Nonlinear Steepening of the Electrostatic Ion Cyclotron Wave

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Electrostatic ion cyclotron waves observed in space at altitudes between 5000 and 8000 km often have a sinusoidal form. Occasionally, however, wave forms having a spiky or sawtooth form indicative of steepening are observed. The nonlinear fluid equations which characterize the electrostatic ion cyclotron wave have traveling-wave solutions with sinusoidal, spiky, and sawtooth forms. The wave form depends on the amplitude and phase velocity of the wave.

We report the observation of the nonlinear steepening of the electrostatic hydrogen cyclotron wave at altitudes between 5000 and 8000 km above the auroral regions of Earth's ionosphere. The observed waves are shown to have a form similar to the nonlinear propagating-wave solutions of the fluid equations which describe the electrostatic ion cyclotron wave.

The electrostatic ion cyclotron wave is one of the low-frequency eigenmodes of a magnetized plasma.[1,2] Such waves may be unstable to current-driven instabilities in the auroral magnetosphere at altitudes above 1000 km,[3] and have often been observed by S3-3 satellite at altitudes between 5000 km and the maximum altitude of the satellite, 8000 km.[4] These waves are usually associated with beams of 0.5- to 16-keV H^+ and O^+ ions flowing out of the auroral regions along magnetic field lines and with the observations of magnetic fluctuations indicative of currents flowing parallel to the magnetic fields.[5]

The S3-3 satellite was equipped with three orthogonally oriented pairs of spheres which made three-component measurements of the electric field. The spheres on two of the pairs were separated by 37 m while the third pair was separated by 6 m. Data from one of these pairs of detectors from three separate hydrogen cyclotron-wave events are shown in Fig. 1. The potential difference has been divided by the separation distance to give the electric field in millivolts per meter. The top part of Fig. 1 shows the usual wave form of the electrostatic hydrogen cyclotron wave. As is clear from the data there was a narrow spectral peak at 140 Hz which, as is expected from the theory of electrostatic ion cyclotron waves,[1] was above the hydrogen cyclotron frequency of 116 Hz. The middle and bottom parts of Fig. 1 also display hydrogen cyclotron waves. Here, however, the hydrogen cyclotron waves are steepened. The data in the middle part of Fig. 1 have a sawtooth from while in the bottom part the wave has steepened into a series of double spikes repeating at frequencies of those hydrogen cyclotron waves at the same altitude which show little nonlinear steepening. The orbital parameters corresponding to the times of these measurements are given in Table I. In each case the electrostatic-ion-cyclotron-wave events lasted for a time less than the 9 to 18 sec that it took to determine the complete pitch-angle distribution of the particles. In the top example the ion detector was fortuitously pointing along the magnetic field downward toward Earth and saw ions with energies between 0.09 and 1.4 keV flowing up. The detector has eight energy steps between 0.09 and 3.9 keV.

We now show that these nonsinusoidal wave forms resemble the traveling-wave solutions of the nonlinear fluid plasma equations describing the electrostatic ion cyclotron wave. It has been shown that traveling-wave solutions for the ion cyclotron wave may have a sawtooth form.[6] Our

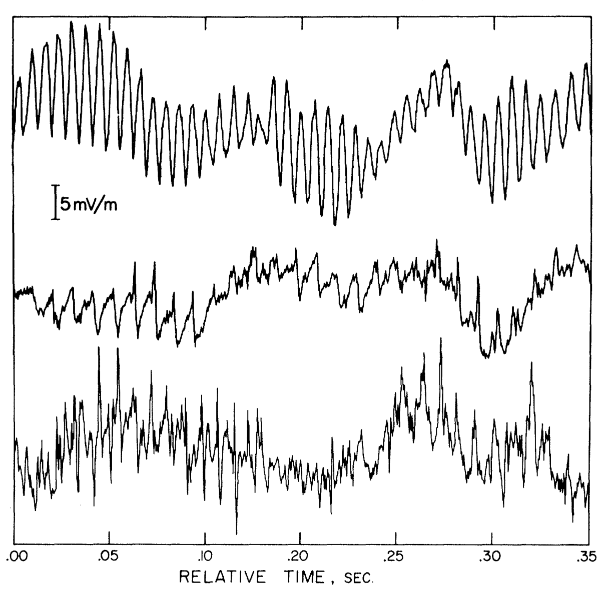

FIG. 1. Three examples of electrostatic ion cyclotron waves observed by the S3-3 satellite. The orbital parameters are given in Table I.

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TABLE I. Orbital parameters corresponding to the data in Fig. 1. The parameters are the orbit number, date, universal time (UT), the local H^+ cyclotron frequency (f_{ci}), altitude (Alt.), magnetic local time (MLT), and invariant latitude (ILA).

	Orbit	Date	UT	f_{ci} (Hz)	Alt. (km)	MLT (h)	ILA (deg)
Top	757	11 Oct. 76	0043:00	116	6199	15.95	73.15
Middle	685	2 Oct. 76	0204:41	84	7349	16.84	67.72
Bottom	757	11 Oct. 76	0043:40	118	6132	15.86	73.58

analysis differs from the previous analysis in that we use $n = n_0 e^{e\varphi/kT}$ instead of the approximation $(n - n_0)/n_0 = e\varphi/kT$ to relate the potential and density variations and in that we consider arbitrary amplitudes instead of making a small- but finite-amplitude assumption.

Following the previous analysis we make the assumption that the ions are cold and that the perturbation is confined to the x-z plane. This gives

$$\frac{dV_x}{dt} + V_x \frac{dV_x}{dx} = -C_s^2 \frac{d\psi}{dx} + \Omega_i V_y,$$

$$\frac{dV_y}{dt} + \frac{V_x dV_y}{dx} = -\Omega_i V_x,$$

for the ion dynamics where $C_s^2 = kT_e/M_i$, $\psi = e\varphi/kT_i$, and Ω is the gyrofrequency. Eliminating V_y from the set of equations, substituting $\xi = (x/V_\varphi - t)\Omega_i$ to look for traveling-wave solutuions, using ion continuity to get $V_x = (\delta n/n)V\varphi$, and using $n = n_0 e^\psi$ to relate the potential to the density fluctuation together with the assumption of quasineutrality give

$$\frac{d}{d\xi}\left[\left(\frac{1}{N^3} - \frac{1}{\beta^2 N}\right)\frac{dN}{d\xi}\right] + N - 1 = 0, \tag{1}$$

where $N = n/n_0$; n and n_0 are the actual density in the wave and the background density, respectively; and $\beta^2 = V_\varphi^2/C_s^2$ where V_φ is the phase velocity perpendicular to the magnetic field.

The above equation has periodic wave solutions provided the maximum amplitude $N_{max} < \beta$ and $\beta > 1$. This ensures that the factor in front of the derivative is finite. To see that the solution in this case must be periodic note that (1) is even in ξ. Thus the solution is even about any maximum or minimum. Therefore, if the solution has at least one maximum and one minimum, it is periodic. If $N_{max} < \beta$, the second derivative is negative when the first derivative is zero and $N > 1$. This ensures a maxim. Likewise, a minimum can be easily shown to exist.

The solutions for a given β are parametrized by the value of N_{max}. For small N_{max} (i.e., $N_{max} \ll \beta$) the solutions are sinusoidal. For N_{max} near β and β only slightly greater than 1 the solutions have a sawtooth form. For N_{max} near β and β large the solutions have a spiky form.

Figure 2 shows numerical solutions for the cases $\beta^2 = 2$ and $N_{max} = 1.2$, $\beta^2 = 1.25$ and $N_{max} = 1.11$, and $\beta^2 = 20$ and $N_{max} = 3.25$. The corresponding electric field, given by $E \propto d\psi/d\xi = d\ln n/d\xi = (1/n)(dn/d\xi)$, is shown in Fig. 3. Note the similarity of the solutions in Fig. 3 to the data in Fig. 1.

Unlike the acoustic case[7] the solution does not steepen with time. Rather steepening may occur as a result of wave growth or of propagation in a nonuniform plasma. The actual values of β and N_{max} are determined by the factor which gives rise to wave growth, that is, the resonant parti-

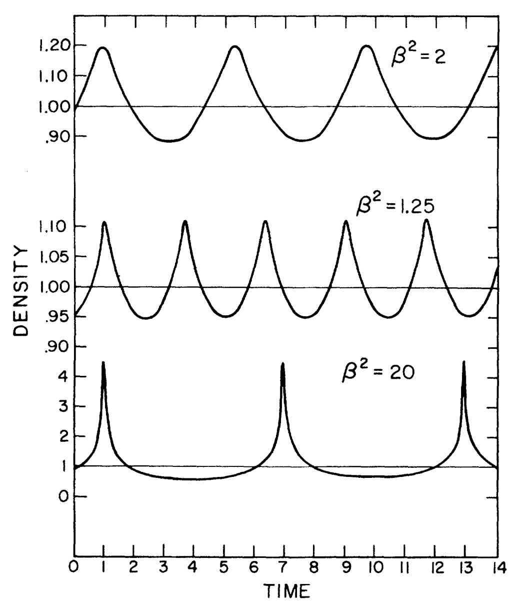

FIG. 2. Numerical solution of the density in a nonlinear electrostatic cyclotron wave.

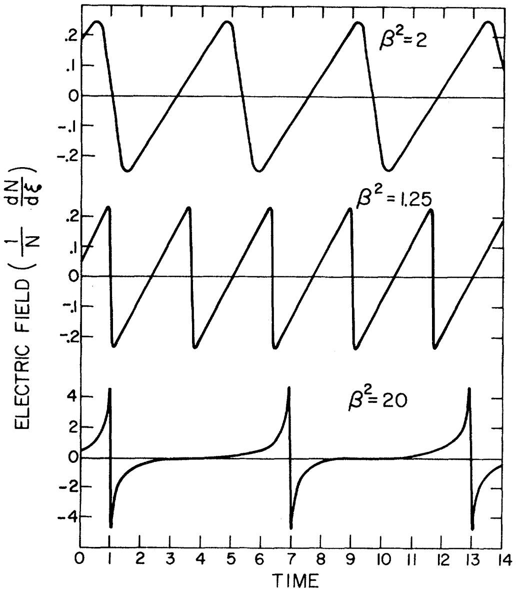

FIG. 3. Numerical solution of the electric field in a nonlinear electrostatic cyclotron wave.

cles which are not considered in our analysis. The nonlinear waves in Fig. 1 do not have a larger electric-field-potential amplitude than the waves having a sinusoidal form. This is consistent since it is the ratio of amplitude to the plasma temperature $e\varphi/kT$ that is important and not the absolute amplitude (φ).

The Fourier transform of a sawtooth or spiky wave has an infinite number of harmonics. The phase and amplitude of the harmonics are determined by the form of the original wave. Harmonics can also be generated linearly in the Vlasov theory of the electrostatic ion cyclotron wave. However, in the absence of nonlinear coupling between the harmonics, such waves would be expected to have relative phases and amplitudes different from those necessary to produce sawtooth or spiky solutions.

We have shown that a simple solution of the equations characterizing electrostatic ion cyclotron wave reproduces the essential features of the observed wave form. A more complete analysis would need to include, among other things, the effects of several traveling waves, arbitrary initial conditions, resonant particles, the linear generation of the higher-order harmonics, and other nonlinear effects.[8]

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Effect of Electrostatic Fields on Charged Reaction Products in Six-Beam Symmetrical Implosion Experiments

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Alpha and proton reaction product spectra have been measured in exploding pusher experiments on the symmetrical illumination six-beam ZETA laser system (Nd-doped phosphate glass). DT ion temperatures and positive energy shifts for α particles and protons were obtained for experiments with incident on-target power between 1 and 2.5 TW. Time-averaged electrostatic potentials have been measured up to 330 kV.

Symmetrical exploding pusher experiments have been conducted on the six-beam ZETA laser system of the Laboratory for Laser Energetics at the University of Rochester.[1] In these experiments 70-psec pulses [full width at half maximum (FWHM)] with a peak power from 1 to 2.5 Tw [in-

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