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Observation of Stationary Vortex Arrays in Rotating Superfluid Helium

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The positions of quantized vortex lines in rotating superfluid helium have been recorded using a photographic technique. The photographs show stationary arrays of vortices. The observed patterns are in good agreement with theoretical predictions.

Since the work of London[1] it has become an accepted notion that superfluidity is a manifestation of quantum mechanics on a macroscopic scale. Pursuing this idea in a quite literal way, Onsager and Feynman[2] tried to deduce the qualitative features of a single macroscopic wave function, $\psi(r)$, which would describe the superfluid state. They concluded that the superfluid velocity v_s was proportional to the gradient of the wave function's phase and that the nodes in $\psi(r)$ marked the position of vortex lines with circulation quantized in units of h/m (h is Planck's constant and m the mass of the helium atom).

This paper reports observation of stationary quantized-vortex-line patterns in rotating He II. These patterns display the nodal structure of the stationary states of $\psi(r)$ and provide a vivid demonstration of the long-range coherence of the superfluid state.

The basic technique[3] to record the line's position utilizes electron bubbles (ions) trapped in the vortex core. This trapped charge is extracted through the fluid's meniscus (where the line meets the free surface) and imaged onto a phosphor screen. The light emanating from this phosphor is conveyed (via coherent fiber optics) to room temperature, amplified in a low-light-level television camera, and recorded on a single frame of a movie film. Figure 1 shows a block diagram of the apparatus and the caption describes the essential points.

Since it takes about 10 sec to charge the vortex lines, we can record the vortex pattern about 6 times each minute. In a typical experiment the steady-state features of a pattern are enhanced by making a multiple exposure of many individual movie frames. This method of photographic signal averaging reduces the transient effects of noise due to the image intensifier's dark current. It also obscures random vortex motion caused by mechanical disturbances.

The sample of superfluid fills a cylindrical bucket of 2 mm diam and 25 mm depth. A small amount (0.8%) of the ^{3}He is added to the ^{4}He to provide some normal fluid damping at the low temperatures required to perform electron optics in the helium vapor. The cylinder's axis is placed near the rotation axis of a rotating dilu-

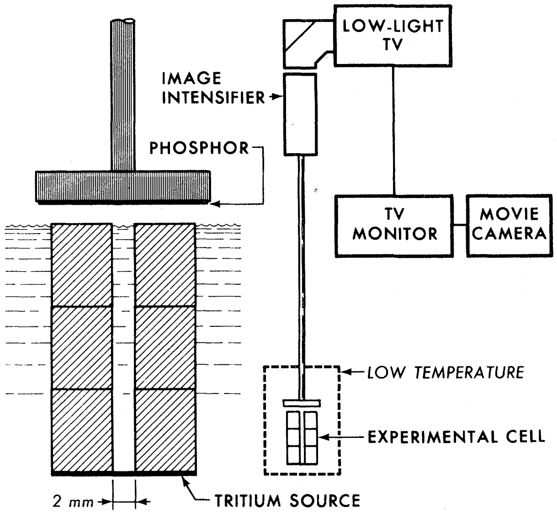

FIG. 1. A block diagram of the apparatus. The cylindrical bucket consists of three sections made by drilling a 2-mm-diam hole in three carbon resistors. A tritium source for production of ions is located at the bottom of the bucket. Voltage differences are applied across each resistor section to produce electric fields for manipulation of the ions. A 700-V potential difference is applied between the phosphor screen and the top of the bucket to accelerate the electrons. An axial magnetic field of 4 kG prevents defocusing of the electrons.

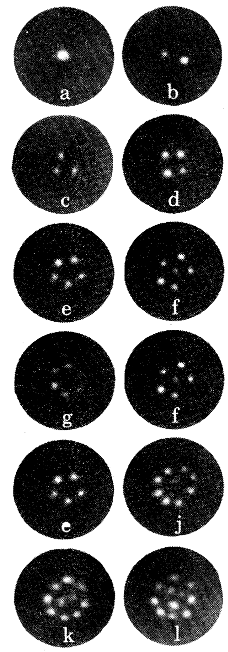

FIG. 2. Photographs of stable vortex arrays. Each photograph is a multiple exposure of 60 consecutive motion-picture frames. The length scale on the photographs is known to an accuracy of approximately 2%. The diameter of the dark circles corresponds to the 2-mm bucket diameter. The arrays were placed symmetrically within the circles. The actual location of the bucket relative to the arrays is not known. The angular velocities were (a) 0.30 $\sec^{-1}$, (b) 0.30 $\sec^{-1}$, (c) 0.40 $\sec^{-1}$, (d) 0.37 $\sec^{-1}$, (e) 0.45 $\sec^{-1}$, (f) 0.47 $\sec^{-1}$, (g) 0.47 $\sec^{-1}$, (h) 0.45 $\sec^{-1}$, (i) 0.86 $\sec^{-1}$, (j) 0.55 $\sec^{-1}$, (k) 0.58 $\sec^{-1}$, and (l) 0.59 $\sec^{-1}$. The two distinct configurations of six vortices appeared alternately at constant angular velocity.

tion refrigerator which maintains sample temperatures near 100 mK.

In previous work[3] with this apparatus the vortex positions could be recorded but stationary states were not observed, presumably because mechanical disturbances perturbed the system. In an attempt to isolate the superfluid from such disturbances several mechanical modifications have been made. Perhaps the most important change is the use of a television system which has eliminated disturbances made in one-shot image collection and has permitted the long-term monitoring of the vortex state.

Several hundred movie sequences have been recorded at various angular velocities up to 1 rad/sec. The earliest films did not usually display any obvious stationary patterns when projected at normal running speeds (16 frames per second). However, the signal-averaging technique often yielded images of patterns displaying a high degree of symmetry[4] (e.g., several concentric circular rings of vortices). After improving the mechanical isolation of the system, stationary patterns became observable in real time and signal averaging is now used only to moderately enhance the stationary features of the pattern.

Figure 2 displays signal-averaged pictures showing the states with vortex numbers $N = 1$ to 11. The theoretical calculations[5] on vortex arrays based on perfectly rectilinear vortices predict patterns which can exist at a given angular velocity ω. Large energy barriers between states will stabilize patterns possessing energy substantially greater than the lowest-lying state. These barriers are so large that one expects transitions between metastable states to almost never occur in a truly isolated system.

The size of a given symmetric configuration depends explicitly on the quantum of circulation. To a good approximation[6] a stationary configuration of N singly quantized vortices at an angular velocity ω will satisfy[7]

$$\sum_{j=1}^{N} r_j^2 = \frac{h}{m} \frac{N(N-1)}{4\pi\omega},$$

where r_j is the distance of the jth vortex from the axis of the cylindrical container. Thus we can, in principle, determine h/m by measuring the positions of the vortices on the photographs. In practice, the measurements are complicated by the fact that the location of the axis of the bucket is not known. We therefore assume that the axis is located at the center of the pattern so that $\sum_{j=1}^{N} r_j^2$ is minimized.

A further complication arises from the fact that several of the patterns shown in Fig. 2 are somewhat distorted from the simple symmetry expected from theory. This distortion may be caused by the vortices being pinned to the sides or bottom of the bucket. No one has been able to quantitatively incorporate such three-dimensional features into vortex configuration calculations. Nevertheless, measurements of those patterns displaying a high degree of symmetry (e.g., $N = 4$ and 5) yield values of h/m which agree with the accepted value within the accuracy of the experiment (approximately 5%). Some of the more distorted patterns give results which differ by as much as 30%. The fact that the length scale of the observed patterns can be deduced from atomic constants demonstrates the macroscopic quantum nature of the He II.

It is interesting that the patterns observed in our long and narrow bucket are so similar to the predictions based on rectlinear (i.e., two-dimensional) vortices. This presumably shows that distortion due to pinning of the lines is not a significant factor at distances far from the pinning centers.

We wish to emphasize that the particular state of the superfluid is determined not by the absolute minimum in the free energy F, but by a combination of past history of the sample and local minima in F. Monotonic acceleration from rest always produces a state with fewer vortices than the absolute equilibrium state. Similarly deceleration can produce states with more vortices than the equilibrium state. For example, in one trial the apparatus was brought from rest to a speed of 0.86 rad/sec at a constant acceleration of 2×10^{-5} rad/sec. During the acceleration, arrays containing one, two, three and finally four vortices successively appeared. Following the acceleration, the angular velocity was held constant for eleven hours. No further transitions were observed even though the equilibrium state is expected to contain 23 vortices at this angular velocity. This highly metastable state might have lasted indefinitely were it not for the near exhaustion of the refrigerator helium supply. Prior to stopping rotation, a perturbation was applied by jarring the cryostat. This resulted in an immediate increase in the number of vortices present within the bucket. The excessive brightness of the signal resulted in a general blurring of the photographs so that the precise number of lines could not be determined.

It is interesting to point out that if quantized vortices exist in superfluid ^{3}He then this vortex photography technique could possibly be used to determine whether the circulation is $h/2m$ as expected for Cooper pairing. In its present form this technique can be used to study vortex dynamics in He II. In particular, it should be possible to directly observe vortex waves involving small numbers of vortices.

In summary, we have recorded arrays of vortices characteristic of stationary states of rotating He II. The observed patterns are well described by hydrodynamic calculations for rectilinear quantized vortices. These patterns provide a striking demonstration of macroscopic quantum coherence in the flowing superfluid.

We wish to thank L. J. Campbell for preprints of his vortex calculations and J. Eisenstein, S. Garret, and G. Swift for many helpful comments. The many contributions of G. A. Williams, in the original design and construction of much of the apparatus are gratefully acknowledged. The low–light-level television camera is generously loaned to us by the Lawrence Livermore Laboratory. One of us (R.E.P.) would like to thank D. F. Brewer and the University of Sussex Low Temperature Group for their hospitality during part of this work. He also thanks the Science Research Council for the award of a Senior Postdoctoral Fellowship. This work was supported by the National Science Foundation (Grant No. DMR 76-82934).

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Spinodal Line and Critical Point of an Acrylamide Gel

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We present direct evidence for the existence of the critical point in an acrylamide gel, the gel consisting of a covalently cross-linked polymer network immersed in an acetone-water mixture. We determined the spinodal line for the gel from the measurements of the scattered light intensity. The spinodal line has a maximum at $T = (10 \pm 1)°C$ and a network volume concentration of $(18 \pm 3)\%$. This corresponds to the critical point.

In 1977, Tanaka, Ishiwata, and Ishimoto reported measurements of the temperature dependence of the intensity and the correlation time of laser light scattered by the concentration fluctuations in an acrylamide gel.[1,2] As the temperature of the gel was lowered, both the intensity and the correlation time of the scattered light increased by a factor of more than 200 and both appeared to diverge at a certain temperature. Based on Flory's formula for the osmotic pressure of a cross-linked gel, they concluded that the temperature of the divergence corresponded to the metastable spinodal temperature, and that the gel would have no critical point. Recently, Tanaka[3] and Fillmore and Tanaka[4] reported the swelling equilibria of a gel which was immersed in acetone-water mixtures. They observed a reversible collapse of the gel network upon changing the acetone concentration or temperature of the system. It was demonstrated that these phenomena could be understood in terms of a mean-field theory based on an extension of Flory's theory. With this theory, Tanaka predicted the existence of the critical point in the gel associated with a phase separation of the covalently cross-linked polymer network and solvent.

In this Letter, we present direct evidence for the existence of the critical point in an acrylamide gel, the gel consisting of a covalently cross-linked polymer network immersed in an acetone-water mixture. Using measurements of the scattered-light intensity, we determined the spinodal temperature at which the network concentrations diverge for various network concentrations of the gel. The network concentrations were varied by adjusting the degree of swelling of the gel. The highly swollen gel corresponds to a low concentration of the network, and the shrunken gel corresponds to a high network concentration. We observed a maximum temperature in the spinodal line which corresponds to the critical point. We also determined the equilibrium concentration of the gel when it is immersed in a large volume of an acetone-water mixture at different temperatures. The combination of both the spinodal line and the swelling equilibrium line gives us the entire phase diagram of the gel. These data are analyzed by the mean-field theory presented previously.[3]

Acrylamide gels were prepared by dissolving 5 g acrylamide, 0.133 g N,N'-methylene-bisacrylamide, 40 mg ammonium persulfate, and 400 μl

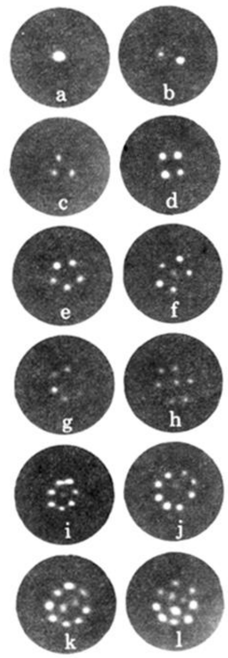

FIG. 2. Photographs of stable vortex arrays. Each photograph is a multiple exposure of 60 consecutive motion-picture frames. The length scale on the photographs is known to an accuracy of approximately 2%. The diameter of the dark circles corresponds to the 2-mm bucket diameter. The arrays were placed symmetrically within the circles. The actual location of the bucket relative to the arrays is not known. The angular volocities were (a) 0.30 sec^{-1}, (b) 0.30 sec^{-1}, (c) 0.40 sec^{-1}, (d) 0.37 sec^{-1}, (e) 0.45 sec^{-1}, (f) 0.47 sec^{-1}, (g) 0.47 sec^{-1}, (h) 0.45 sec^{-1}, (i) 0.86 sec^{-1}, (j) 0.55 sec^{-1}, (k) 0.58 sec^{-1}, and (l) 0.59 sec^{-1}. The two distinct configurations of six vortices appeared alternately at constant angular velocity.