range of 5×10^7 to 2×10^8 cm^{-3}, and assuming that the neutral-hydrogen gas velocity, v_φ^0, in the core of the plasma is in the range $0<v_\varphi^0<v_\varphi/2$, we arrive at 40 msec $<\tau_\varphi^{cx}(0)<300$ msec. Since charge exchange transports ion energy as well as ion momentum, we must also have $\tau_\varphi^{cx}>\tau_{Ei}\approx 20-40$ msec. Thus, charge exchange falls short of explaining the damping by a significant, but not large, factor. The other two processes, ripple damping and perpendicular viscosity, are much slower by comparison. A simple estimate gives confinement times 2×10^3 and 2.5 sec against these two processes, respectively.

More detailed measurements of the radial profiles of the rotation—by utilizing lines of various ions with ionization potentials in the 600–1300-eV range—appears to be feasible by the Doppler-shift method. Such measurements, especially when including the decay of the rotation after the end of the injection, or during intermittent injection, should allow quite detailed interpretation of the local plasma dynamics under various conditions.

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Modification of Plasma Solitons by Resonant Particles

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Experimental and numerical results are compared with new theoretical results describing soliton propagation and deformation in a strongly magnetized, plasma-loaded waveguide.

Experimental results concerning nonlinear waves in plasmas have in several cases demonstrated a striking agreement with propagation characteristics predicted by the Korteweg–de Vries (KdV) equation. Thus Ikezi, Taylor, and Baker[1] have demonstrated soliton collisions and recurrence phenomena in experiments with ion acoustic waves. Propagation of KdV solitons was also demonstrated for electron waves in a magnetized, cylindrical, plasma-loaded waveguide.[2] The overall agreement between experiment and theory for these cases has stimulated the interest for considering even finer details obtained by modifying the KdV equation in various ways. Particular attention has been paid to finding an adequate representation for the Landau damping. We considered the propagation of nonlinear pulses in a strongly magnetized, plasma-loaded waveguide.[2] A kdV equation appropriate for this system and modified to take into account the effect of resonant particles was derived. By means of a recently developed perturbation-theory[3] solutions for this equation were obtained and compared with experimental results.

The experiment was carried out in a cesium plasma produced by surface ionization on a hot (~2000 K) tantalum plate of diameter 3 cm. The 120-cm-long plasma column was confined radially by a magnetic field of 0.4 T and surrounded by a grounded brass cylinder with inner diameter 4 cm. Plasma densities were 10^6–10^7 cm^{-3} and $T_e \sim 0.2$ eV. Pulses were excited by a 30-cm-long brass tube terminating the waveguide. This setup is particularly well suited for exciting large-amplitude pulses.[4,5] Potential variations were detected by a Langmuir probe connected directly to a high-impedance capacitive amplifier (1 MΩ, 2pF). A slot in the waveguide permitted 85-cm axial movement of the probe. A numerical simulation of the experiment was performed using a particle-in-cell (PIC) method. A leap-frog scheme was applied for the movement of 5×10^4 particles. At each step, the electric potential, φ, was calculated from Poisson's equation in a form appropriate for the present problem, $\varphi_{xx} - \varphi/a^2 = e(n-n_0)/\epsilon_0$, with $a \simeq$ (plasma radius)/2.4 and with n_0 the density of the immobile ion background. Only the lowest-order radial eigenmode was considered. The accuracy of the program was checked by calculating the total energy at each time step. Within ~20 plasma periods, $2\pi/\omega_p$, energy was conserved to 3%. The program is described by Turikov.[6]

In the theoretical analysis we employ a modified KdV equation of the form[7]

$$\frac{\partial U}{\partial t} + \left[C_0 + \alpha(\tfrac{3}{2}C_0)U\right]\frac{\partial U}{\partial x} + \beta\frac{\partial^3 U}{\partial x^3} = -\tfrac{1}{2}C_0^3\frac{\partial N_R}{\partial x} \quad (1)$$

with $C_0^2 = \omega_p^2 a^2 + 3v_T^2/2$, $v_T^2 = 2T_e/m$, $U = -e\varphi/m$, $\beta = \frac{1}{2}C_0^3/\omega_p^2$; $\alpha \simeq 0.72$ originates from the expansion in radial eigenmodes.[4,5] The number of resonant particles, N_R, is given by

$$N_R = \frac{1}{n_0}\int_{\mathrm{res}}[f(x,v,t) - f_0(v)]\,dv,$$

where f and f_0 are perturbed and unperturbed electron velocity distribution functions, respectively, while $n_0 = \int f_0\,dv$. The subscript "res" indicates integration over the resonant particles.

In order to investigate the modification caused by N_R of the soliton solution to the ordinary KdV equation we consider the right-hand term of Eq. (1) as a perturbation and apply a recently developed theory[3] to the soliton solution

$$U = U_0(t)[\mathrm{sech}^2([x - x_0(t)]/\delta(t)) + w(x,t)],$$

where the time variation of the amplitude, U_0, and the width, δ, are controlled by the perturbation. The term $w(x,t)$ accounts for a deformation of the soliton. When evaluating N_R we note that there are two characteristic time scales for the problem: (i) a time of resonant interaction $\tau_R = \delta/(2U_0)^{1/2}$ and (ii) an unperturbed soliton time[3] defined as $\tau_s = 8C_0\delta/U_0$.

For $t \ll \tau_R$ we derive[5,8]

$$N_R = (f_0'/n_0)\,\mathrm{P}\int_{-\infty}^{\infty}dx'\,U(x',t)(x'-x)^{-1}, \quad (2)$$

where $f_0' = df_0/dv$ at $v = v_{ph}$, i.e., a term similar to the one derived by Ott and Sudan[8] for a related problem. For $t \gg \tau_R$ we obtain

$$N_R = -\nu\int_0^{(1-\chi(x,t))^{1/2}} d\eta\,[F(\mu + \nu(\eta^2+\chi)^{1/2}) - F(\mu - \nu(\eta^2+\chi)^{1/2})]\,\mathrm{sgn}[x - x_0(t)], \quad (3)$$

where the following dimensionless quantities are introduced: $\nu = (2U_0/v_T^2)^{1/2}$, $\mu = v_{ph}/v_T$, $v_{ph} = C_0 + U_0/2C_0$, $\chi(x,t) = U(x,t)/U_0$, and $F(v/v_T) = f_0(v)v_T/n_0$. In the following we shall assume that $f_0(v)$ is Maxwellian. Rather than present a full derivation of (3) we shall give later a simple, illustrative physical interpretation of one of our main results concerning soliton damping derived from this perturbation term.

We now note[3] that the influence of the perturbation on the nonlinear evolution will be important only for $t > \tau_s$. Since $\tau_R/\tau_s \sim U_0^{1/2}/C_0 \ll 1$, however, we considered only (3). In order to apply the perturbation analysis,[3] we bring Eq. (1) into the standard form $\psi_t - 6\psi\psi_\xi + \psi_{\xi\xi\xi} = \epsilon R[\psi]$, $\epsilon R[\psi] = 2^{-7/3}\omega_p^{4/3}\partial N_R/\partial\xi$. We have used the substitutions $x - C_0 t = \beta^{1/3}\xi$ and $U = -4C_0\beta^{1/3}\psi$. The main results may be summarized as follows: An approximate asymptotic solution may be written as $\psi = \psi_s - 2\kappa^2 w(z,t)$ where $\psi_s = -2\kappa^2 \operatorname{sech}^2 z$ with $z = \kappa = \kappa(t)[\xi - \xi_0(t)]$, where the amplitude varies (in our case, damps) according to

$$\frac{d\kappa}{dt} = -\frac{\epsilon}{4\kappa}\int_{-\infty}^{\infty} R[\psi_s]\operatorname{sech}^2 z\, dz.$$

The function $w(z,t)$ accounts for the deformation of the soliton accompanying the damping. In general, it consists of a plateau with length proportional to t which forms behind the soliton. Behind the plateau, an oscillatory, rapidly damped tail develops. An important parameter for the plateau is

$$q = (4\kappa^5)^{-1}\int_{-\infty}^{\infty} R[\psi_s]\tanh^2 z\, dz.$$

The sign of ϵq gives the sign of the plateau. The *amplitude* of the plateau is given by $w_- = -\frac{1}{2}\epsilon q$. Applying these results to (1) and (4) we find the damping[9]

$$d\nu/dt = (8\mu^4/\tau_s\nu^3)A(\mu,\nu) \tag{4}$$

and the plateau amplitude

$$U_- = (U_0 4\mu^4/\nu^4)[B(\mu,\nu) - A(\mu,\nu)], \tag{5}$$

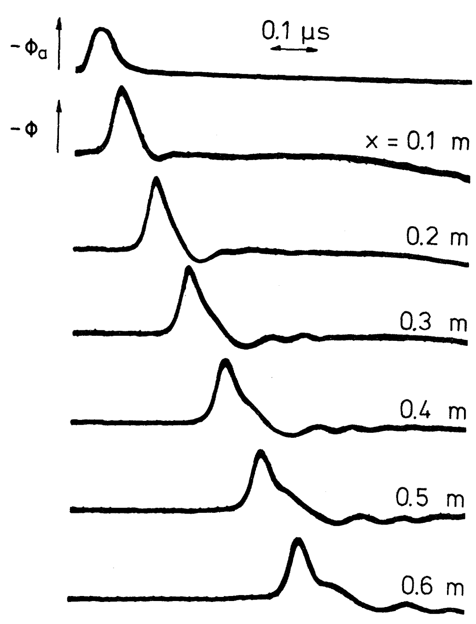

FIG. 1. Oscilloscope traces showing soliton damping and the developments of the plateau and an oscillatory tail. φ_a is the applied potential.

where

$$A(\mu,\nu) = \frac{4}{\nu^2}\left(\int_{\mu}^{\mu+\nu} d\sigma(\sigma-\mu)^2 F(\sigma) + \int_{\mu}^{\mu-\nu} d\sigma(\sigma-\mu)^2 F(\sigma)\right)$$

and

$$B(\mu,\nu) = 2\left(\int_{\mu}^{\mu+\nu} d\sigma F(\sigma) + \int_{\mu}^{\mu-\nu} d\sigma F(\sigma)\right).$$

The remaining quantities were defined in connection with Eq. (4). Note that in our case $\operatorname{sgn}(\epsilon q) = \operatorname{sgn}(\psi_s)$.

Figures 1–2 and 3–4 show experimental and numerical results, respectively. Experimentally (Fig. 1) we observe the development of the plateau with the right sign and oscillatory tail in full qualitative agreement with theoretical predictions. Also the soliton damping (Fig. 2) is in qualitative agreement with theory. In order to increase C_0 (i.e., decrease the number of resonant particles) these measurements were carried out at a plasma density somewhat higher than in the experiment by Saeki *et al.*[4] The electron hole discussed in that work[4] cannot be described by a simple KdV equation as (1) and it is irrelevant in the present connection. In the numerical simulation, where the input parameters are well defined, we are able to make an accurate numerical comparison. Also here we observe the formation of a plateau (Fig. 3) with length proportional to t. The electrons reflected by the soliton are seen on the phase-space diagram. The measured ratio U_-/U_0 is compared with theory in Fig. 4(a). Very small amplitudes are not included because (i) the inherent shot-noise in the PIC model makes amplitude esti-

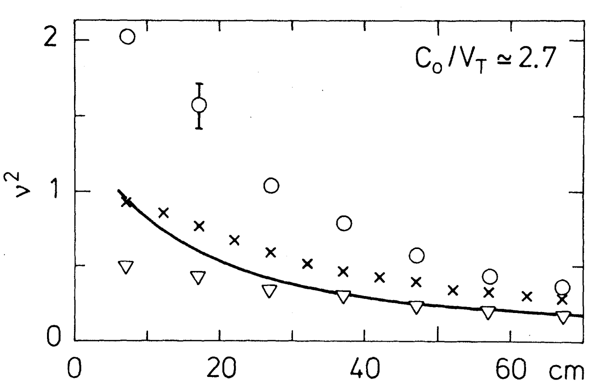

FIG. 2. Soliton damping for different initial amplitudes.

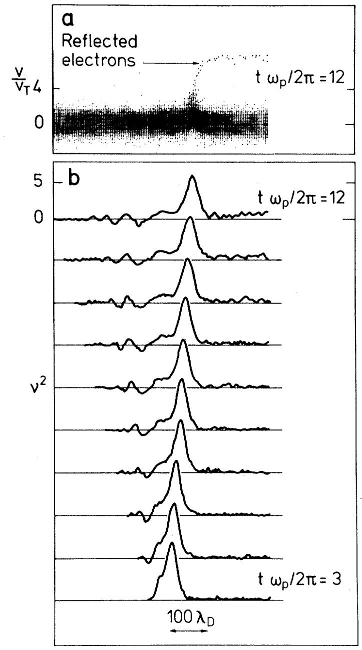

FIG. 3. Numerical results showing the evolution of the perturbed soliton shown in a frame of reference moving with C_0, where $C_0/V_T=4.14$; (a) phase space, (b) normalized potential.

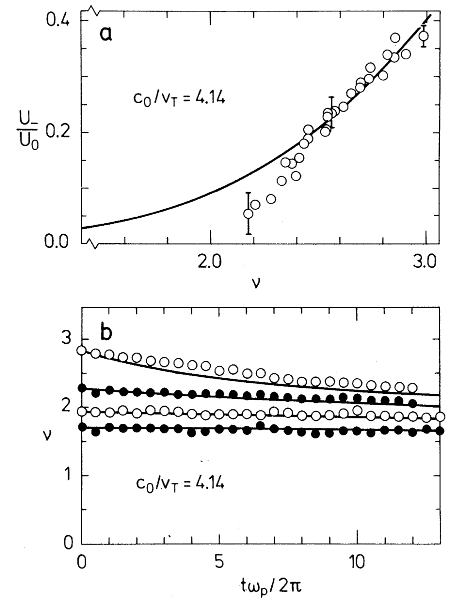

FIG. 4. (a) Relative plateau amplitude U_-/U_0 as a function of normalized soliton amplitude, from the simulations. (b) Soliton damping, for different initial amplitudes. Full lines indicate theoretical dependence.

mates rather uncertain, and (ii) the criterion $t > \tau_s$ becomes unobtainable within the time span of the code. Finally, in Fig. 4(b), we investigate the temporal damping of the soliton. By varying the plasma density in both experiment and simulation we find that when $C_0/v_T \lesssim 2$ the plateau is no longer fully developed[4,5] and appears rather like an evanescent "tail." Obviously the number of reflected particles becomes too large to be correctly described by a perturbation theory. We note, however, that in spite of the disappearing plateau, the damping continues to show good agreement with theory. Krivoruchko *et al.*[10] have observed soliton damping and obtained a theoretical expression valid for very small amplitudes. Such a result can also be obtained from our Eq. (4) by a series expansion of the integrand in $A(\mu, \nu)$.

Our conclusions may be summarized as follows: Experimental results are in full qualitative agreement with theory. In particular, we note that large pulses have a large damping rate, as expected (Fig. 2). We are unable to make a precise numerical comparison with the experimental results since the solitons in our setup move "upstream," towards the ionizing plate, and we are not able to measure the electron velocity distribution function in this direction. This information is essential to the theory. We note, however, that by assuming a Maxwellian distribution with $T_e = 0.2$ eV and determining the absolute soliton amplitude by measuring its Mach number, we find agreement within a factor 2; see full line and crosses, Fig. 2. The numerical simulations (Figs. 3 and 4) give a fully satisfactory quantitative agreement with theoretical results. We may thus conclude that our modified KdV equation together with the applied perturbation analysis is capable of describing even the fine details of the experiment in question.

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[9]The physical contant of this result may be understood as follows: In the first approximation, for small k and T_e, the energy density of Trivelpiece-Gould waves is $\epsilon_0 k_\perp^2\varphi^2$. The total energy of the soliton is thus $W=\epsilon_0 k_\perp C_0(8/3)\varphi_m^{3/2}(m/e)^{1/2}$, where we have used the analytic soliton relation with $\varphi_m\equiv\max\varphi$. Electrons with initial velocity v, reflected by the soliton with velocity u, receive (or give up) an amount of energy $2mu(u-v)$. The flux of such electrons towards the soliton is $|v-u|f_0(v)dv$. Letting φ_m be time dependent we obtain (4) by equating $-dW/dt$ with the rate of net energy gain of all the reflected particles calculated from these expressions. The effect of transient particles can be shown to be negligible for $t>\tau_R$ and for not too strong dampings; see S. M. Krivoruchko, Ya. B. Fainberg, V. D. Shapiro, and V. I. Shevchenko, Zh. Eksp. Teor. Fiz. 67, 2092 (1974) [Sov. Phys. JETP 40, 1039 (1975)].

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Observation of Stationary Vortex Arrays in Rotating Superfluid Helium

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The positions of quantized vortex lines in rotating superfluid helium have been recorded using a photographic technique. The photographs show stationary arrays of vortices. The observed patterns are in good agreement with theoretical predictions.

Since the work of London[1] it has become an accepted notion that superfluidity is a manifestation of quantum mechanics on a macroscopic scale. Pursuing this idea in a quite literal way, Onsager and Feynman[2] tried to deduce the qualitative features of a single macroscopic wave function, $\psi(r)$, which would describe the superfluid state. They concluded that the superfluid velocity v_s was proportional to the gradient of the wave function's phase and that the nodes in $\psi(r)$ marked the position of vortex lines with circulation quantized in units of h/m (h is Planck's constant and m the mass of the helium atom).

This paper reports observation of stationary quantized-vortex-line patterns in rotating He II. These patterns display the nodal structure of the stationary states of $\psi(r)$ and provide a vivid demonstration of the long-range coherence of the superfluid state.

The basic technique[3] to record the line's position utilizes electron bubbles (ions) trapped in the vortex core. This trapped charge is extracted through the fluid's meniscus (where the line meets the free surface) and imaged onto a phosphor screen. The light emanating from this phosphor is conveyed (via coherent fiber optics) to room temperature, amplified in a low-light-level television camera, and recorded on a single frame of a movie film. Figure 1 shows a block diagram of the apparatus and the caption describes the essential points.

Since it takes about 10 sec to charge the vortex lines, we can record the vortex pattern about 6 times each minute. In a typical experiment the steady-state features of a pattern are enhanced by making a multiple exposure of many individual movie frames. This method of photographic signal averaging reduces the transient effects of noise due to the image intensifier's dark current. It also obscures random vortex motion caused by mechanical disturbances.

The sample of superfluid fills a cylindrical bucket of 2 mm diam and 25 mm depth. A small amount (0.8%) of the ^{3}He is added to the ^{4}He to provide some normal fluid damping at the low temperatures required to perform electron optics in the helium vapor. The cylinder's axis is placed near the rotation axis of a rotating dilu-

