}\sum_{i\neq j}\theta(x_i-y_i)\delta(x_k-y_k)\frac{y_i}{x_i}\left(\frac{\delta_{h_i\bar{h}_j}}{x_i+x_j} - \frac{\Delta}{y_i-x_i}\right)$$
$$= V(y_i,x_i)\,[\Delta\tilde{\varphi} \equiv \tilde{\varphi}(y_i,Q) - \tilde{\varphi}(x_i,Q)],$$

is the interaction between each pair of quarks due to exchange of a single gluon [Fig. 1(b)]. The Kronecker delta $\delta_{h_i\bar{h}_j}$ is 1 (0) when quark helicities are antiparallel (parallel). As in the meson case, the infrared singularity at $y_i = x_i$ is cancelled because the baryon is a color singlet. (In detail, the cancellation is due to self-energy corrections on the external quark legs.)

Any solution of the evolution equation can be expressed in terms of the eigenfunctions of V:

$$\varphi(x_i,Q) = x_1x_2x_3\sum_{n=0}^{\infty} a_n\tilde{\varphi}_n(x_i)\exp(-\gamma_n\xi), \quad [\tfrac{3}{2}C_F/\beta - \gamma_n]x_1x_2x_3\tilde{\varphi}_n = V\tilde{\varphi}_n. \tag{4}$$

The coefficients a_n may be determined from the soft wave function[4]:

$$a_n\left(\ln\frac{\lambda^2}{\Lambda^2}\right)^{-\gamma_n} = \int_0^1 [dx_i]\,\tilde{\varphi}_n(x_i)\varphi(x_i,\lambda).$$

The leading eigenvalues γ_n and eigenfunctions $\tilde{\varphi}_n(x_i)$ for helicity-$\frac{1}{2}$ and -$\frac{3}{2}$ baryons[5] are given in Table I. (See Ref. 2 for further details.) In practical applications it is usually simpler to integrate the evolution numerically [beginning with $\varphi(x_i,\lambda)$ at $\xi = \ln\ln(\lambda^2/\Lambda^2)$] as opposed to using expansion (4). However, from Eq. (4) and Table I, we can find the asymptotic wave function for very large Q^2:

$$\varphi(x_i,Q) \to C x_1x_2x_3 \begin{cases} \chi[\ln(Q^2/\Lambda^2)]^{-2/3\beta}, & |h| = \frac{1}{2}, \\ \chi[\ln(Q^2/\Lambda^2)]^{-2/\beta}, & |h| = \frac{3}{2}, \end{cases} \tag{5}$$

where C is determined by the qqq wave function at the origin, and h is the total helicity. Since asymptotically φ is symmetric under interchange of the x_i's, Fermi statistics demands that the corresponding flavor-helicity wave functions must be completely symmetric under particle exchange—i.e., identical to those assumed in the symmetric SU(6) quark model.[6]

The magnetic form factor $G_M(Q^2)$ for nucleons is given by Eq. (1), where T_B is computed from the sum of all minimally connected diagrams for $\gamma^* 3q \to 3q$ [see Fig. 1(a)]. We find $(h_1 = h_3 = -h_2 = h)$[7]

$$T_B = 64\pi^2\left[\frac{C_B\alpha_s(Q^2)}{Q^2}\right]^2\left[\sum_{j=1}^{3} e_j T_j(x_i,y_i) + (x_i \leftrightarrow y_i)\right], \tag{6}$$

where

$$T_1 = T_3(1 \leftrightarrow 3) = \frac{1}{x_2 x_3(1-x_3)}\,\frac{1}{y_2 y_3(1-y_1)} - \frac{1}{x_3(1-x_1)^2}\,\frac{1}{y_3(1-y_1)^2},$$

$$T_2 = -\frac{1}{x_1 x_3(1-x_1)}\,\frac{1}{y_1 y_3(1-y_3)},$$

and e_j is the electromagnetic charge (in units of e) of particle j. Convoluting with wave function (4), we obtain the QCD prediction for the large-Q^2 behavior of G_M:

$$G_M(Q^2) = \frac{32\pi^2}{9}\,\frac{\alpha_s^2(Q^2)}{Q^4}\sum_{n,m} b_{n,m}\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-\gamma_n-\gamma_m}[1+O(\alpha_s(Q^2), m/Q)]\,. \tag{7}$$

For very large Q^2, the $n=m=0$ term dominates and we find

$$G_M(Q^2) \to \frac{32\pi^2}{9}\,C^2\,\frac{\alpha_s^2(Q^2)}{Q^4}\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-4/3\beta}(e_{\parallel}-e_{-\parallel}), \tag{8}$$

where $e_{\parallel}$ ($e_{-\parallel}$) is the mean total charge of quarks with helicity parallel (antiparallel) to the nucleon's helicity (in the fully symmetric flavor-helicity wave functions). For protons and neutrons we have

$$e_{\parallel}^{p}=1,\quad e_{-\parallel}^{p}=0,\quad \text{and } e_{\parallel}^{n}=-e_{-\parallel}^{n}=-\tfrac{1}{3}.$$

The constants C are generally unknown for baryons; however, by isospin symmetry $C_p = C_n$ and thus QCD predicts the ratio of form factors as $Q^2\to\infty$:

$$G_M^{\,n}(Q^2)/G_M^{\,p}(Q^2) \to -\tfrac{2}{3}. \tag{9}$$

This is remarkably close to the measured ratio $G_M^{\,n}/G_M^{\,p} \cong \mu_n/(1+\mu_p) = -0.685$, which remains roughly constant through the range of data ($0<Q^2<3$ GeV^2). Notice also that $G_M^{\,p}$ is positive and $G_M^{\,n}$ negative in the limit (8), which is consistent with data [e.g., $G_M^{\,p}<0$ as $Q^2\to\infty$ would imply a zero in the form factor at some finite Q^2, since $G_M^{\,p}(0)\equiv 1+\mu_p>0$]. Both the sign and magnitude of the ratio (9) are nontrivial consequences of QCD; they depend upon the detailed behavior of T_B and $\varphi(x_i,Q)$ as $Q^2\to\infty$. For comparison, note that in a theory with scalar or pseudoscalar gluons, diagrams in which the struck quark has antiparallel helicity vanish. Thus scalar QCD predicts a ratio $G_M^{\,n}/G_M^{\,p}\to e_{\parallel}^{n}/e_{\parallel}^{p} = -\frac{1}{3}$.

The predictions for $G_M(Q^2)$ in the subasymptotic domain depend on the $n,m\neq 0$ terms in Eqs. (4) and (7). To indicate the extent of this dependence, we plot $Q^4 G_M^{\,p}(Q^2)$ in Fig. 2(a) beginning with two very different *Ansätze* for the low-energy wave function: $\varphi(x_i,\lambda)$ sharply peaked at $x_i=\frac{1}{3}$ for small λ, and $\varphi(x_i,\lambda)\propto x_1x_2x_3$ for all λ (i.e., no evolution). Primarily because of the factors of α_s in Eq. (6), both theoretical curves fall faster

TABLE I. Eigensolutions of the evolution equation (3) for $|h|=\frac{1}{2}(\tilde{\varphi}^{\uparrow\downarrow\uparrow})$ and $|h|=\frac{3}{2}(\tilde{\varphi}^{\uparrow\uparrow\uparrow})$ baryons (see Ref. 5). A procedure for systematically determining all $\tilde{\varphi}_n$ is given in Ref. 2.

	b_n	N	$a_{00}^{(n)}$	$a_{10}^{(n)}$	$a_{01}^{(n)}$	$a_{20}^{(n)}$	$a_{11}^{(n)}$	$a_{02}^{(n)}$
$\tilde{\varphi}_n^{\uparrow\downarrow\uparrow}$	-1	120	1					
	2/3	1260		1	-1			
	1	420	2	-3	-3			
	5/3	756	2	-7	-7	8	4	8
	7/3	34020		1	-1	$-4/3$		4/3
	5/2	1944	2	-7	-7	14/3	14	14/3
$\tilde{\varphi}^{\uparrow\uparrow\uparrow}$	0	120	1					
	3/2	420	1	-3				
	3/2	420	1		-3			
	7/3	5760	1	$-7/2$	$-7/2$	7/2	7/2	7/2
	17/6	3024	1	$-7/2$	$-7/2$	2	8	2
	17/6	34020		1	-1	$-4/3$		4/3
	$\gamma_n=(2b_nC_B+\frac{3}{2}C_F)/\beta$					$\tilde{\varphi}_n=N^{1/2}\sum_{i,j}a_{ij}^{(n)}x_1^{\,i}x_3^{\,j}$		

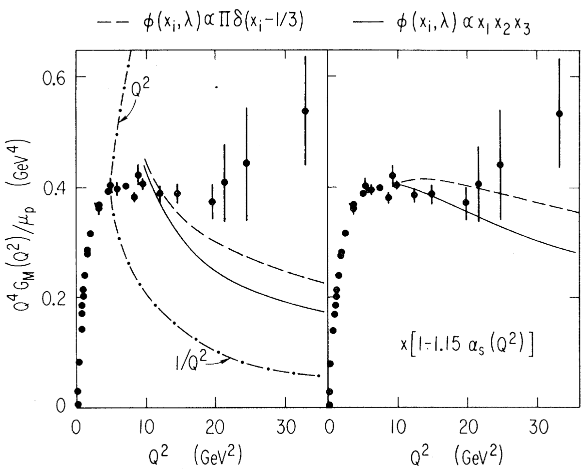

FIG. 2. (a) QCD predictions ("leading log" approximation) for the proton's magnetic form factor under two different *Ansätze* for the low-energy ($\lambda^2=4$ GeV^2) wave function φ [$\varphi\propto x_1x_2x_3$ corresponds to Eq. (7) with $C=0.26$ GeV^2]. A large QCD scale parameter ($\Lambda^2=1$ GeV^2) is used here since, effectively, q is shared by three constituents, thereby reducing the momentum controlling scaling violations. The data are from Ref. 6. (b) The theoretical curves in Fig. 2(a) are multiplied by $1-1.15\times\alpha_s(Q^2)$ and renormalized to indicate the potential significance of nonleading (but calculable) corrections. The solid curve can also be obtained directly from Eq. (7) by choosing a scale parameter $\Lambda^2=0.000\,15$ GeV^2 and $C=1.4$ GeV^2.

than the data[8]—though not as fast as a full power of $1/Q^2$. Nonleading terms could well be important for $Q^2\lesssim 25$ GeV^2, as is illustrated in Fig. 2(b) where the curves from Fig. 2(a) are multiplied by $1-1.15\alpha_s(Q^2)$ and renormalized to fit the data.[9] These corrections can and in fact must be computed before a definitive comparison with the data is made. The ratio of neutron to proton form factors, which is perhaps less sensitive to such corrections, is roughly independent (i.e., to $\pm 10\%$ in this range of Q^2) of the choice of wave function, at least for wave functions $\varphi(x_i,Q)$ having little oscillatory behavior as the x_i are varied. In scalar QCD, both wave functions result in the same curve.

As is the case for mesons, form factors for processes in which the baryon's helicity is changed ($\Delta h\neq 0$), or in which the initial or final baryon has $h>1$, are suppressed by factors of m/Q, where m is an effective quark mass. (Crossing and the $\Delta h=0$ rule imply that form factors for particles with opposite helicity dominate for q^2 timelike.) Thus the helicity-flip nucleon form factor is predicted to fall roughly as $F_2\to mM/Q^6$. The reaction $e^+e^-\to\Delta^+\Delta^-$ and $ep\to e\Delta$ are dominated by baryons with $|h_\Delta|=\frac{1}{2}$; the e^+e^- cross section for production of $|h_\Delta|=\frac{3}{2}$ pairs of deltas with $|h_\Delta|=\frac{3}{2}$ and $\frac{1}{2}$ is suppressed. Most of these predictions test the vector nature of the gluon, e.g., transitions $ep\to e\Delta(|h_\Delta|=\frac{3}{2})$ are not suppressed in scalar QCD.

The techniques outlined above for studying asymptotic form factors can clearly be extended to the computation of any exclusive process involving large transverse momentum exchange between color singlets, e.g., the fixed angle amplitude for the process $AB\to CD$. For $p_\perp$ sufficiently large, the wave functions tend to their asymptotic form and the cross section becomes

$$\frac{d\sigma}{dt}(AB\to CD)\to\left[\frac{\alpha_s(p_\perp^2)}{p_\perp^2}\right]^{n-2}\left(\ln\frac{p_\perp^2}{\Lambda^2}\right)^{-2\Sigma i\gamma_i} f(\theta_{\mathrm{c.m.}}), \qquad (10)$$

where for mesons $\gamma_i=0$, $-4/3\beta$ (for $|h|=0,1$) and for baryons $\gamma_i=-2/3\beta$, $-2/\beta$ (for $|h|=1/2$, $3/2$). The normalization is, in principle, fixed by form-factor data. Contributions due to the pinch singularities discussed by Landshoff[10] are suppressed by Sudakov form factors.[1,11] Consequently, these contributions fall faster than any power of t and can be neglected relative to (10) except possibly when $s\gg|t|$.

It should be emphasized that the specific integral power Q^{-4} predicted for G_M in Eq. (7) reflects both the scale invariance of the internal quark-quark interactions, and the fact that the minimal spin-$\frac{1}{2}$ color-singlet wave function contains three quarks. Thus both the dynamics and symmetry properties of QCD are directly tested. Furthermore, the spin dependence of quark-quark interactions can be tested at short distances by studying the helicity dependence of elastic and transition form factors. The fact that the data for the ratio $G_M{}^n/G_M{}^p$ are close to the predicted asymptotic value appears to be a striking success for QCD. We also note that it should be possible to relate the normalization and structure of the wave function $\varphi(x,\lambda)$ at large distances to wave functions used in the study of baryon spectroscopy.

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[4]Since V is symmetric under the interchange $x \leftrightarrow y$, the eigenvalues γ_n are real and the eigenfunctions are orthogonal with respect to weight $x_1x_2x_3$. Convergence of the expansion (4) is assured by the boundary conditions satisfied by bound-state wave functions describing composite particles (see Ref. 1). These conditions also insure that F_B is dominated by short-distance phenomena and, consequently, that the "leading log" approximation is justified for large Q^2.

[5]The anomalous dimensions γ_n in this table have been verified by M. Peskin with use of a different method (private communication).

[6]Note that we are *not* assuming SU(6) symmetry. The fact that the coordinate-space wave functions become symmetric at short distances is a dynamical consequence of the theory.

[7]T_B is Lorentz and gauge invariant. It is most easily computed in the Breit frame ($\vec{p}\,' = -\vec{p}$) where only three independent amplitudes need be computed [Fig. 1(a)]. This method is used by E. M. Levin, Yu. M. Shabelsky, V. M. Shekter, and A. N. Solomin, to be published.

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[9]As is well understood, higher-order corrections can to a large extent be absorbed into a redefinition of Λ^2. Choosing $\Lambda^2 = 0.000\,15$ GeV^2 in Eq. (7) reproduces the solid curve in Fig. 2(b).

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Model of Confinement for Gauge Theories

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It is shown that if the Lagrangian density for a gauge field is taken as an appropriate nonlinear function of the usual Lagrangian density, there results a simple classical model of confinement that has nothing to do with the non-Abelian character of the gauge group. Plane-wave solutions are suppressed, suggesting confinement of the gauge quanta as well.

It is generally felt[1] that the basic strong interactions among quarks should be mediated by a non-Abelian "gluon" gauge field, coupled to the color quantum numbers. Among the desired characteristics of such a theory are (1) renormalizability,[2] (2) asymptotic freedom,[3] (3) confinement of quarks, and (4) confinement of gluons. Of these, the standard form of quantum chromodynamics (QCD) provides the first two, and the hope has been that (3) and (4) may follow from the as yet unplumbed complications of the theory. I explore here a possible alternative route to confinement, making use of the arbitrariness in choice of Lagrangian allowed by the requirements of Lorentz and gauge invariance. The resulting theory gives strong indications of satisfying (2), (3), and (4), and a reasonable expectation of satisfying (1) as well. It will be noticed that the essential features of the discussion apply equally, indeed more readily, to the case of an Abelian gauge field.

The conventional gauge-invariant Lagrangian density in the absence of sources is

$$l = \tfrac{1}{4}\vec{\mathrm{F}}_{\alpha\beta}\cdot\vec{\mathrm{F}}^{\beta\alpha} \tag{1}$$

$$= \tfrac{1}{2}(|\vec{\mathrm{E}}|^2 - |\vec{\mathrm{H}}|^2), \tag{2}$$

where

$$\vec{\mathrm{F}}_{\alpha\beta} = \partial_\beta \vec{\mathrm{b}}_\alpha - \partial_\alpha \vec{\mathrm{b}}_\beta + g\vec{\mathrm{b}}_\alpha \times \vec{\mathrm{b}}_\beta, \tag{3}$$

and by analogy with electrodynamics, we refer to the polar and axial vector parts of $\vec{\mathrm{F}}^{\alpha\beta}$ as $\vec{\mathrm{E}}_i$ and $\vec{\mathrm{H}}_i$, respectively. We propose the Lagrangian density

$$L = S(l) - 4\pi\vec{\mathrm{b}}_\alpha\cdot\vec{\mathrm{J}}^{\,\alpha}, \tag{4}$$

where the form of the function $S(l)$ will be chosen to accomplish our various ends. The resulting energy density is

$$W = |\vec{\mathrm{E}}|^2 S'(l) - S(l) - \vec{\mathrm{b}}^{\,i}\cdot\vec{\mathrm{J}}^{\,i}, \tag{5}$$

where the repeated index i is summed from 1 to 3. The necessary and sufficient conditions on $S(l)$ for positive semidefiniteness of the source-free

