fect we have assumed that the average momentum transferred to the nucleus prior to the emission of a 50-MeV $\alpha$ particle is $k_\gamma = 70$ MeV/$c$. The number of nucleons, $p$, sharing the momentum at the time of $\alpha$ emission was varied to obtain the best fit to the angular distribution data. With use of the experimentally determined slopes for $\sigma(E_\alpha)$ the fits shown in Fig. 3 are obtained with $p = 6$. Similar fits can be obtained for $E_\alpha = 30$ MeV by varying $k_\gamma$ and $p$ in a qualitatively reasonable way. In view of the fair agreement observed, it seems plausible that the exciton model could describe the $\alpha$-particle angular distributions.

In conclusion, we have observed a significant preequilibrium component in the $(e, \alpha)$ reaction which may be described by a two-step or few-step process in which only a few nucleons share the initial excitation energy prior to particle emission.

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# Magnetic Moments in Calcium Isotopes via a Surface-Interaction Experiment

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A rotation of the angular correlation of deexcitation $\gamma$ rays from $^{40}$Ca and $^{44}$Ca was observed in a tilted foil geometry. The signs and magnitudes of the magnetic moments of the $2_1^+$ level of $^{44}$Ca and of the $3_1^-$ level of $^{40}$Ca were determined to be $g = -0.28 \pm 0.11$ and $g = +0.52 \pm 0.18$, respectively. The experiment demonstrates that polarization of deeply bound electronic configurations can be appreciable and that this technique can be used as a quantitative measure of magnetic moments of picosecond nuclear levels.

The magnetic moments of the $2_1^+$ levels of even Ca isotopes have not been measured hitherto, but it is evident that their values and signs can provide significant information regarding the shell-model structure of low-lying levels in this mass region. We report here the measurement of the magnetic moment of the $2_1^+$ level of $^{44}$Ca. The experiment utilizes the "tilted foil" hyperfine interaction, i.e., the interaction between the nuclear level and a polarized electronic ensemble associated with high-velocity ions emerging from a surface whose normal does not lie along the beam direction.[1,2] The polarization of electronic configurations manifested in the emittance of circularly polarized light has been extensively studied in the transmission (tilted-foil) geometry[3] and in the reflection (grazing-angle) geometry.[4] For deeply bound atomic levels such polarization can be observed via their hyperfine interaction with the nucleus. A rotation of the angular correlation of decay $\gamma$ rays from an excited nuclear level is a signature of a polarized electronic ensemble and provides a measure of the sign and magnitude of the magnetic moment of this level. The first two experiments utilizing this phenomenon were carried out for the $2_1^+$ level of $^{18}$O and $3_1^-$ level of $^{16}$O, confirming the negative and positive signs, respectively, of the magnetic moments of these levels.[1,2] In the present experiment, we have extended these measurements to the $^{44}$Ca and $^{40}$Ca isotopes. A better understanding of the phenomenology of the polarization in such geometries[4] makes it possible now to obtain the magnitude of $g(2_1^+)$ of $^{44}$Ca by comparison to the known[5] $g(3_1^-)$ of $^{40}$Ca. An independent measurement using the integral perturbation of an unpolarized electronic ensemble was also carried out for the $2_1^+$ and $3_1^-$ levels of $^{44}$Ca and $^{40}$Ca, respectively.

An $\alpha$ beam from the tandem Van de Graaff accelerator at the Weizmann Institute was used to excite the $3_1^-$ and $2_1^+$ levels of $^{40}$Ca and $^{44}$Ca via the $(\alpha, \alpha')$ reaction on isotopically enriched targets. Backscattered $\alpha$ particles were detected in a 100-$\mu$m annular surface-barrier detector. Decay $\gamma$ rays in coincidence with the inelastic $\alpha$ groups populating either the $3_1^-$ or the $2_1^+$ levels were measured in four movable 12.5-cm × 12.5-cm NaI (Tl) counters. Random coincidences were monitored and subsequently subtracted from each $\gamma$ spectrum. A narrow slit subtending 2° was placed in front of the particle detector, perpendicular to the plane of the $\gamma$ counters, in order to sharpen the $\alpha$-$\gamma$ correlation and to prevent shadowing the particle counter by the target assembly in the tilted position.[1] The targets consisted of $^{40}$Ca or $^{44}$Ca evaporated inside the experimental chamber on a 10-$\mu$g/cm$^2$ carbon foil. Measurements of the excitation function and of the angular correlation were carried out with the target at the same 70° angle to the beam direction subsequently used in the precession experiments. The target thickness at this position was measured by monitoring the positions and widths of the $\alpha$ energy peaks. An effective thickness of $240 \pm 40$ $\mu$g/cm$^2$ was thus obtained, ensuring that the calcium ions did indeed recoil out of the target and thus experience the full hyperfine interaction in vacuum. The bombardment energies were chosen to maximize the yield and reduce interference from other levels, and were 16.17 MeV for $^{40}$Ca and 13.10 MeV for $^{44}$Ca. The recoil velocities of the $^{40}$Ca and $^{44}$Ca ions corresponding to these energies are very close and result in similar electronic environments for both isotopes.

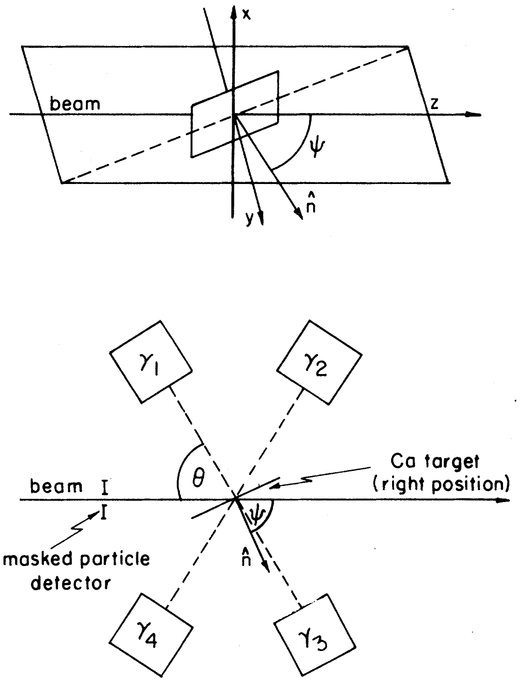

FIG. 1. Schematic view of the experimental arrangement. The target is shown in the "right" position. The normal $\hat{n}$ is at $\psi = 70°$ to the beam direction and the electronic polarization is in the direction of the $+x$ axis.

The next step in the experiment was the measurement of the angular precession due to tilting of the target at an angle of $\psi = \pm 70°$ to the beam direction. The experimental arrangement was essentially similar to that described earlier[1,2] and is shown in Fig. 1. The $\gamma$ counters were placed at angles where the logarithmic derivatives $W^{-1}dW/d\theta$ are large. The precession of the angular correlation for $^{40}$Ca ions recoiling into a solid silver backing was also measured to serve as a check against various systematic errors; for recoil into solid, there should be no effect due to target tilting since the ions stop inside the backing and do not experience the surface interaction. The results of the precession experiments are given in Table I. We define the double ratios $\rho_{ij} = [(W_i^R/W_i^L)(W_j^L/W_j^R)]^{1/2}$ where $W_i^R$, for example, is the number of counts in detector $i = 1-4$ with the target tilted in the "right" direction (see Fig. 1). The precession angle $\Delta\varphi$ is given by

$$\Delta\varphi = (W^{-1}dW/d\theta)^{-1}(1-\rho)/(1+\rho),$$

where

$$\rho = (\rho_{14}\rho_{32})^{1/2}.$$

The absence of a measurable precession in the

TABLE I. Summary of the experimental results. $\theta$ is the angle of the NaI(Tl) counters in the precession measurements. $\rho$ and $\Delta\varphi$ are defined in the text.

| | $\theta$ | $\frac{1}{W}\frac{dW}{d\theta}$ | $\rho_{14}$ | $\rho_{32}$ | $\rho_{13}$ | $\rho_{24}$ | $\rho$ | $\Delta\varphi$ (mrad) |
|---|---|---|---|---|---|---|---|---|
| $^{44}$Ca (into vacuum) | 68° | $-3.53(5)$ | 1.020(12) | 1.020(15) | 0.984(13) | 0.986(15) | 1.020(9) | $+2.8\pm1.3$ |
| $^{40}$Ca (into vacuum) | 52.5° | $-2.06(13)$ | 0.956(12) | 0.974(12) | 0.985(12) | 1.004(12) | 0.965(8) | $-8.6\pm1.9$ |
| $^{40}$Ca (into silver) | 52.5° | $-4.60(20)$ | 1.006(18) | 0.992(18) | 1.007(18) | 1.007(18) | 0.999(13) | $-0.1\pm1.3$ |

$^{40}$Ca recoil-into-silver experiment and the values of the cross ratios $\rho_{13}$ and $\rho_{42}$ in all experiments demonstrate the quality of the measurements and the lack of significant systematic errors. The respective signs of $\Delta\varphi$ for $^{40}$Ca and $^{44}$Ca directly yield $g(^{40}\mathrm{Ca}, 3_1^-) > 0$ (Ref. 6) and $g(^{44}\mathrm{Ca}, 2_1^+) < 0$. The determination of the relative signs is model independent and the determination of the individual signs depend only on the well-established sense of polarization in a tilted-foil geometry.

In order to deduce the magnitude of the magnetic moment, one has to utilize information regarding the electronic configurations and polarization of Ca ions recoiling into vacuum at about $v/c \simeq 0.01$ (with charge states $3^+$–$6^+$). The intermediate-ionization model has been successfully used for various ions and in particular for Ca ions at a close recoil velocity.[5] For the present experiment, we use a slight modification of this model and assume that Ca ions with charge $10$-$n$ ($4 \leqslant n \leqslant 7$) populate only $M$-shell configurations of the type $3s^2p^{n-2}$ and $3s^13p^{n-1}$. The hyperfine magnetic fields at the nucleus are taken from Ref. 5 and the perturbation of the angular correlation is calculated for each allowed $LSJ$ term. Various modifications of the intermediate-ionization model have been tried with no significant change in the results presented below.

It can be shown that the perturbed angular distribution (for a single $LSJ$ term) is given by

$$W_p(\theta) = \textstyle\sum_k A_k [G_k P_k(\theta) + H_k P_k{}^1(\theta)], \tag{1}$$

where $A_k$ are the unperturbed correlation coefficients and $P_k$ and $P_k{}^1$ are the Legendre and associated Legendre polynomials; $G_k$ is the well known integral perturbation attenuation coefficient

$$G_k = \frac{1}{2J+1} \textstyle\sum_{FF'} (2F+1)(2F'+1) \begin{Bmatrix} F & F' & K \\ I & I & J \end{Bmatrix}^2 \dfrac{1}{1+(\omega_{FF'}\tau)^2}\,; \tag{2}$$

$I$ and $J$ are the angular momenta of the nucleus and the electrons, $\bar{F} = \bar{I} + \bar{J}$,

$$\omega_{FF'} = \tfrac{1}{2}[F(F+1) - F'(F'+1)]\tilde{\omega},$$

$$\tilde{\omega} = -(g\mu_N/\hbar)H(0)a(LSJ),$$

$a(LSJ)$ is a geometrical coupling factor, $H(0)$ is the field at the nucleus, and $\tau$ is the mean life of the nuclear level; $H_k$ is the coefficient reflecting the presence of atomic polarization and its time-integral form is given by

$$H_k = \frac{3p\cos(\bar{J}, \bar{L})}{k(k+1)\tilde{\omega}\tau[J(J+1)]^{1/2}}(1 - G_k), \tag{3}$$

where

$$\cos(\bar{J}, \bar{L}) = \frac{1}{2}\,\frac{J(J+1) + L(L+1) - S(S+1)}{[J(J+1)L(L+1)]^{1/2}} \qquad (J, L \neq 0),$$

and $p = \langle L_x \rangle / [L(L+1)]^{1/2}$ ($L \neq 0$) is the polarization fraction. $G_k$ and $H_k$ have to be averaged over all $LSJ$ terms. The polarization fraction $p$ may be regarded as an average polarization of the electronic ensemble. However, recent experiments and theoretical considerations indicate that $p$ is approximately constant for all electronic terms at a given velocity and tilt angle.[4] It should be noted that in the limit $\tilde{\omega}\tau \to 0$ one obtains

$$H_k \xrightarrow[\tilde{\omega}\tau \to 0]{} p\cos(J, L)\tilde{\omega}\tau[J(J+1)]^{1/2}$$

independent of $I$ and $k$ and resembling a classical precession. In this approximation the perturbed angular distribution (Eq. 1) reduces to

$$W_p(\theta) = W_0(\theta)\,[1 - W_0{}^{-1}(dW_0/d\theta)\Delta\varphi],$$

where

$$W_0(\theta) = \textstyle\sum_k A_k P_k(\theta) \text{ and } \Delta\varphi = H_k.^7$$

However, for $\tilde{\omega}\tau \geqslant 1$ (as in the case for $^{40}$Ca), this approximation no longer holds and one has to use the full formalism of Eqs. 1–3.

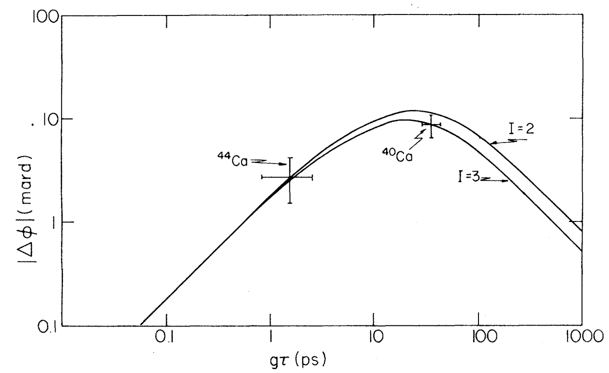

FIG. 2. The absolute value of the angular precession $\Delta\varphi$ calculated by using the electronic model described in the text and Eqs. 1–3 with a polarization fraction of $p = 0.077$.

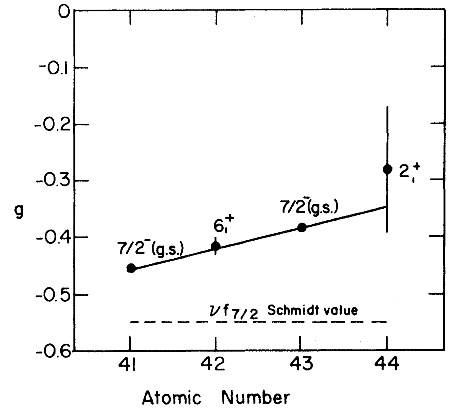

FIG. 3. $g$ factors of levels in Ca isotopes beyond $^{40}$Ca. The $^{44}$Ca result is from the present experiment; the other results are from Ref. 9.

The results of calculations for the $3_1^-$ and $2_1^+$ levels of $^{40}$Ca and $^{44}$Ca using the electronic model and Eqs. (1)–(3) are presented in Fig. 2. For $^{40}$Ca one can use the value for the $g$ factor of $g = 0.55 \pm 0.11$ (see below) and the mean life[8] $\tau = 68 \pm 3$ ps to obtain $g\tau(^{40}\mathrm{Ca}, 3_1^-) = 37.4 \pm 7.7$ ps. The fraction $p$ can now be treated as a free parameter and can be deduced from $\Delta\varphi$ to be

$$p = \langle L_x \rangle / [L(L+1)]^{1/2} = 0.077 \pm 0.018.$$

This value of $p$ in turn leads to $|g\tau|(^{44}\mathrm{Ca}, 2_1^+) = 1.6 \pm 1.0$, and with $\tau = 4.2 \pm 0.3$ ps (Ref. 8) we obtain $g(^{44}\mathrm{Ca}, 2_1^+) = -0.38 \pm 0.23$. The error includes the statistical errors in the $^{40}$Ca and $^{44}$Ca precession measurements as well as the uncertainties in the magnetic moment of the $3^-$ level. As mentioned above, the procedure adopted here is quite insensitive to the particulars of the electronic model employed.

An additional experiment was carried out to determine $|g(^{44}\mathrm{Ca}, 2_1^+)|$. The ratio $R$ of counts at $\theta = 45°$ to counts at $\theta = 90°$ was separately measured for recoil into silver and recoil into vacuum. This ratio is sensitive to the magnitude of the hyperfine interaction and therefore determines the magnetic moment (if one assumes the above electronic model). We obtain $R = 14.60 \pm 1.03$ and $R = 13.06 \pm 0.78$ for the two cases, respectively, yielding $|g| = 0.25 \pm 0.12$. This result is independent of the precession measurement and is in agreement with it. Averaging the two results for $^{44}$Ca, we obtain $g(^{44}\mathrm{Ca}, 2_1^+) = -0.28 \pm 0.11$. Taking the data for the unperturbed and perturbed correlation for $^{40}$Ca (Table I), we obtain $|g(^{40}\mathrm{Ca}, 3_1^-)| = 0.52 \pm 0.18$, in good agreement with the value $g = 0.56 \pm 0.13$ of Ref. 6. The value $g = 0.55 \pm 0.11$ used above for the determination of $g(^{44}\mathrm{Ca}, 2_1^+)$ is the average of these two results.

Negative $g$ factors of nuclear $2^+$ levels are rare and can, in fact, be expected only in quite pure shell-model configurations of neutrons in stretched angular momentum states ($p_{3/2}$, $d_{5/2}$, $f_{7/2}$, etc.); even small admixtures will, in general, increase appreciably the algebraic value of the $g$ factor. In the calcium nuclei beyond $^{40}$Ca, $f_{7/2}$ neutrons are predominant, and the negative value of $g(^{44}\mathrm{Ca}, 2_1^+)$ indicates that the $f_{7/2}{}^4$ configuration is quite pure in this case.

It is interesting to compare the measured value of $g(^{44}\mathrm{Ca}, 2_1^+)$ to other $g$ factors of low-lying levels in Ca isotopes beyond $^{40}$Ca. The $g$ factors of the $\frac{7}{2}^-$ ground states of $^{41}$Ca and $^{43}$Ca were measured to be $g = -0.456$ and $g = -0.380$.[9] The $g$ factor of the $6_1^+$ level of $^{42}$Ca was measured to be $g = -0.415 \pm 0.015$.[9] Our result agrees with the general trend (cf. Fig. 3) of negative $g$ factors of monotonically decreasing magnitude, indicating enhanced importance of small admixtures of configurations other than $f_{7/2}$ for the low-lying levels in the heavier Ca isotopes.

The polarization fraction of $p = 0.08$ is quite appreciable and can be comparable to the effects observed in atomic experiments for outer configurations. Comparison between polarizations for outer configurations (at low velocity) and for inner configurations (at high velocity) should provide information regarding the surface-interaction mechanism. The tilted-foil method can become a powerful *quantitative* technique for *measuring signs and magnitudes* of magnetic moments of picosecond nuclear levels.

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# Investigation of Noncentral Proton-Proton Interaction at Low Energy

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The analyzing power of proton-proton scattering has been measured at 6.14 MeV in the angular range $7.5 \leq \theta_{\rm lab} \leq 20°$ with an accuracy of $\pm 3\times 10^{-4}$. Phase shifts are deduced from an analysis of the cross-section and polarization data. The spin-orbit and tensor $P$-wave phase-shift combinations are determined in a model-independent way to be $\Delta_{LS} = 0.139°$ $\pm 0.31°$ and $\Delta_T = -0.488° \pm 0.023°$.

The study of the proton-proton interaction is one of the most fundamental problems in nuclear physics. In the low-energy region ($E_p \leq 10$ MeV), where the $S$-wave scattering is dominant, most efforts have been made to measure the differential cross section with high accuracy.[1-6] In this energy region the $P$-wave contribution to proton-proton scattering is small and comes mainly from the interference of Coulomb and nuclear amplitudes. It has been shown by Sher, Signell, and Heller[7] that the $S$-wave phase shift and the central $P$-wave phase-shift combination $\Delta_c = \frac{1}{9}[\delta(^3P_0) + 3\delta(^3P_1) + 5\delta(^3P_2)]$ can be extracted from low-energy differential-cross-section data. For a determination of the noncentral $P$-wave phase-shift combinations $\Delta_T = \frac{5}{72}[-2\delta(^3P_0) + 3\delta(^3P_1) - \delta(^3P_2)]$ and $\Delta_{LS} = \frac{1}{12}[-2\delta(^3P_0) - 3\delta(^3P_1) + 5\delta(^3P_2)]$, additional polarization measurements are necessary. Recent high-precision analyzing-power measurements have been performed at 10 MeV (Ref. 8) and 16 MeV.[9] It was the aim of the present investigation to continue these measurements to lower energies and to extract the $P$-wave splitting and hence the noncentral $P$-wave phase-shift combinations $\Delta_T$ and $\Delta_{LS}$ in an unambiguous way. In this Letter we present an analyzing-power measurement at 6.14 MeV accurate to $\pm 3 \times 10^{-4}$.

The experiment was performed with the polarized proton beam of the Universität Erlangen Lamb-shift source and the 6-MV model EN tandem accelerator. The beam polarization (60–65%) was monitored continuously with a $^4$He polarimeter mounted behind the Faraday cup. Since a measurement at extreme forward angles was in-

TABLE I. Experimental values of the analyzing power for proton-proton scattering at 6.141 MeV.

| $\theta_{\rm c.m.}$ (deg) | $10^4 A(\theta)$ |
|---|---|
| 15 | $-5.0 \pm 4.0$ |
| 20 | $-9.1 \pm 2.9$ |
| 25 | $-9.1 \pm 2.7$ |
| 30 | $-9.4 \pm 3.3$ |
| 35 | $-6.4 \pm 2.8$ |
| 40 | $-3.3 \pm 3.6$ |