and Z bosons will have to be accompanied by comparably massive fermions. Nonetheless, the familiar corrections of perturbation theory to the tree graph amplitudes, proportional to g^2m^2/M^2, remain small.

Besides the bound-state Higgs particles that get absorbed, there will be at least one Higgs particle which does not disappear from the spectrum. This bound state will be essentially indistinguishable from an elementary Higgs field. Conceivably it could be almost exclusively a bound state of either heavy quarks or heavy leptons. Then the Higgs particle would decay primarily to hadrons or quarks, respectively.

In the standard SU(2) ⊗ U(1) model with elementary Higgs scalars, the mass of this physical Higgs field is given by

$$M_{\rm H}^2/M_W^2 = 4\lambda/g^2,$$

where λ is the quartic Higgs coupling. In our dynamical model, this coupling can be calculated as a loop effect, and to lowest order is given by

$$\lambda = 3g^2 \tan^2\theta,$$

so that

$$M_{\rm H}^2 = 12 \tan^2\theta \, M_W^2.$$

This completes the calculation of all the mass ratios in the standard SU(2) ⊗ U(1) model in terms of the gauge couplings.

It seems difficult to show that the irregular solution is actually the one that minimizes the vacuum energy in weak-interaction models. This is tantamount to proving that dynamical symmetry breaking actually occurs. However, it is clear that there already exist weak isodoublets of quarks and leptons with large ultraviolet masses. Whether these ultraviolet masses are described by the irregular solution or not, they will contribute to the W and Z masses from the ultraviolet regions of the integrals (3a) and (3b) [although heavier fermions are probably required to saturate the sum rule (6)]. The existence of these massive fermion doublets is sufficient to ensure the $\Delta I = \frac{1}{2}$ rule.

We would like to thank the Aspen Center for Physics for hospitality. One of us (H.P.) thanks V. I. Zakharov and M. B. Voloshin for discussions about the weak $\Delta I = \frac{1}{2}$ rule.

This work was supported in part by the U. S. Department of Energy under Contract Grant No. EY-76-C-02-2232B.*000.

Note added.—After completion of this work, we became aware of work by S. Englert and R. Brout, Phys. Lett 49B, 77 (1973), containing results similar to ours but with a different emphasis.

[1]A. A. Migdal and A. M. Polyakov, Zh. Eksp. Teor. Fiz. 51, 135 (1966) [Sov. Phys. JETP 24, 91 (1967)].

[2]R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973); J. M. Cornwall and R. E. Norton, Phys. Rev. D 8, 3338 (1973).

[3]H. Pagels, Rockefeller Report No. COO-2232B-180, 1979 (unpublished).

[4]L. Susskind, SLAC Report No. SLAC-PUB-2142 (Phys. Rev. D, to be published).

[5]S. Weinberg, Phys. Rev. D 19, 1277 (1979).

[6]V. I. Zakharov, M. B. Voloshin, and L. Susskind, to be published.

[7]The regular and irregular solutions are discussed by K. Lane, Phys. Rev. D 10, 2605 (1974); H. Pagels, Phys. Rev. D 19, 3080 (1979), and Rockefeller University Report No. COO-2232B-186 (to be published).

Partons and Their Transverse Momenta in Quantum Chromodynamics

Davison E. Soper
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403

(Received 30 August 1979)

It is argued that the parton distribution function $\mathcal{P}_{a/A}(x, \vec{k}_T; \zeta)$ is well defined and useful in quantum chromodynamics. Here $\zeta = (2P_A \cdot n)^2/(-n^2)$ arises as a variable because of the use of the axial gauge $n \cdot A = 0$. An approximation for $\mathcal{P}_{a/A}(x, k_T; \zeta)$ is given which is obtained from an approximate solution of the Bethe-Salpeter equation in the axial-gauge ladder model.

There has recently been great progress[1,2] in deriving an improved version of the parton model (including scaling violations) from quantum chromodynamics (QCD). One can now derive from QCD the improved version of those results in which, in the original parton model, only the parton distributions $\mathcal{P}_{a/A}(x) = \int d^3k_T \, \mathcal{P}_{a/A}(x, \vec{k}_T)$ integrated over transverse momenta appear. In this paper, I adapt the line of argument of Ref. 2 to include the Drell-Yan[3] process at measured Q_T, a process that is sensitive to the full parton distribution $\mathcal{P}_{a/A}(x, \vec{k}_T)$. This argument is quite straightforward and follows the program outlined by Collins.[4] I then present a model for (the nonsinglet part of) $\mathcal{P}_{a/A}(x,\vec{k}_T)$ that may be valid in QCD for large $\vec{k}_T$. This model is based on an approximate solution[5] of the renormalization-group-improved ladder-model Bethe-Salpeter equation for the flavor nonsinglet Green's function to find a far-off-shell quark in a hadron.

The conclusions that I will draw are as follows:

(a) The parton distributions $\mathcal{P}_{a/A}(x,\vec{k}_T;\zeta)$ are well defined in QCD, before or after integration over $\vec{k}_T$.

(b) These distributions enter the leading-order formula for high-mass dimuon production in just the way predicted by the Drell-Yan parton model for this process.

(c) The preceding conclusions depend on the use of the axial gauge $n_\mu A^\mu(x)=0$, where the gauge vector n_μ remains fixed as viewed in the reference frame in which the momenta of the problem are being scaled up. The parton distribution functions depend on the variable $\zeta_A \equiv (2P_A\cdot n)^2/(-n^2)$. That is, in contrast with the original parton model, the distribution of partons in a hadron depends on how fast the hadron is moving. Of course a proton is a proton no matter how fast it is moving. What changes as $(P_A\cdot n)^2$ increases is the definition of a parton.

(d) Also in contrast with the original parton model, $\mathcal{P}_{a/A}(x,\vec{k}_T;\zeta_A)$ does not fall to zero very quickly as k_T^2 becomes large: roughly speaking, $\mathcal{P}\propto k_T^{-2}$. However, $\mathcal{P}$ falls to zero when $k_T^2\approx\zeta_A$.

Let us begin with the discussion of dimuon production: $A+B\to\mu^+ +\mu^- +X$, where A and B are spinless or unpolarized hadrons. Choose the c.m. frame with the z axis along the beam direction. Then $P_A^+ = P_B^- = (S/2)^{1/2}$, $P_A^- = P_B^+ = \vec{P}_A^T = \vec{P}_B^T = 0$. (Hadron masses are neglected.) Let Q^μ denote the dimuon momentum. I consider the limit $S\to\infty$ with Q^2/S, Q^+/Q^-, and $\ln^2(Q^2/Q_T^2)/\ln(Q^2/\mu^2)$ fixed. In order to make use of the regularity theorem of Ref. 2, I work in the spacelike axial gauge $n_\mu A^\mu(x)=0$, $n^2=-1$, with the components of n_μ in the c.m. frame fixed as $S\to\infty$. As a convenience I choose $n^+=n^-=2^{-1/2}$, $\vec{n}_T=0$.

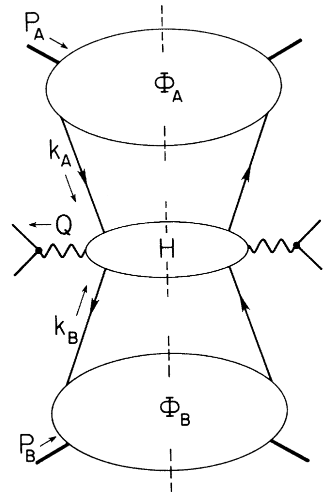

FIG. 1. Class of diagrams for dimuon production considered in this Letter.

I assume[4] that graphs for dimuon production of the form shown in Fig. 1 are the most important in $S\to\infty$ limit. By definition, the hard-scattering function H is amputated on all four legs and is two-particle irreducible in both the A and B channels. In order to simplify Fig. 1, use the renormalization group to rewrite H as

$$H(k_A,k_B,Q;g)=e^{-4t}\exp\{-4\int_0^t d\tau\,\gamma_F[g(t)]\}H[e^{-t}k_A,e^{-t}k_B,e^{-t}Q;g(t)], \tag{1}$$

where $e^{2t}=S/\mu^2$ and μ provides a reference momentum scale. In the Drell-Yan limit, $e^{-t}Q_\mu$ is fixed. The momentum $e^{-t}k_A{}^\mu$ is integrated over but [with use of some information about $\Phi_A(k_A)$ discussed in Ref. 5 and below] we can surmise that the most important integration region is that in which $e^{-t}k_A{}^\mu$ is a finite, nearly lightlike vector in the + direction. Similarly, $e^{-t}k_B{}^\mu$ is "usually" a finite, nearly lightlike vector in the − direction.

Since $g(t)\propto 1/t\to 0$ as $t\to\infty$, the higher-order terms in the expansion of H in powers of $g(t)$ can be neglected compared with the lowest-order term unless the higher-order terms contain compensating factors of t. The regularity theorem of Ellis, Georgi, Machacek, Politzer, and Ross[2] assures us that in the axial gauge, H is not singular as $(e^{-t}k_A)^2$ or $(e^{-t}k_B)^2$ tends to zero, so that no compensating factors like $\ln[(e^{-t}k_A)^2/\mu^2]\propto t$ can occur. The higher-order terms in H can, however, be singular when $e^{-t}k_A{}^\mu$, $e^{-t}k_B{}^\mu$, $e^{-t}Q^\mu$, and n^μ all lie in the same plane: $\vec{k}_A{}^T=\vec{k}_B{}^T=\vec{Q}^T=0$. Such "coplanar" singularities are not discussed in the regularity theorem of Ref. 2. In the first-order correction to H such a singularity does indeed occur. (I thank J. Collins for a helpful discussion of this point.) The singularity is strong enough to modify the lowest-order cross section if Q_T^2 is too small compared with Q^2, but has a negligible effect in the "modestly large" Q_T^2 limit in which $\ln^2(Q_T^2/Q^2)/\ln(Q^2/\mu^2)$ is fixed as $Q^2 \to \infty$. This region is of physical interest because, at least in the model proposed here, most of the cross section at large Q^2 comes from "modestly large" Q_T^2. (I will discuss the first-order correction to H and its effect in more detail elsewhere.) Thus, although a proof to all orders is lacking, it seems plausible that H can be replaced by the lowest-order term in its perturbative expansion:

$$W^{\mu\nu} \sim \sum_a e_a^2 \int \frac{d^4k_A}{(2\pi)^4}\frac{d^4k_a}{(2\pi)^4}(2\pi)^4\delta^4(k_A+k_B-Q)\exp\{-4\int d\tau\gamma_F[g(\tau)]\}\operatorname{Tr}[\Phi_{a/A}(k_A)\gamma^\mu\Phi_{\bar{a}/B}(k_B)\gamma^\nu]. \quad (2)$$

We now use two simple and highly plausible pieces of information about the functions $\Phi_{a/A}(k_A)$ and $\Phi_{\bar{a}/B}(k_B)$ (see Ref. 4 and below). First, in the important integration region discussed earlier, $k_A{}^\mu \approx x_A P_A{}^\mu$, the Dirac structure of $\Phi_{a/A}(k)$ is $\Phi_{a/A} \sim \not{P}_A\varphi$, so that $\Phi_{a/A}(k_A) \sim \gamma^- \frac{1}{4}\operatorname{Tr}[\gamma^+\Phi_{a/A}(k_A)]$. Second, the range over which $k_B{}^-$ can vary before $\Phi_{\bar{a}/B}(k_B)$ vanishes is large [of order $P_B{}^- = (S/2)^{1/2}$]. On the other hand, the range over which $k_A{}^-$ can vary before $\Phi_{a/A}(k_A)$ vanishes is small. [It is not as small as one might think; we will see later that the effective cutoff imposed by $\Phi_{a/A}(k_A)$ is $|k_A{}^-| < |k_A{}^T|$.] Therefore we can replace $\int dk^-\Phi_{a/A}(k^-)\Phi_{\bar{a}/B}(Q^- - k^-)$ by $\int dk^-\Phi_{a/A}(k^-)\Phi_{\bar{a}/B}(Q^-)$. The corrections to these two approximations are smaller than the terms retained by powers of Q_T^2/S. Similar results hold, of course, with the roles of A and B, $+$ and $-$, interchanged.

These replacements give for the dimuon cross section

$$\frac{d\sigma}{d^4Q d\Omega} \sim \frac{8\pi\alpha^2}{3SQ^2}\left(\frac{3}{16\pi}(1+\cos^2\theta)\right)\sum_a e_a^2 \int dk_A{}^T \int dk_B{}^T \delta(\vec{k}_A{}^T + \vec{k}_B{}^T - Q^T)\mathcal{P}_{a/A}(x_A, \vec{k}_A{}^T; \zeta_A) \times \mathcal{P}_{\bar{a}/B}(x_B, \vec{k}_B{}^T; \zeta_B), \quad (3)$$

where $x_A\ Q^+/P_A{}^+$, $x_B = Q^-/P_B{}^-$. Here

$$\mathcal{P}_{a/A}(k_A{}^+/P_A{}^+, \vec{k}_A{}^T; \zeta_A) \equiv \exp\{-2\int_0^t d\tau\, \gamma_F[g(\tau)]\}\frac{2}{(2\pi)^3}\int \frac{dk_A{}^-}{2\pi}\ \frac{1}{4}\operatorname{Tr}[\gamma^+\Phi_{a/A}(k_A)], \quad (4)$$

where $\zeta_A = (2P_A\cdot n)^2/(-n^2)$ and $t = \frac{1}{2}\ln(\zeta_A/\mu^2)$. With our choice of n_μ, $\zeta_A = \zeta_B = S$. If $\mathcal{P}(x, \vec{k}_T; \zeta)$ is interpreted as the parton distribution function, this is exactly the parton-model prediction[3,6] for the cross section.

The interpretation of $\mathcal{P}(x, \vec{k}_T; \zeta)$ as the parton distribution function can be supported by another argument. In canonical field theory quantized on the surface $x^+ = 0$,[7] the operator $\int dx^- d\vec{x}_T \exp[i(k^+x^- - \vec{k}_T \cdot \vec{x}_T)]\bar{\psi}(0)\gamma^+\psi(0, \vec{x}_T, x^-)$ counts the number of quarks with momentum k^+, $\vec{k}_T$ at "time" $x^+ = 0$. The expectation value of this operator in hadron state $|P_A\rangle$ is $\mathcal{P}_{a/A}(x, \vec{k}_T; \zeta)$ as given by Eq. (4) with γ_F set equal to zero.

In view of the role played by $\mathcal{P}(x, \vec{k}_T; \zeta)$ it is evidently of interest to try to estimate this function for large $k_T{}^2$ in quantum chromodynamics. For this purpose we first need detailed information on the function $\Phi(k)$, which I have evaluated approximately ($\Phi^{\rm NS}$, for the flavor nonsinglet channel) in a recent paper.[8] There I argue that, in an appropriate large-k^μ region, the Bethe-Salpeter equation that $\Phi^{\rm NS}$ obeys can be approximated by the ladder-model Bethe-Salpeter equation with a running coupling constant at the vertices. I then solve this equation using "leading log" approximations that are valid in the limit $\zeta \to \infty$, $-2k\cdot P \to \infty$ with $-2k\cdot P < \zeta$ and with $x \equiv k^+/P^+$, $y \equiv k^2/2k\cdot P$, and $\ln^2[\zeta/(2k\cdot P)]/\ln(\zeta/\mu^2)$ fixed. In this limit, $1/\ln(\zeta/\mu^2) \ll 1$, $1/\ln[(-2k\cdot P)/\mu^2] \ll 1$, and $\ln[\zeta/(-2k\cdot P)]/\ln(\zeta/\mu^2) \ll 1$ but $(-2k\cdot P) \ll \zeta$. The conjecture is advanced in Ref. 5 that the ladder model itself is a good approximation under these same conditions, but this conjecture remains unproved.

Let us now approximately evaluate the integral (4) for $\mathcal{P}^{\rm NS}(x, \vec{k}_T; \zeta)$ with the approximate solution from Ref. 5 inserted for $\Phi^{\rm NS}(k)$. To keep the discussion simple, I drop all the terms in Φ that turn out not to contribute in the $\zeta \to \infty$ limit. I also modify the function $\Phi(k)$ that emerged naturally from the ladder model to a slightly different function that is more convenient for our present purposes and is equivalent to that of Ref. 5 in the limit described in the preceding paragraph. The result is

$$\mathcal{P}^{\rm NS}(x, \vec{k}_T; \zeta) \sim \frac{4}{3}\frac{2}{\pi b\ln(\zeta/\mu^2)}\mathrm{P}\int_0^\infty \frac{d(-2k\cdot P)}{-2k\cdot P}\frac{1+x^2/y^2}{k_T{}^2 - (-2k\cdot P)^2/\zeta}e^{-F}\mathcal{P}^{\rm NS}(y;\zeta),$$
$$F = \tfrac{4}{3}(2/b)\ln^2[\zeta/(-2k\cdot P)]/\ln(\zeta/\mu^2). \quad (5)$$

Here $b = 11 - \frac{2}{3}N_f$, P indicates a principal-value prescription, and $\mathcal{P}^{NS}(y;\zeta)$ is the nonsinglet part of the probability to find a parton a with momentum fraction y in hadron A, as measured in deep-inelastic lepton scattering with a probe of spacelike virtualness $-q^2 = \zeta$.

In the integration region $-2k\cdot P < (k_T^2\zeta)^{1/2}$ the integration is essentially $\int d\ln(-2k\cdot P)$ over a wide range of $\ln(-2k\cdot P)$ with an integrand that is slowly varying (until a lower cutoff to be discussed in a moment is reached). When $(k_T^2\zeta)^{1/2} < -2k\cdot P$, the integration gains another factor of $(-2k\cdot P)^{-2}$ and is quickly cut off. Thus the denominator factor in (5) can be replaced by $(1/k_T^2) \times \theta((k_T^2\zeta)^{1/2} - (-2k\cdot P))$. Consider now the variable $y \equiv k^2/2k\cdot P = x + k_T^2/(-2k\cdot P)$. Over most of the integration region $k_T^2 \ll -2k\cdot P$ and thus $y \approx x$. But when $-2k\cdot P$ approaches k_T^2, y suddenly [on a $\ln(-2k\cdot P)$scale] increases to 1. At this point the integration is cut off because $\mathcal{P}(y;\zeta) = 0$ when $y > 1$. Thus we replace y by x everywhere and supply a factor $\theta[(-2k\cdot P) - k_T^2]$. (Notice that problems arise if x is very near 1.)

These approximations give

$$\mathcal{P}^{NS}(x,\vec{k}_T;\zeta) \sim \mathcal{P}^{NS}(x;\zeta)(1/k_T^2)G(k_T^2,\zeta), \qquad (6)$$

where

$$G(k_T^2,\zeta) = \tfrac{4}{3}(4/\pi b)[\ln(\zeta/\mu^2)]^{-1/2}\int_{z/2}^{z} dx \exp[-\tfrac{4}{3}(2/b)x^2],$$
$$z = \ln(\zeta/k_T^2)[\ln(\zeta/\mu^2)]^{-1/2}. \qquad (7)$$

We should, of course, only trust the function $G(k_T^2,\zeta)$ for large k_T^2. Nevertheless, it is reassuring to note that the normalization integral for G is $\int d^3k_T k_T^{-2}G(k_T^2,\zeta) = 1$, exactly. Thus $\int d^3k_T \times \mathcal{P}(x,\vec{k}_T;\zeta) = \mathcal{P}(x;\zeta)$. The behavior of $G(k_T^2,\zeta)$ is illustrated in Fig. 2.

The general conclusions that I draw from these results have been stated already in the introductory paragraphs. Here I wish to add a few technical comments. First, the general parton-model formula for dimuon production, Eq. (3) with no approximation introduced for $\mathcal{P}(x,\vec{k}_T;\zeta)$, appears to be valid to first order in $1/\ln(S/\mu^2)$ for "modestly large" Q_T^2. Second, it seems plausible that the approximate formula (6) for $\mathcal{P}(x,\vec{k}_T;\zeta)$ is valid in the limit $\zeta \to \infty$ with x and $\ln^2(\zeta/k_T^2)/\ln(\zeta/\mu^2)$ fixed, but this conjecture remains speculative until more is known about the corrections to the axial gauge ladder model used in obtaining $\Phi(k)$. Third, the factorization of the $\vec{k}_T$ dependence in Eq. (6) appears to be an artifact of the approximations used. I do not expect this feature to hold

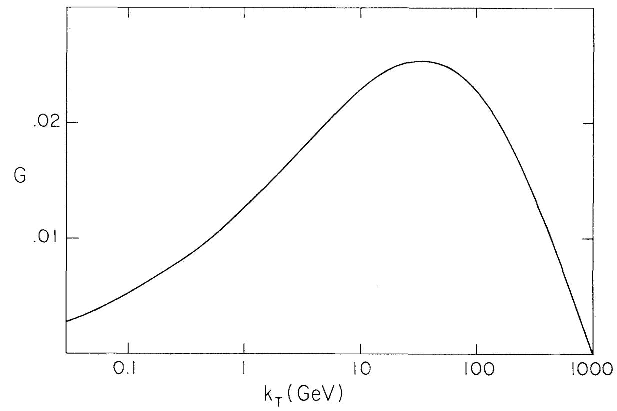

FIG. 2. The parton transverse momentum distribution obtained from the ladder model, $G(\vec{k}_T^2,\zeta) = k_T^2\mathcal{P}^{NS}(x,\vec{k}_T;\zeta)/\mathcal{P}^{NS}(x;\zeta)$, for $\zeta = 1$ TeV^2, $\mu^2 = 0.1$ GeV^2, $N_f = 4$.

in general. Fourth, the Drell-Yan process has been studied, with use of the ladder model in axial gauge, by Dokshitser, D'Yakonov, and Troyan.[1] If one inserts Eq. (6) for $\mathcal{P}(x,\vec{k}_T;\zeta)$ into the general formula (3) for the Drell-Yan cross section, the result, after some further manipulations and approximations, is essentially that of these authors. Dokshitser, D'Yakonov, and Troyan,[1] however, give a dramatically different interpretation to this result. Fifth, Parisi and Petronzio[9] have obtained a slightly different form for $d\sigma/d^4Q$ by use of an independent-soft-gluon emission model. Sixth, it may be difficult to measure the slowly varying part of Q_T^2 dependence of $d\sigma/d^4Q$ for dimuon production, but the asymptotic power behavior,[7] Q_T^{-2}, should be detectable in future experiments at high enough energy so that there is a substantial range of Q_T^2 available between 1 GeV^2 and Q^2.

I would like to thank J. Gunion, R. Hwa, S. Brodsky, A. Mueller, and J. Collins for helpful conversations. This work was supported, in part, by the U. S. Department of Energy under Contract No. EY-76-S-06-2230.

[1] Y. Dokshitser, D. D'Yakonov, and S. Troyan, Proceedings of the Thirteenth Winter School of the Leningrad B. P. Konstantinov Institute of Nuclear Physics, Leningrad, 1978 (unpublished) [available in English as SLAC Report No. SLAC-TRANS-183 (unpublished)].

[2] R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer, and G. G. Ross, Nucl. Phys. B152, 285 (1979); see also S. B. Libby and G. Sterman, Phys. Rev. D 18, 3252 (1978), and the papers cited in Ellis *et al.* and in D. E. Soper, University of Oregon Report No. OITS-116 (un-

published).
[3]S. D. Drell and T. M. Yan, Phys. Rev. Lett. 25, 316 (1970), and Ann. Phys. (N. Y.) 66, 578 (1971).
[4]J. C. Collins, Phys. Rev. Lett. 42, 291 (1979).
[5]D. E. Soper, see Ref. 2.
[6]G. Farrar, Nucl. Phys. B77, 429 (1974).
[7]J. B. Kogut and D. E. Soper, Phys. Rev. D 1, 2901 (1970); J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev. D 3, 1382 (1971).
[8]D. E. Soper, Phys. Rev. Lett. 38, 461 (1977).
[9]G. Parisi and R. Petronzio, CERN Report No. TH. 2627-CERN (unpublished); see also G. Fox and S. Wolfram, California Institute of Technology Report No. CALT-68-723 (unpublished), and C. Y. Lo and J. D. Sullivan, University of Illinois, Urbana, Report No. ILL-TH-79-22 (unpublished).

First Estimate of the Higher-Order Terms in the Scaling-Violation Pattern of Quantum Chromodynamics

Moshe Moshe
Physics Department, Tel-Aviv University, Ramat-Aviv, Israel
(Received 20 August 1979)

It is shown that the special leading-logarithm structure of the solution to the renormalization-group equation enables one to obtain a reasonable estimate of the still unknown terms of order $(\ln q^2)^{-\gamma-2}$. Implications of this result on future tests of quantum chromodynamics at short distances are discussed.

In the last two years[1-3] much effort has been put into calculating the precise pattern of scaling violations.[4] Indeed, as quantum chromodynamics (QCD) emerges as the theory of strong interactions, one can foresee in the future very precise tests of the theory by carrying out of experiments that are intended to check not only its qualitative features but also its precise quantitative predictions. What exactly would be the theory's "$g-2$" test is still an open and interesting question, but it may very well be one test or another of its well-understood short-distance behavior.

The exact evaluation of high-order terms in the scaling-violation pattern in QCD amounts to calculating the high-order loop contributions to the renormalization-group functions $\beta(g)$ and $\gamma(g)$ and the coefficient C_i^n. Though the calculations are in principle straightforward, they are extremely tedious and are an extraordinary task already at the two-loop level. Fortunately, this task had been achieved in the valuable calculations of Refs. 1 and 2. The number of diagrams that had to be calculated approaches one hundred and it will grow by about one order of magnitude at the three-loop level. Therefore, it may very well be that for a long time to come the calculations of Refs. 1 and 2 will be the most detailed ones available. Thus, if the complete three-loop calculations are to be missing for a while, it would be interesting to have a semirough estimate of what we can expect for the contributions of the next order term which is of order $(\ln q^2)^{-\gamma-2}$.

The typical scaling-violation pattern of the structure-function moments in deep-inelastic lepton scattering has the form[5,6]

$$M_n^{\,i}(-q^2/\Lambda^2)=\int_0^1 d\xi\,\xi^{n-1}F_i(\xi,q^2/\Lambda^2) \simeq C_i^{\,n}(-q^2/\mu^2,g^2)\langle h|O^{(n)}|h\rangle, \qquad (1)$$

where only the leading twist-two operator $O^{(n)}$ is shown and only a single such term appears on the right-hand side of Eq. (1) if i is the flavor nonsinglet part. The matrix element

$$\Gamma_{JJ}(q^2)\sim\int d^4x\,e^{iq\cdot x}\langle h|J_{(x)}J_{(0)}|h\rangle$$

of the two currents (weak or electromagnetic) in the target state $|h\rangle$ satisfies a renormalization-group equation and thus $C_i^{\,n}(-q^2/\mu^2,g)$ is expressed in the following form[7]:

$$\begin{aligned}C_i^{\,n}(-q^2/\mu^2,g^2)&=C_i^{\,n}(-q^2/\Lambda^2)=A_i^{\,n}(g_0^{\,2})^{d_n}\{1+g_0^{\,2}[C_{1,0}^{\;n,i}+C_{1,1}^{\;n,i}\ln g_0^{\,2}]\\&\quad+g_0^{\,4}[C_{2,0}^{\;n,i}+C_{2,1}^{\;n,i}\ln g_0^{\,2}+C_{2,2}^{\;n,i}\ln^2 g_0^{\,2}]+\dots\}\\&=A_i^{\,n}(g_0^{\,2})^{d_n}\sum_{k=0}\sum_{j\le k}C_{k,j}^{\;n,i}g_0^{\,2k}(\ln g_0^{\,2})^j,\end{aligned} \qquad (2)$$

where

$$g_0^{\,2}=[\beta_0\ln(-q^2/\Lambda^2)]^{-1},\quad d_n=\gamma_n^{(0)}/2\beta_0,\quad \Lambda^2=\mu^2\exp\left\{-\frac{\beta_1}{\beta_0^{\,2}}\ln g^2+\frac{1}{\beta_0}\left[-\frac{1}{g^2}+\sum_{l=1}\frac{f_l}{l}g^{2l}\right]\right\},$$