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Pair Correlations and the Condensate Fraction in Superfluid ^{4}He

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By applying the method of Hyland, Rowlands, and Cummings to pair-correlation functions obtained from new high-accuracy neutron-diffraction measurements, the condensate fraction, n_0, in superfluid ^{4}He has been determined for seven temperatures in the range $1.00 \leq T \leq 2.15$ K. The results are very well described by the relation $n_0(T) = n_0(0)[1-(T/T_\lambda)^\alpha]$, with $n_0(0) = 0.133 \pm 0.012$ and $\alpha = 6.2 \pm 1.6$.

It is generally believed that the unique properties of superfluid ^{4}He result from the macroscopic occupation of the zero-momentum state, and numerous theoretical calculations[1-7] suggest that the fraction of atoms in this state, n_0, is in the range 0.08 to 0.13 at $T=0$.

In principle, the most direct method for the experimental determination of n_0 is, as proposed by Hohenberg and Platzman,[8] by means of neutron inelastic scattering at large wave-vector transfer, Q, where the dynamic structure factor $S(Q,\omega)$ reflects the momentum distribution of the atoms. In practice, however, the scattering at Q values accessible at present is distorted by a number of effects[9] and attempts[10-14] to determine n_0 by this method have yielded contradictory results with values ranging from 0.02 to 0.17 at $T \simeq 1.2$ K.

An alternative method in which the condensate fraction is determined through its effect on the spatial correlations of the atoms as reflected in the static pair-correlation function $g(r)$ was proposed some years ago by Hyland, Rowlands, and Cummings.[15] In this Letter we report the first critical experimental test of this method and obtain values of n_0 for seven temperatures which give a reasonably complete picture of its temperature dependence. Our values are consistent with the most recent theoretical estimates[5,7] but not with most of the values inferred from the high-Q experiments.[10-14]

The basic hypothesis of Hyland, Rowlands, and Cummings[15] is that the temperature variations of $g(r)$ at large r for superfluid ^{4}He are caused primarily by atoms condensing into the zero-momentum state where they no longer contribute to the spatial correlations of the atoms which give rise to the oscillations in $g(r)$. On the basis of *ad hoc*, although physically resonable, assumptions they then obtain

$$g(r) - 1 = (1-n_0)^2[g^*(r) - 1], \tag{1}$$

where $g(r)$ and n_0 refer to some temperature below T_λ, and $g^*(r)$ refers to a temperature T^* just above T_λ. The quantity $g(r)-1$, which is a measure of the spatial correlations, is thus proportional to $(1-n_0)^2$, the normalized probability that the two atoms are not in the condensate. A necessary condition for the validity of (1) is that $r > r_0$, where r_0 is the value above which the one-particle density matrix effectively attains its asymptotic limit, n_0.

There have been two previous attempts to determine n_0 from Eq. (1). Cummings, Hyland, and Rowlands[15] found from existing x-ray results that $n_0 \simeq 0.1$ at $T = 1.4$ K with an uncertainty of a factor of 2 or 3. Later, Raveché and Mountain[16] used neutron-diffraction results for pressurized liquid ^{4}He to obtain values of 0.1 at $T = 1.86$ K

(with $T_\lambda = 2.135$ K) and 0.02 at $T = 1.94$ K (with $T_\lambda = 2.05$ K). Again the data were not sufficiently accurate to provide a critical test of the theory or to yield very precise values for n_0. Further, their n_0 values probably were somewhat distorted because their T^* values ($\simeq 2.85$ K) were so far above T_λ.

We have recently completed an extensive neutron-diffraction study of liquid ^{4}He at saturated vapor pressure.[17] A full account of this work will be published elsewhere. Here we shall simply remark that the static structure factor $S(Q)$ has been determined for $0.8 \leq Q \leq 10.8$ Å^{-1} at eleven temperatures in the range $1.00 \leq T \leq 4.27$ K with an average statistical precision of 0.8% and with a residual systematic error which is estimated to be less than 1%. The results are in excellent agreement with the very precise x-ray results of Hallock[18] which cover the range $0.133 \leq Q \leq 1.125$ Å^{-1}. From the combined sets of results we have determined $g(r)$ for each temperature by Fourier inversion. Taking $T^* = 2.27$ K, our lowest temperature above T_λ, we then used Eq. (1) to obtain n_0 for each temperature below T_λ.

To be physically meaningful, Eq. (1) must give n_0 values which are independent of r for $r > r_0$ and this implies stringent limitations on the behavior of $g(r)$ which can be tested experimentally. For example, the nodes of $g(r) - 1$ should be independent of temperature below T^*. From Table I we see that for $T \leq 2.27$ K the nodes are indeed constant to within 0.02 Å on the average. The small variations appear to be random and can be attributed to residual experimental error. There is no indication of a systematic trend resulting from the small ($\simeq 0.7\%$) density variation for $T < T^*$. There are, however, definite shifts at higher T where the density varies more rapidly.

TABLE I. Values of r for which $g(r) = 1$.

T (K)	r (Å)				
1.00	2.96	4.53	6.16	7.77	9.34
1.38	2.96	4.53	6.18	7.77	9.44
1.77	2.99	4.52	6.13	7.70	9.38
1.97	2.96	4.52	6.15	7.76	9.34
2.07	2.97	4.52	6.18	7.76	9.35
2.12	2.96	4.53	6.18	7.77	9.38
2.15	2.97	4.55	6.17	7.78	9.37
2.27	2.96	4.53	6.18	7.78	9.41
3.00	2.98	4.56	6.19	7.79	9.45
3.60	2.98	4.59	6.21	7.76	9.39
4.27	2.99	4.60	6.23	7.84	9.26

As illustrated in Fig. 1, the small differences in the positions of the nodes produce systematic variations and spurious singularities in the n_0 curves obtained from Eq. (1). To within the uncertainties caused by these differences, the results for $r \geq 6$ Å are, however, consistent with constant n_0 values. The results for region I suggest that r is not large enough for Eq. (1) to be applicable and hence that $r_0 \geq 6$ Å. Theoretical estimates[2,3,5,7] of r_0 range from 4 to 6 Å. We have, therefore, used only results from regions II and III in determining n_0. The horizontal dashed lines in Fig. 1 indicate the values (0.131 and 0.124) of n_0 for 1.00 K obtained by averaging over the ranges indicated by the lengths of the lines. These ranges are centered approximately on the extrema of $g(r)$ so as to minimize the effect of the differences in the positions of the nodes. The overall average, $n_0 = 0.127$, is listed in Table II and shown in Fig. 2 together with the values for the other temperatures obtained by the same proceudre. At each temperature, regions II and III give essentially the same value for n_0 and these values are not sensitive to the range of r (centered as indicated above) over which the averaging is carried out. The errors are, however, quite sensitive to the range, and the values assigned in Table II simply indicate the precision with which n_0 can be determined from the results in the chosen ranges of r. Any effects of the small density changes for $T < T^*$ (which are ignored by the theory) or of our T^* being 0.1° above T_λ are probably well within the assigned errors.

For an ideal Bose-Einstein gas,[19]

$$n_0(T) = n_0(0)[1 - (T/T_\lambda)^\alpha], \quad (2)$$

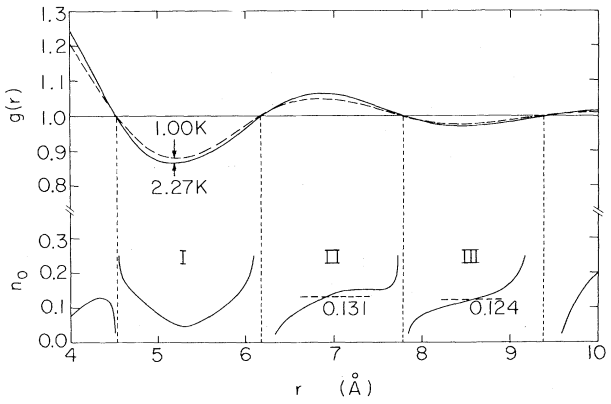

FIG. 1. The pair-correlation function $g(r)$ at 1.00 K and 2.27 K and the corresponding values of n_0 for 1.00 K determined from Eq. (1).

with $n_0(0) = 1$ and $\alpha = \frac{3}{2}$. While this equation has little justification for superfluid ^{4}He, it does provide a convenient means of parametrizing our results and of obtaining an estimate of $n_0(0)$ for comparison with theoretical calculations. A least-squares fit of Eq. (2) to the results of Table II with $T_\lambda = 2.17$ K gives $n_0(0) = 0.133 \pm 0.012$ and $\alpha = 6.2 \pm 1.6$. The corresponding solid curve in Fig. 2 is clearly a good description of our results. The crosses in Fig. 2 show the results of theoretical calculations[1-7] the most recent of which give $n_0(0)$ in the range 0.11 to 0.13 in substantial agreement with our estimate. Our results are not consistent with the work of Chela-Flores[20] who, using Eq. (2), obtained $n_0(0) = 0.023$ and $\alpha = 3$, nor with the work of Hyland and Rowlands[21] who proposed another form for $n_0(T)$ which gave $n_0(0) = 0.1$ but such a rapid drop with increasing T that $n_0(1.2\ \mathrm{K})$ was predicted to be 0.0033.

The asymptotic behavior of $n_0(T)$ as $T \to 0$ is given by[22] $n_0(T) = n_0(0)[1 - (T/T_0)^2]$, where $(k_B T_0)^2 = 12\hbar^3 c\rho/m$. Here c is the sound velocity, ρ the number density, and m the atomic mass so that $T_0 = 7.61$ K. Taking $n_0(0) = 0.133$, this relation gives agreement with the solid curve in Fig. 2 to within 1% for $T \leq 1.3$ K. This strongly suggests that $n_0(T)$ does not vary significantly below 1 K and that the accuracy of our estimate of $n_0(0)$ is not limited by the degree of validity of Eq. (2). Near T_λ one expects[23] that $n_0(T) \propto (T_\lambda - T)^{2\beta}$. Our results do not, however, determine β very precisely and we refrain from quoting a value.

In the method of Hohenberg and Platzman[8] mentioned earlier, the condensate fraction is expected to give rise to a narrow central peak, having integrated intensity n_0, in a scattered-neutron distribution at sufficiently large Q. Such a peak has never been clearly resolved in existing experiments,[10-14] but several values for n_0 (shown as open symbols in Fig. 2) have been inferred from various analyses of the line shapes and linewidths of the distributions. The large discrepancies do not, in our opinion, indicate any basic disagreement among the experiments themselves, but simply reflect differences and inadequacies in the analyses. As discussed in detail by Martel *et al.*,[9] the distributions in the Q range, $5 \leq Q \leq 20$ Å^{-1}, of the existing measurements are seriously distorted by interference effects and final-state interactions, and a distinct condensate peak may be observable only for $Q \geq 100$ Å^{-1}. Woods and Sears[14] attempted to minimize the effects of these distortions by symmetrizing the distributions and averaging over a range of Q values. They then deduced the value $n_0 = 0.069 \pm 0.008$ at 1.1 K from the difference between the observed scattering at 1.1 K and 4.2 K. The disagreement between their result and our present value may be due to the failure of their procedure to achieve its objective completely or to the use of 4.2 K as a base line rather than a temperature near T_λ. (The use of $T^* = 4.2$ K in the present work would have led to negative values for n_0.)

In this Letter we have demonstrated that $g(r)$ can be determined with sufficient precision to allow the extraction of accurate values for $n_0(T)$. To within the experimental uncertainties, we

TABLE II. Values of the condensate fraction determined from Eq. (1) with $T^* = 2.27$ K.

T (K)	n_0
1.00	0.127 ± 0.020
1.38	0.140 ± 0.034
1.77	0.138 ± 0.061
1.97	0.050 ± 0.024
2.07	0.031 ± 0.029
2.12	0.028 ± 0.029
2.15	-0.001 ± 0.019

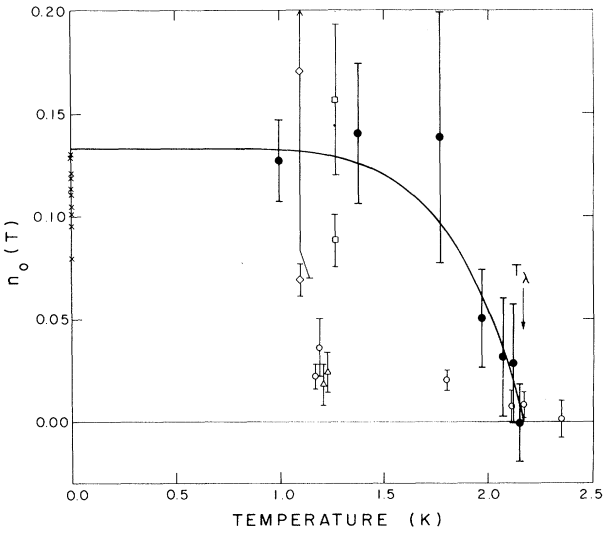

FIG. 2. The condensate fraction is superfluid ^{4}He. Solid circles show our new results and the curve represents a least-squares fit of Eq. (2) to these results. The crosses at $T = 0$ are the results of theoretical calculations (Refs. 1–7). The open symbols are values obtained by neutron inelastic scattering, upper diamond (Ref. 10), lower diamond (Ref. 14), squares (Ref. 11), triangles (Ref. 12), and circles (Ref. 13).

find that Eq. (1) does indeed give $n_0(T)$ values which are independent of r for $r \geq 6$ Å, and, further, that these values exhibit a very reasonable temperature dependence and are consistent with the best theoretical estimates of $n_0(0)$. This certainly lends credence to Eq. (1) and leads us to believe that our new values are the most realistic estimates of $n_0(T)$ obtained to date. They should, however, still be treated with caution since existing derivations[15,16] of Eq. (1) rely heavily on physical intuition. It would be desirable to obtain the relation between $g(r)$ and n_0 from a systematic theory that clearly establishes its limits of validity, and we hope that the success of the present study will stimulate further theoretical work in this direction.

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Selective Transmission of High-Frequency Phonons by a Superlattice: The "Dielectric" Phonon Filter

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The zone folding in the acoustic-phonon dispersion curve of GaAs/AlGaAs superlattices is demonstrated using superconducting tunnel junctions as sources and detectors of quasimonochromatic phonons. We present data on selective transmission of high-frequency phonons due to narrow band reflection determined by the superlattice period.

Recently, there has been much activity in the area of high-frequency–phonon propagation studies in a variety of crystalline solids and liquid helium.[1] This is in large part due to the development of thin-film superconducting tunnel junctions[2-5] as sources and detectors of such phonons. Aside from tunnel junctions, bulk impurity levels tunable by magnetic field[6] or stress[3] have been used as phonon spectrometers. The use of bulk structures has, however, been limited because of degradation[7,8] of the high-frequency phonons at the interface to the medium under study.