stress due to interactions $\tilde{S}$ have diagonal components S_1, S_2, and S_3. Then the difference in strain energy between the two orientations is $2\xi = (2S_1 - S_2 - S_3) - (-S_1 + 2S_2 - S_3) = 3(S_1 - S_2)$. ξ is the parameter which enters Eq. (3).

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Crossover from First-Order to Continuous Phase Transition Induced by Symmetry-Breaking Fields

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The crossover from first order to continuous transition induced by a symmetry-breaking field g in an $n = 2$ cubic model, for which a stable fixed point is not accessible, is studied. It is shown that unlike previously studied cases, the (g, T) phase diagram is rather complicated, exhibiting critical end points, tricritical, and fourth-order critical points. It is suggested that this phase diagram be studied experimentally in ferroelectric $Tb_2(MoO_4)_3$ by applying uniaxial and shear stresses.

It has recently been shown that a variety of systems, which are predicted to exhibit continuous phase transitions within mean-field theory, yield first-order transitions due to critical fluctuations.[1-7] Within the renormalization-group approach, this may occur either when the appropriate model does not possess a stable fixed point[1-4] or when the stable fixed point is not physically accessible.[5,6] However, by applying a symmetry-breaking field g, the dimensionality of the order parameter is reduced. The system may then flow to a stable fixed point, and a continuous transition is restored.[8-11] This situation has been observed experimentally[12] in MnO and more recently[13] in $RbCaF_3$. These systems exhibit a first-order phase transition which becomes second order when a sufficiently strong uniaxial stress is applied.

The (g, T) phase diagram associated with various models has been studied by renormalization-group techniques,[9] perturbation theory,[9] high-temperature expansions,[11] and Monte Carlo calculations.[11] The models which were studied were found to exhibit a relatively simple phase diagram displaying a phase transition line with a tricritical point at a finite, nonzero g [see Fig. 1(a)]. In the present Letter, we show that in certain cases, depending upon the symmetry of the system and the field g, the phase diagram is more complicated, as shown in Figs. 1(b) and 2. These phase diagrams exhibit critical end points, tricritical, and fourth-order critical points. The calculation is performed for an $n = 2$ component vector model with cubic anisotropy, which is the appropriate model for the ferroelectric transition in tetragonal[14] $Tb_2(MoO_4)_3$. It is predicted that the phase diagram of Fig. 2 should be observed experimentally by applying uniaxial and shear stresses in various directions in the x-y plane. We believe that similar phase diagrams should be observed in some of the physical systems which do not possess a stable fixed point, such as[1-4] UO_2, MnO, Cr, and Eu, by applying a magnetic field or a uniaxial stress in certain directions. We shall discuss this problem in a future publication.[15]

Consider an $n = 2$ component cubic model described by the following Landau-Ginzburg-Wilson (LGW) Hamiltonian:

$$H = \int \mathcal{H} d^d x, \tag{1a}$$

$$\mathcal{H} = -\tfrac{1}{2} r(\varphi_1^2 + \varphi_2^2) - \tfrac{1}{2}[(\nabla\varphi_1)^2 + (\nabla\varphi_2)^2] - u(\varphi_1^4 + \varphi_2^4) - v\varphi_1^2\varphi_2^2. \tag{1b}$$

For stability of the free energy, we require $u > 0$ and $2u + v > 0$. The critical behavior associated with this model has been studied by several authors.[16] It has been shown that in $d = 4 - \epsilon$ dimen-

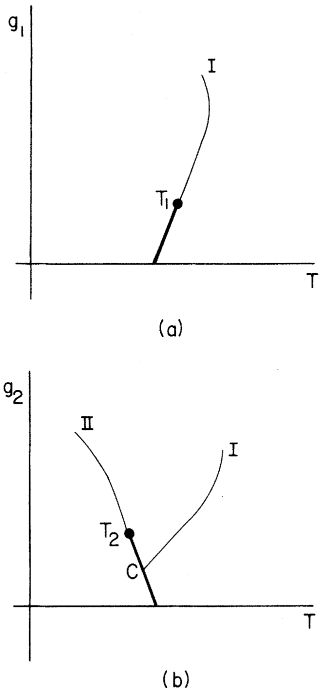

FIG. 1. Schematic (g,T) phase diagrams associated with the $n=2$ cubic model which lies in region (a) with symmetry-breaking fields g_1 and g_2 defined by Eqs. (3) and (4). Thin lines represent continuous transitions, thick lines represent first-order transitions, T_1 and T_2 are tricritical points, and C is a critical end point.

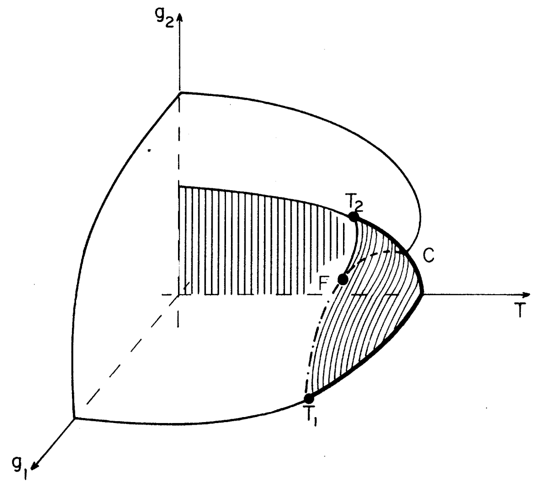

FIG. 2. Schematic (g_1,g_2,T) phase diagram. Thin lines are continuous transitions; thick lines and shaded areas are first-order transitions; dash-dotted lines are tricritical points; and dashed lines are critical end points. T_1 and T_2 are tricritical points, C is a critical end point, and F is a fourth-order critical point. The line T_2F is the wing critical line associated with the tricritical point T_2. The critical lines in the g_1-T, g_2-T, and g_1-g_2 planes and the curve T_1FC form a boundary of a critical surface.

sions ($\epsilon>0$) the isotropic fixed point ($v^*=2u^*$) is stable. However, by examining the flow diagram of this model on the critical manifold it is discovered that, although the model possesses a stable fixed point, there are two regions in the (u,v) plane satisfying $u>0$ and $2u+v>0$ which lie outside its domain of attraction (see, e.g., Refs. 5, 6, and 9). The two regions are given by

$$\text{(a) } v>6u>0 \text{ and (b) } 0>v>-2u. \tag{2}$$

Therefore, if the initial physical Hamiltonian lies in one of these regions, the stable fixed point is not accessible, and the transition is expected to be first order.[5,6] The effect of a symmetry-breaking term

$$g_1(\varphi_1^2-\varphi_2^2) \tag{3}$$

on the transition has recently been studied,[9] if we assume that the initial Hamiltonian lies in region (a). It has been found that the (g_1,T) phase diagram exhibits a tricritical point, as shown in Fig. 1(a). Here we analyze the effect of a symmetry-breaking field

$$g_2\varphi_1\varphi_2. \tag{4}$$

It is readily seen, by applying a 45° rotation in the (φ_1,φ_2) plane, that this problem is equivalent to one which is described by the same Hamiltonian (1) but which lies in region (b) with a symmetry-breaking field g_1. We therefore consider the Hamiltonian

$$\mathcal{H}=-\tfrac{1}{2}r_1\varphi_1^2-\tfrac{1}{2}r_2\varphi_2^2-\tfrac{1}{2}[(\nabla\varphi_1)^2+(\nabla\varphi_2)^2]-u(\varphi_1^4+\varphi_2^4)-v\varphi_1^2\varphi_2^2, \tag{5}$$

with $r_1=r-g$, $r_2=r+g$, and $-2u<v<0$. We analyze the phase diagram associated with this model in the limit of large symmetry-breaking field $g>1$ and for $v,u\ll 1$, $v+2u\ll O(v,u)$. For $g\gtrsim 1$ there exists a critical line I defined by $r_1\simeq O(u,v)$ which separates the disordered phase $\langle\varphi_1\rangle=\langle\varphi_2\rangle=0$ from an ordered phase in which $\langle\varphi_1\rangle\neq 0$ and $\langle\varphi_2\rangle=0$. Inside the ordered phase, there exists a transition line II below which $\langle\varphi_2\rangle$ becomes nonzero. To study this phase transition we define a shift in the order parameter φ_1:

$$\varphi_1\equiv M+\sigma,\quad M^2=r_1/4u. \tag{6}$$

In terms of σ and φ_2 the Hamiltonian (5) takes the form

$$\mathcal{H} = -\tfrac{1}{2}\bar{r}_1\sigma^2 - \tfrac{1}{2}\bar{r}_2\varphi_2^2 - \tfrac{1}{2}[(\nabla\sigma)^2 + (\nabla\varphi_2)^2] - w_1\sigma^3 - w_2\sigma\varphi_2^2 - u(\sigma^4 + \varphi_2^4) - v\sigma^2\varphi_2^2, \quad (7)$$

where $\bar{r}_1 = 2|r_1|$, $\bar{r}_2 = r_2 = 2vM^2$, $w_1 = 4uM$, and $w_2 = 2vM$. Far from the transition to the disordered phase, i.e., for $r_1 < -1$, one can integrate over the σ variable and obtain an effective Ising-like Hamiltonian

$$\mathcal{H}_{\rm eff} = -\tfrac{1}{2}\bar{r}\,\varphi_2^2 - \tfrac{1}{2}(\nabla\varphi_2)^2 - u_4\varphi_2^4 - u_6\varphi_2^6 - \ldots. \quad (8)$$

The parameters which appear in this Hamiltonian can be calculated with use of a diagrammatic expansion in u and v. We find that to second order in u and v the parameter u_4 is given by

$$u_4 = u - (1/4u)v^2 - 4B(\bar{r}_1)v^2 + O(v^3, u^3), \quad (9a)$$

where

$$B(\bar{r}_1) = \int_{0<|q|<1} \frac{1}{(\bar{r}_1 + q^2)^2}\frac{d^dq}{(2\pi)^d}. \quad (9b)$$

By calculating the appropriate diagrams for $\bar{r}$ and u_6, one obtains $\bar{r} = \bar{r}_2 + O(u,v)$, and $u_6 \simeq O(u^3, v^3) > 0$. In three or more dimensions the effective Hamiltonian (8) yields a continuous transition for $u_4 \gtrsim 0$, a first-order transition for $u_4 \lesssim 0$ (with $u_6 > 0$) and a tricritical point at $\bar{r} \simeq O(u_6)$ and $u_4 \simeq O(u_6)$. The tricritical point can, therefore, be located to leading order in u and v by solving the equations $\bar{r}_2(r,g,u,v) = u_4(r,g,u,v) = 0$. The integral $B(\bar{r}_1)$ is a decreasing function of $\bar{r}_1$, and it approaches zero as $\bar{r}_1 \to \infty$. Therefore, for large $\bar{r}_1$ one has $u_4 > 0$, and the transition is continuous. However, as $\bar{r}_1$ decreases, u_4 changes sign and the transition becomes first order. The system hence exhibits a tricritical point at $\bar{r}_1 = \bar{r}_{1,t}$ given by

$$B(\bar{r}_{1,t}) = \tfrac{1}{16}(2 - v/u)(2u + v/v^2). \quad (10)$$

Since the expression is valid for $\bar{r}_{1,t} \gtrsim 1$ [with $B(\bar{r}_{1,t}) \lesssim O(1)$], the existence of the tricritical point has been established only for $0 < 2u + v \lesssim v^2$. However, the same result is expected to be valid even for $2u + v > O(v^2)$ (with $v < 0$), since by applying renormalization-group transformation in $d = 4 - \epsilon$ dimensions, the Hamiltonian flows to the region $0 < 2u + v \lesssim O(v^2)$, where Eq. (10) can be satisfied. A similar analysis for the critical line I shows that no tricritical point exists on this line in the limit of large symmetry-breaking field g. This suggests that the critical line I should terminate in a critical end point[17] as shown in Fig. 1(b).

In order to analyze the phase diagram in the three-dimensional space (g_1, g_2, T), we first outline the calculation which shows that there exists a fourth-order critical point at nonzero fields g_1 and g_2. The various thermodynamic surfaces which appear in the (g_1, T) and (g_2, T) planes can then be connected in a simple way to yield the phase diagram of Fig. 2. We consider a Hamiltonian which lies in region (a). The phase diagram associated with region (b) is obtained by interchanging g_1 and g_2 in Fig. 2. To verify the existence of a fourth-order critical point, we first perform a rotation in the (φ_1, φ_2) plane so as to diagonalize the quadratic term. The following Hamiltonian is obtained:

$$\mathcal{H} = -\tfrac{1}{2}\tilde{r}_1\psi_1^2 - \tfrac{1}{2}\tilde{r}_2\psi_2^2 - \tfrac{1}{2}[(\nabla\psi_1)^2 + (\nabla\psi_2)^2] - \tilde{u}(\psi_1^4 + \psi_2^4) - \tilde{v}\psi_1^2\psi_2^2 - \tilde{w}\psi_1\psi_2(\psi_1^2 - \psi_2^2), \quad (11)$$

where ψ_1 and ψ_2 are the rotated φ's, and the parameters $\tilde{r}_1$, $\tilde{r}_2$, $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ are functions of r, g_1, g_2, u, and v. Assuming large symmetry-breaking fields, we take $\tilde{r}_1 \simeq 0$ and $\tilde{r}_2 \simeq 1$. Integrating over the ψ_2 variable one obtains an effective, Ising-like Hamiltonian

$$\mathcal{H}_{\rm eff} = -\tfrac{1}{2}\tilde{r}\,\psi_1^2 - \tfrac{1}{2}(\nabla\psi_1)^2 - \tilde{u}_4\psi_1^4 - \tilde{u}_6\psi_1^6 - \tilde{u}_8\psi_1^8 - \ldots, \quad (12)$$

where $\tilde{r}$, $\tilde{u}_4$, $\tilde{u}_6$, and $\tilde{u}_8$ can be calculated diagrammatically for small u and v. To leading order in u and v, the fourth-order critical point is defined by the equations

$$\tilde{r}(r, g_1, g_2, u, v) = \tilde{u}_4(r, g_1, g_2, u, v) = \tilde{u}_6(r, g_1, g_2, u, v) = 0,$$

with $\tilde{u}_8 > 0$. We have solved these equations, and thus demonstrated the existence of a fourth-order critical point. This solution will be discussed in more detail in a future publication.[15]

The model described in this paper is directly applicable to the phase transition in[14] $Tb_2(MoO_4)_3$. This is a tetragonal crystal which exhibits a first-order ferroelectric transition associated with a zone-boundary mode $\vec{q} = [\tfrac{1}{2}, \tfrac{1}{2}, 0]$. The transition is described by the LGW Hamiltonian[14]

$$\mathcal{H} = -\tfrac{1}{2}r(\varphi_1^2 + \varphi_2^2) - \tfrac{1}{2}[(\nabla\varphi_1)^2 + (\nabla\varphi_2)^2] - u(\varphi_1^4 + \varphi_2^4) - v\varphi_1^2\varphi_2^2 - w\varphi_1\varphi_2(\varphi_1^2 - \varphi_2^2) + O(\varphi^6). \quad (13)$$

This Hamiltonian possesses an extra fourth-order term, w, which does not appear in the Hamiltonian (1). However, this term is a redundant variable. By applying an appropriate rotation in the (φ_1, φ_2) plane, the Hamiltonian (13) can be transformed into a model of the form given by Eq. (1). By applying uniaxial and shear stresses along various direction in the x-y plane, the fields g_1 and g_2 are realized and the (g_1, g_2, T) phase diagram can be mapped.

The analysis presented in this paper can be easily extended to the case of the $n=3$ cubic $RbCaF_3$, and the phase diagram (2) is expected to be realized in this crystal. It is also expected that similar phase diagrams should also occur in systems with no stable fixed point, such as UO_2, MnO, Cr, and Eu. This can be achieved by applying a symmetry-breaking field which favors an ordering different from the one favored by the fourth-order anisotropic terms.[15]

In summary, we have demonstrated that the crossover from first order to continuous phase transition induced by symmetry-breaking fields can lead to quite complicated and interesting phase diagrams. We suggest that these phase diagrams be tested experimentally in real physcial systems.

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Gauge Wheel of Superfluid ^3He

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A change in the phase of the order parameter of ^3He-A can be undone by a subsequent rotation. We investigate the dynamical consequence of this broken relative gauge and rotational symmetry. In particular a wheel rotated in the liquid acts as a "gauge transformer" driving a superflow. Such experiments provide a very direct probe of this unusual feature of the order parameter, at the same time measuring the orbital quantum number of the pairs.

The defining property of a superfluid is the broken gauge symmetry; that is, the phase of a wave function becomes a significant and macroscopic variable. The Anderson-Brinkman-Morel (ABM) state, generally accepted to describe the ^3He-A phase,[1] breaks this gauge symmetry in a

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