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Cosmological Constraints on Superweak Particles

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Previous work has used the primordial abundance of ^{4}He to infer limits on the number of neutrinos with full-strength neutral-current weak interactions. By accounting for the quark-gluon constituents of hadrons, we extend the analysis to earlier times and higher temperatures and densities and, therefore, to considerably weaker interactions. The maximum number of new, superweakly interacting, light ($\lesssim$ MeV) particles is between $\sim$1 and $\sim$20.

Recent work has emphasized the value of using the primordial abundance of ^{4}He to provide a constraint on the number of types (flavors) of light ($\lesssim$1 MeV), stable or almost stable ($\tau \gtrsim 1$ sec) neutral leptons.[1-3] The most recent analysis by Yang *et al.*[4] has limited the number of two-component neutrinos with full-strength, neutral-current weak interactions to $\lesssim$3. This result has several striking consequences of importance for unified theories. For example, if the "usual" left-handed neutrinos (ν_e, ν_μ, ν_τ) have right-handed counterparts, the right-handed neutrinos cannot have full-strength, neutral-current weak interactions. In addition, this limit suggests that there are no further lepton flavors beyond e, μ, and τ; if there is a connection between quark and lepton doublets, then there may be no new quarks beyond u, d, s, c, t, and b. The constraint which emerges from the previous analysis does not apply to new particles whose interaction strength is considerably weaker than that given by the Weinberg-Salam theory. It is therefore of importance to extend the analysis to include light fermions (e.g., neutrinos, gravitinos) and/or bosons (e.g., gravitons) of arbitrary interaction strength. That extension is the subject of this Letter.

Let us first recall the connection between the number of new light particles and the primordial abundance of ^{4}He. The abundance of ^{4}He emerging from big-bang nucleosynthesis is most sensitive to the competition between the expansion rate of the Universe (t^{-1}) and the rates of the standard, charged-current, weak interactions such as $e^- + p \rightleftarrows n + \nu_e$, $e^+ + n \rightleftarrows p + \bar{\nu}_e$. At early times the expansion rate and the total mass-energy density are related by $t^{-1} \propto \rho^{1/2}$. Furthermore, since the early Universe is radiation dominated, we have $\rho \propto T^4$; the proportionality constant in this relation depends on, and increases with, the number of species of relativistic particles. Thus, the more types of light particles present, the faster the Universe expands, and the less time there is available for the usual Fermi processes listed above to convert neutrons into protons prior to nucleosynthesis at $T \approx 10^9$ K. Since virtually all neutrons present are incorporated in ^{4}He (as a result of the gap at mass 5), more neutrons means more ^{4}He (for a detailed discussion see Ref. 4 and the review by Schramm and Wagoner[5]).

At the time the neutron-to-proton ratio is being established prior to nucleosynthesis ($T \gtrsim 10^{10}$ K), the neutrinos which have full-strength, neutral-current weak interactions are in equilibrium with the photon background. The contribution to the total density due to such neutrinos depends on the total number of spin states. In contrast, particles which interact more weakly will have decoupled earlier and are no longer in equilibrium with the photon background. The contribution to the energy density due to such particles now depends, in addition to the number of spin states, on the fourth power of the ratio of their temperature to the photon temperature. Therefore, the decoupled particles contribute less to the energy density than they would have, had they remained in equilibrium.

Before proceeding to the analysis, note that the primordial abundance of ^{4}He is a weak function of the nucleon density (increasing with increasing nucleon density). A reasonable lower limit to the nucleon density is[6] $\rho_N \gtrsim 10^{-31}$ g cm^{-3}. With this density and with three two-component neutrinos (ν_e, ν_μ, ν_τ), the predicted ^{4}He abundance is ≈ 0.25.[4] In Yang *et al.*[4] it was concluded that observations suggest that the primordial abundance was $\lesssim 0.25$. Thus, any increase in the nucleon density and/or the number of neutrinos will lead to excess ^{4}He. We now proceed to make this result more quantitative.

In the early Universe, relativistic particles dominate the total energy density. During the expansion, as the temperature drops, more and more species become nonrelativistic ($T < m$), and their disappearance heats the remaining, interacting, relativistic particles. Those particles which have decoupled because of the weakness of their interactions do not share this energy; their contribution to the total energy density, relative to that of the interacting particles, decreases. Still, such decoupled particles may have some small effect on the physics of the early Universe such as, for example, in the primordial abundance of ^{4}He which is a sensitive probe of the early expansion rate.[1-5]

Consider, first, those relativistic particles ($m < T$) which, through interactions, are in equilibrium. Their contribution to the density is

$$\rho(T) = \tfrac{1}{2} g(T) \rho_\gamma(T); \quad g = g_B + \tfrac{7}{8} g_F. \tag{1}$$

In Eq. (1), $\rho_\gamma = aT^4$ is the photon density and g_B (g_F) is the number of boson (fermion) spin states. The entropy in these interacting particles, in a specific but arbitrary comoving volume V, is

$$S \sim (\rho/T) V \sim g(\rho_\gamma/T) V \sim g(T) N_\gamma(T). \tag{2}$$

When the gas is heated, the number of photons (and other interacting particles) in V increases; conservation of entropy shows how to relate the number of photons at different times: $N_{\gamma 1}/N_{\gamma 2} = g_2/g_1$.

Next, consider particles, A, which have decoupled at some temperature T_d [i.e., $(n_A \langle \sigma_A v \rangle t)_{T_d} \approx 1$]. As the Universe expands, $T_A \sim V^{-1/3}$ and, for $T_A < T_d$, $N_A =$ const. In contrast, N_γ is increasing as a result of the heating each time that $T < m_i$; thus, below T_d, $T_\gamma \geq T_A$. Suppose that $T_i = m_i (\leq T_d)$ is the lowest temperature at which $T_A = T_\gamma$; then for $T_A < T_i$,

$$\left(\frac{T_A}{T_\gamma}\right)^3 = \frac{g(T_\gamma)}{g(T_i)} = \frac{N_\gamma(T_i)}{N_\gamma(T_\gamma)}. \tag{3}$$

For $T \geq T_i$, $T_A = T_\gamma$ and $N_\gamma(T_d) = N_\gamma(T_i)$. For example, neutrinos with full-strength neutral-current weak interactions (ν_e, ν_μ, ν_τ) decouple when $T_d \approx 1$ MeV; the gas is next heated by the annihilation of $e^\pm$ pairs below $T_i = m_e$.

At any epoch, both interacting and decoupled particles contribute to the total density:

$$\rho_T = \rho + \rho_A = \tfrac{1}{2}(g + g_A)\rho_\gamma = \xi^2 \rho, \tag{4}$$

where

$$g_A = \sum_B g_{AB} \left(\frac{T_{AB}}{T_\gamma}\right)^4 + \frac{7}{8} \sum_F g_{AF} \left(\frac{T_{AF}}{T_\gamma}\right)^4, \tag{5a}$$

$$\xi^2 = \rho_T/\rho = 1 + g_A(T)/g(T). \tag{5b}$$

For the radiation-dominated early Universe, the age-versus-temperature relation is

$$t\, T_{\mathrm{MeV}}^2 = (2.42 \text{ sec})(\xi^2 g)^{-1/2}. \tag{6}$$

As an illustration of the application of these results, consider the "ordinary" neutrinos: ν_e, ν_μ, ν_τ. Suppose that there are no other decoupled particles present and define $T_0 \ll T_i = m_e$ and T

$\geq T_d \approx 1$ MeV. Then

$$g_0 = g_\gamma = 2,\ g_{A0} = \frac{7}{8} g_\nu \left(\frac{T_\nu}{T_\gamma}\right)_0^4 = \frac{21}{4}\left(\frac{T_\nu}{T_\gamma}\right)_0^4, \tag{7a}$$

$$g_i = g_\gamma + \tfrac{7}{8} g_e = \tfrac{11}{2},\quad g_{Ai} = \tfrac{7}{8} g_\nu = \tfrac{21}{4}, \tag{7b}$$

$$g(T) = g_\gamma + \tfrac{7}{8}(g_e + g_\nu) = \tfrac{43}{4},\quad g_A(T) = 0. \tag{7c}$$

Notice that for $T \geq T_d$, $g_A = 0$ because all particles are assumed to be in equilibrium. From Eqs. (2)–(6) we find that

$$N_{\gamma 0} = (g_i/g_0) N_{\gamma i} = \tfrac{11}{4} N_{\gamma i} = \tfrac{11}{4} N_{\gamma d} = \tfrac{11}{43} g(T) N_\gamma(T), \tag{8a}$$

$$(T_\nu/T_\gamma)_0 = (N_{\gamma i}/N_{\gamma 0})^{1/3} = (\tfrac{4}{11})^{1/3},\quad g_{A0} = \tfrac{21}{4}(\tfrac{4}{11})^{4/3} = 1.36, \tag{8b}$$

$$[t(\mathrm{sec})\, T_{\mathrm{MeV}}^2]_0 = 1.32,\quad [t(\mathrm{sec})\, T_{\mathrm{MeV}}^2]_{>m_e} = 0.74. \tag{8c}$$

Now, suppose that there are additional particles, A, which have decoupled earlier ($T_d > 1$ MeV). To repeat the above analysis, we need to count the number of spin states [$g(T)$]; the particle physics simplifies this task considerably. Early on, the density is so high that hadrons overlap and the Universe is filled with quarks and gluons rather than the multitude of mesons, baryons, and resonances.

In another paper[7] we consider, in some detail, the quark-hadron transition; here, we present a brief, qualitative analysis. At relatively low temperatures ($T \gtrsim m_\pi$) and densities there is an ideal gas of the few, lightest hadrons (mostly pions); at much higher temperatures ($T \lesssim 1$ GeV) and densities there is an ideal gas of quarks and gluons. In between there is a nonideal gas of strongly interacting particles. Wagoner and Steigman[8] have shown that the interaction energy is comparable to the thermal energy ($\sim T$) for $T \lesssim 0.4$ GeV; a lower limit to the quark-hadron transition temperature (T_c) may be obtained as follows. For $T \gtrsim 0.2$ GeV, the density of pions and other hadrons is so large that they overlap and the quark constituents are no longer confined to specific hadrons. In agreement with our more detailed estimate[7] we find $0.2 \lesssim T_c \lesssim 0.4$ GeV. Our subsequent results do not depend crucially on the precise value of T_c nor on the choice of quark masses. For constituent masses,[9] we have $m_u \approx m_d \approx T_c \lesssim m_s$, whereas for current-algebra masses[10], $m_u < m_d \ll m_s \lesssim T_c$. Only for a decoupling temperature in the vicinity of T_c do we need to know T_c and m_s with some precision; for $T_d \ll T_c$ and, for $T_d \gg T_c$, our results are unchanged.[11]

In addition to the photon, eight massless gluons, and $W^\pm$, Z^0, $\pi^\pm$, and π^0 we include three families of color-triplet quarks and leptons (i.e., u, d, e, and ν_e; ...). Our results are summarized in Table I where we present, as a function of the decoupling temperature, the photon ratio $N_{\gamma 0}/N_{\gamma d} = \frac{11}{43} g(<T_d)$ and the age-temperature relation [see Eq. (6)].

The primordial abundance of ^{4}He depends sensitively on the expansion rate in the vicinity of $T \approx 1$ MeV.[1-5] For $T \approx 1$ MeV, $g = \frac{43}{4}$ and the "speed-up" parameter is [see Eqs. (4)–(6)]

$$\xi^2 = [t(T)/t'(T)]^2 = \rho_T/\rho = 1 + \frac{7}{86}\sum_{\mathrm{F}} g_{A\mathrm{F}}\left(\frac{T_{A\mathrm{F}}}{T}\right)^4 + \frac{8}{86}\sum_{\mathrm{B}} g_{A\mathrm{B}}\left(\frac{T_{A\mathrm{B}}}{T}\right)^4. \tag{9}$$

The ratio of temperatures follows from Eqs. (3) and (8a):

$$\left(\frac{T_A}{T}\right)_{\mathrm{nucl}} = \left(\frac{11}{4}\right)^{1/3}\left(\frac{T_A}{T}\right)_0 = \left[\frac{11}{4}\left(\frac{N_{\gamma d}}{N_{\gamma 0}}\right)\right]^{1/3}. \tag{10}$$

With the requirement that the ^{4}He abundance be $Y \lesssim 0.25$, Yang *et al.*[4] find that $1 \leq \xi \leq 1.074$.

Consider, then, ΔN_ν "new" two-component neutrino "flavors" which interact "superweakly" and have decoupled at $T_d > 1$ MeV. In this case, $g_{A\mathrm{F}}$

TABLE I. Photon numbers, expansion rate, and the permitted number of new neutrinos.

T_d [a]	$N_{\gamma 0}/N_{\gamma d}$ [b]	$t(\mathrm{sec}) T_{\mathrm{MeV}}$ [c]	ΔN_ν [d]
$m_e - m_\mu$	2.75	0.74	0.9
$m_\mu - m_\pi$	3.65	0.64	1.4
$m_\pi - T_c$	4.41	0.58	1.8
$T_c - m_s$	13.1	0.34	7.6
$m_s - m_c$	15.8	0.31	9.7
$m_c - m_\tau$	18.5	0.28	12.0
$m_\tau - m_b$	19.4	0.28	12.8
$m_b - m_t$	22.1	0.26	15.1
$m_t - m_W$	24.8	0.25	17.1
$m_W -$	27.1	0.24	19.9

[a] The decoupling temperature, T_d, may be anywhere in the range indicated. These results are for constituent masses (Ref. 9).

[b] The ratio of photons now to those at decoupling; see Eq. (3).

[c] See Eq. (6).

[d] The maximum permitted number of "new," two-component neutrinos.

$=2\Delta N_\nu$ and $g_{AB}=0$ so that we obtain the constraint

$$\Delta N_\nu \lesssim 0.94[\tfrac{4}{11}(N_{\gamma 0}/N_{\gamma d})]^{4/3}. \quad (11)$$

In Table I we also give the limit to ΔN_ν as a function of the decoupling temperature.

If the "new" neutrinos interact superweakly because they couple to a heavier W' ($m_{W'} > m_W$), the decoupling temperature depends on $m_{W'}$. The cross section for $e^+ + e^- \rightleftarrows \nu' + \bar{\nu}'$ varies as $\sigma' \sim T^2 m_{W'}^{-4}$ and the reaction rate $\Gamma = n_e \langle \sigma' v\rangle \sim T^5 m_{W'}^{-4}$. Since the expansion rate is $t^{-1} \sim T^2$, the decoupling temperature $T_d \sim m_{W'}^{4/3}$. For neutrinos coupled to the W, decoupling is at ≈ 1 MeV so that $T_d\,(\mathrm{MeV}) \approx (m_{W'}/m_W)^{4/3}$.

For example, for $m_{W'} \lesssim 32 m_W$, $T_d \lesssim m_\mu$ and it follows from Table I that at most one "new" two-component neutrino is allowed. Another consequence of the results in Table I is that if the usual left-handed neutrinos have right-handed counterparts, the right-handed neutrinos must decouple when $T_d > T_c \gtrsim 0.2$ GeV; this suggests that $m_{W_R} \gtrsim 53 m_{W_L}$.

Similar constraints follow for other new particles. For example, for gravitinos, $N_A = (\Delta N_\nu)_{\max} \approx 20$ and, for gravitons, $N_A = \frac{7}{8}(\Delta N_\nu)_{\max} \approx 17$.

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CP Noninvariance in the Decays of Heavy Charged Quark Systems

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Within the context of a six-quark model combined with quantum chromodynamics we study the asymmetry in the decay of heavy charged mesons into a definite final state as compared with the charge-conjugated mode. We find that, in decays of mesons involving the b quark, measurable asymmetries may arise. This would present the first evidence for *CP* noninvariance in charged systems.

To date, the observation of *CP* nonconservation[1] has been limited to electrically neutral mesons. Effects in such systems are dominated by particle-antiparticle mixing in their mass and width matrices.[2] A striking prediction of *CP* nonconservation is that the decay rate of a particle into a definite final state can differ from the rate of the antiparticle decaying into the corresponding charge-conjugated state, namely[3] $\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$; of course, the *TCP* theorem guarantees that the total widths are identical.

In this paper, we present, in the context of definite models of *CP* nonconservation and the strong interactions, calculations for such asymmetries involving the decays of heavy charged mesons. We find, that although small, such an

