Cosmological Origin of the Grand-Unification Mass Scale

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The origin of the universe as a quantum phenomenon leads to a self-consistently generated space-time structure in which the mass of the created particles is $O(\kappa^{-1/2})$. We interpret the origin of the universe as a phase transition in which the grand unified symmetry is spontaneously broken.

A fascinating consequence of the hypothesis of grand unification is the appearance of the Planck mass ($m \sim \kappa^{-1/2} \sim 10^{18}$ GeV) as the scale of breakdown of the unified symmetry into the strong and weak-electromagnetic sectors. This scale has come up in three independent ways (none of which has anything to do with gravity): (1) fitting to the lower bound of the proton lifetime to get the leptoquark mass, (2) with use of renormalization-group estimates to get the Weinberg angle, and (3) to get the ratio of strong to weak coupling constants.[1]

In the present work, it is shown that the hypothesis of the creation of the universe as a quantum phenomenon[2] leads to an approximate eigenvalue condition for the mass of the particles that are created: to wit, $\kappa^{1/2}m = O(1)$. It is then natural to associate these particles with the lepto-quarks. At the end of this note we show how the very special structure of the very early universe could give rise naturally to an estimate for the total number of produced particles which is consistent with the observed value ($\sim 10^{90}$). Furthermore, we suggest the relevance of this special structure to the understanding of matter-antimatter imbalance in terms of CP-nonconserving effects in lepto-quark decay.[3]

We restate the case from the beginning.[2] If there is no cosmological constant, present experimental evidence points towards an open universe. In such a universe the only way to avoid a mathematical singularity in which the energy density becomes infinite (the mathematical "big bang," not to be confused with the early hot fireball) is that matter is produced in the early universe. This is permitted by the conservation laws of general relativity. The production mechanism of I is that of a self-consistent process: A small amount of mass produces space-time variation of the cosmological gravitational field which in turn produces more particles, the whole being a cooperative process. If the matter so produced tends to a constant energy density σ,[4] then the space-time structure of the very early universe tends to that of a de Sitter space. [Subsequent to this stage, incoherence develops, production stops, and the more conventional cosmological models take over.]

Adhering to the notation of I, the de Sitter metric is given by $g_{\mu\nu} = e^{\lambda}[g_{\mu\nu}]_{\text{Minkowski}}$, where

$$e^{\lambda} = (1 - \tau^2/\tau_\infty^{\ 2})^{-2}, \quad \tau_\infty^{\ 2} = 12/\kappa\tau, \quad 0 < \tau < \tau_\infty, \qquad (1)$$

where τ is the kinematic time of I [$\equiv (t^2 - r^2)^{1/2}$]. e^{λ} is the background field which is responsible for the generation of matter through the equations of motion. For a boson field operator ψ, this is (neglecting spin and charge complications) $(\Box + m^2 e^{\lambda})\psi = 0$. This equation is to be solved in the domain $\tau > 0$, subject to the condition that there be no matter at $\tau = 0$. As τ tends to τ_∞, it is shown below that the energy density so generated tends to a constant, thereby ensuring consistency with the initial *Ansatz* (1) in this asymptotic region. Clearly, for earlier times this procedure is not consistent. A mechanism for restoring complete self-consistency will be presented in the last paragraph.

The usual harmonic decomposition on the hyperboloid[5] τ is

$$\psi = (\sqrt{2}\,\tau)^{-1}\int_0^\infty dk \sum_{lm} \Phi_{klm}(r,\theta,\varphi)\xi_k(m\tau)a_{klm}^{\ +} + \text{H.c.};$$

Φ_{klm} are defined on the unit hyperboloid, parametrized by coordinates, r, θ, φ. The resulting equation for ξ_k is

$$[\tau^2 d^2/d\tau^2 + \tau\, d/d\tau + k^2 + m^2\tau^2 e^{\lambda}]\xi_k(m\tau) = 0, \quad 0 < \tau < \tau_\infty, \tag{2}$$

subject to the Wronskian condition $\tau(\xi_k^*\partial_\tau\xi_k - \xi_k\partial_\tau\xi_k^*) = 2i$, required to satisfy the canonical commutation relations for ψ. The solution is $\xi_k = \alpha\varphi_k + \beta\varphi_k^*$, $|\alpha|^2 - |\beta|^2 = 1$, and

$$\varphi_k(x) = (x^{ik}/\sqrt{k})(1-x^2)^{1/2+i\nu}F(\tfrac{1}{2}+i\nu, \tfrac{1}{2}+i(\nu+k); 1+ik; x^2), \tag{3}$$

where $x \equiv \tau/\tau_\infty$, $\nu \equiv \frac{1}{2}(m^2\tau_\infty^2 - 1)^{1/2}$, and F is the hypergeometric function. The condition that there be no matter at $\tau = 0$ is $\beta = 0$.[6]

The number of particles produced in mode k is $\langle N_k(x)\rangle = \frac{1}{2}|\Omega_k\varphi_k|^2$, where $\Omega_k = [2K_k(x)]^{-1/2}[x\partial/\partial x - iK_k(x)]$, $K_k(x) \equiv [k^2 + m^2\tau_\infty^2 x^2/(1-x^2)^2]^{1/2}$. The corresponding total number density is

$$n(x) = [\tau_\infty x/(1-x^2)]^{-3}\int_0^\infty k^2\, dk \langle N_k(x)\rangle/2\pi^2.$$

As $\tau \to \tau_\infty$ we have

$$\lim_{x\to 1} n(x) = \lim_{x\to 1} \frac{1}{\tau_\infty{}^3 2\pi^2}\int_0^\infty p^2\, dp\, N(p/(1-x^2)), \tag{4}$$

where $N(p/(1-x^2)) \equiv \langle N_{p/(1-x^2)}(x)\rangle$. It is therefore required to evaluate $\lim_{k\to\infty;\, x\to 1}\varphi_k(x)$ for $k(1-x^2) = p$, p fixed. The limit is obtained either from the defining series of F or directly from (2) in the asymptotic region:

$$\varphi_k(x) \underset{x\to 1}{\sim} \left(\frac{\pi}{2i}\right)^{1/2}\frac{(1-x)^{1/2}}{\sinh\pi\eta}\left\{e^{\eta\pi/2}J_{-i\eta}(k(1-x)) - e^{-\eta\pi/2}J_{i\eta}(k(1-x))\right\}. \tag{5}$$

One verifies that $\lim_{x\to 1}N(p/(1-x^2)) \equiv \bar{N}(p)$ exists. It is also straightforward to show that $\bar{N}(p) \sim p^{-6}$ for large p and that there is no divergence in (4) for $p \to 0$. Thus $n(x)$ is a rapidly convergent integral and tends to a constant in the asymptotic region $x \to 1$. Similarly the more interesting quantity, the energy density, tends to constancy [see Eq. (7) below].

The above depends on the classical treatment of the gravitational field which can be valid only for $m\tau > 1$. Hence it is interesting to evaluate the energy density $\sigma(1) = \lim_{x\to 1}\sigma(x)$ for large $m\tau_\infty$. In this limit we are led to study $\varphi_k(x)$ for $k(1-x^2) = p = 2\eta q$ as $x \to 1$ and then $\eta \to \infty$ at fixed q. Using the Debye expansion[7] for $J_{-i\eta}$ in (5), one obtains

$$\varphi_k(x) \underset{\substack{x\to 1\\ \eta\to\infty}}{\sim} (1-x)^{1/2}e^{i\eta\gamma}\frac{e^{-i\eta T}}{(\eta T)^{1/2}}\left[1 + \frac{u_1(T^{-1})}{i\eta} - \frac{u_2(T^{-1})}{\eta^2} + O\left(\frac{1}{\eta^3}\right)\right], \tag{6}$$

where $T \equiv \coth\gamma \equiv [1 + k^2(1-x)^2/\eta^2]^{1/2}$. One finds from this form

$$\bar{N}(2\eta q) \underset{\eta\to\infty}{\sim} (1/16\eta^2)(1+q^2)^{-3};$$

this estimate is obtained from $\frac{1}{2}|\Omega_k\varphi_k|^2$, where it suffices to use only the leading term in (6). The corresponding energy density $\sigma_\infty(1)$ is

$$\sigma_\infty(1) \underset{\eta\to\infty}{\sim} \lim_{x\to 1}\frac{1}{[\tau e^{\lambda/2}]^4}\int_0^\infty k^2\, dk\, K_k(x)\langle N_k(x)\rangle/2\pi^2 = \frac{1}{\eta^2}\frac{m^4}{32\pi^2}\int_0^\infty\frac{q^2\, dq}{(1+q^2)^{5/2}} = \frac{1}{\eta^2}\frac{m^4}{96\pi^2}. \tag{7}$$

The zero-point energy has been subtracted in (7).[8] The prefactor containing the metric comes from the conformal scaling by the Robertson-Walker scale factor $R = \tau e^{\lambda/2}$ from the unit hyperboloid in Minkowski space to the corresponding hyperboloid in the de Sitter space.

The factor η^{-2} in (7) is the source of the eigenvalue condition $\kappa^{1/2}m = O(1)$. Comparing (7) to (1) we see that η can be eliminated leaving $(\kappa^{1/2}m)_{\eta\to\infty} = 12\sqrt{2}\,\pi$. We have verified that $\kappa^{1/2}m$ which is the only parameter of the theory, remains, in fact, of order 1 when $m\tau_\infty$ varies in the range ∞ to 1 while $\tau(1)$ varies according to (1).

A further consistency check is possible. The de Sitter space being a solution of Einstein's equation requires the equation of state $t = 4\sigma$, where t is the trace of the energy-momentum. As $\eta \to \infty$,

$$t_\infty(1) \underset{\eta\to\infty}{\sim} \lim_{x\to 1} m^2 e^{-\lambda}\langle\psi^2\rangle$$

which give rise to the subtracted value[8]

$$\lim_{x\to 1} m^e(\eta e^{\lambda/2})^{-2}\int_0^\infty k^2\,dk[|\varphi_k|^2-K_k^{-1}]/4\pi^2.$$

This is evaluated with use of Eq. (6); the correction u_1 and u_2 now have to be included. The result is

$$t_\infty(1)=\frac{1}{\eta^2}\,\frac{m^4}{32\pi^2}\int_0^\infty\frac{q^2(1+6q^2)dq}{(1+q^2)^{7/2}}=\frac{1}{\eta^2}\,\frac{m^4}{24\pi^2}=4\sigma_\infty(1). \tag{8}$$

We have thus shown that I must be complemented by the statement that the particles produced in the very early universe have mass $\sim\kappa^{-1/2}$. We then identify the very early universe as a quantum state of equal numbers[9] of lepto-quarks and anti-lepto-quarks (L). These will decay into baryons, leptons, antibaryons, antileptons, and bosons of all sorts. One may ask two questions at this point. How many L's are produced? Can they conceivably decay in such a way to give a matter-antimatter imbalance?

The key to the answer to these questions lies in the structure of the de Sitter space, the seat in which these events take place before giving way to the more conventional Friedman space after the decay. A de Sitter space is characterized asymptotically by a constant Hubble constant, i.e., $d\ln R/d\tau^*\equiv H=2/\tau_\infty$. Here τ^* is the proper time given by $d\tau^*/d\tau=e^{\lambda/2}$. We see that for a de Sitter space R tends to an exponential increase in the proper time $R/R_0\to\exp2(\tau^*-\tau_0^*)/\tau_\infty$. τ_0^* is the time when particles begin to be significantly produced; $R_0/\tau_0^*=O(1)$. As we have pointed out production stops for times τ^* for which incoherence develops. This will happen within a relaxation time τ_R^* which is expected to be comparable to the decay time (say $\sim 10^3 m^{-1}$). We shall show below that each L decays into $O(1)$ ordinary particles (and not 10^{18} as one might naively expect). The number of presently observed particles is thus $O([R(\tau_R^*)/R_0]^3)$. Thus $\ln N_{\text{observed}}\simeq 6\tau_R^*/\tau_\infty$ and it is not unreasonable that this should give the required magnitude of $O(10^2)$ (recall $\ln N_{\text{observed}}\simeq 200$).

Concerning the decay of the L's, the exponentially rapid increase of the scale factor R/R_0 implies that the momentum of the emitted particles will be scaled down exponentially—on the scale of the lepto-quark mass! Within a few lepto-quark Compton times the momentum of a decay product will become O(1 GeV) which we take to be the mass scale of ordinary particles. There is thus no time to cascade a great abundance of particles. It is then conceivable that CP-nonconserving effects of $O(10^{-8})$ can give rise to the required matter-antimatter imbalance in the decay.[3]

Our last point concerns the complete consistency of the approach. In the first instance we have verified approximate consistency everywhere by matching the de Sitter space to Minkowski space on a hyperboloid $\tau=\tau_0$, where τ_0 was taken to be $O(m^{-1})$; the mass parameter in Minkowski space as well as the matter density is taken to be zero. We interpret this calculation in terms of a phase transition in which we envisage the "edge of the universe" as the phase boundary (a skin $0\le\tau\le\tau_0$), through which the mass changes from zero to its self-consistent value. Symmetry is fully realized outside the universe and is spontaneously broken within. In this sense it is satisfying that the self-consistent mass which we find is indeed that which characterizes the scale of spontaneous breaking of the grand unified symmetry. We conceive the complete problem in terms of quantum mechanical tunneling from one phase to another and we hope that complete consistency can be realized at a semiclassical level as in the droplet model.[10] We should not be surprised if spontaneously broken conformal symmetry plays a key role in realizing this program and that in so doing the nature of gravity in quantum theory will be illuminated.

[1]J. C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973), and Phys. Rev. Lett. 31, 661 (1973), and Phys. Rev. D 10, 275 (1974); H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974). For a more recent estimate, see A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B135, 66 (1978). In this note we do not commit ourselves to a particular grand unified scheme and hence not to a precise value of the lepto-quark mass.

[2]R. Brout, F. Englert, and E. Gunzig, Ann. Phys. (Paris) 115, 78 (1978) (hereafter referred to as I). References to cosmology and to other authors' work in particle production and the origin of the universe as a quantum fluctuation are contained therein. R. Brout, F. Englert, and E. Gunzig, to be published.

[3]This idea seems to be due to M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978), and 42, 746(E) (1979). More

recent work on the subject can be found in S. Dimopoulos and L. Susskind, Phys. Rev. D 18, 4500 (1978), and Stanford University Report No. ITP-616 (unpublished); B. Toussaint, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 19, 1036 (1979); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Phys. Lett. B80, 360 (1979); S. Weinberg, Harvard University Report No. HUTP 78/A040 (unpublished).

[4]In I, the demonstration that σ tends to constancy was faulty in that the passage from 3.24 to 3.26 was incorrect because the functions are too sensitive to k. The present paper supplants this development.

[5]See I and A. A. Grib, S. G. Mamayev, and V. M. Mostepanenko, Gen. Relat. Gravitat. 7, 533 (1976).

[6]This is a rather subtle point. In fact, $\beta=0$ is the condition that $\langle N_k(\tau)\rangle=0$ at $\tau=0$, whereas the physical condition is that there be no particles in states of fixed momentum. k is not momentum and the difference between the two gives rise to effective thermal effects in the k representation. These die out exponentially and do not affect our asymptotic estimates. A detailed treatment of these effects will be covered in a paper now in preparation in collaboration with J.-M. Frère and C. Truffin.

[7]See *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. Stegun (Dover, New York, 1964), p. 366, Formula 9.37: $u_1(T^{-1})=(1/24)(3T^{-1}-5T^{-3})$, $u_2(T^{-1})=(1/1152)(81T^{-2}-462T^{-4}+385T^{-6})$.

[8]The subtraction procedure will be presented in detail in the paper in preparation (Ref. 6). Suffice it to say for the present that our subtraction is equivalent to setting to zero the renormalized cosmological constant in a manner consistent with conformal symmetry in the context of dimensional regularization. In this respect we acknowledge informative conversations with J.-M. Frère, R. Gastmans, and C. Truffin. See also B. de Wit and R. Gastmans, Nucl. Phys. B128, 294 (1977).

[9]See I for the conservation of charge for ψ non-Hermitian.

[10]S. Coleman, Harvard University Report No. HUTP 78/A004 (unpublished).