PHYSICAL REVIEW LETTERS

VOLUME 43 17 DECEMBER 1979 NUMBER 25

Weak $\Delta I = \frac{1}{2}$ Rule and the Dynamical Higgs Mechanism

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(Received 21 September 1979)

We show that the weak $\Delta I = \frac{1}{2}$ rule, $m_W^2/m_Z^2 = \cos^2\theta$, in an $SU_L(2) \otimes U(1)$ gauge model can be obtained from a dynamical Higgs mechanism. This is independent of weak isospin breaking in the fermion mass spectrum. A consequence of this mechanism is a sum rule which implies the existence of heavy quarks or leptons with ultraviolet masses comparable to the W and Z weak boson masses, ~30–100 GeV.

A dramatic experimental success of the standard $SU_L(2) \otimes U(1)$ Weinberg-Salam model is the prediction

$$M_W^2/M_Z^2 = \cos^2\theta, \qquad (1)$$

for the charged to neutral gauge-boson mass ratio in terms of the weak mixing angle θ. This relation, experimentally valid to a few percent, is a consequence of placing the elementary Higgs field in the $(\frac{1}{2}, 1)$ representation and use of the tree approximation. Loop corrections to (1) are of $O(g^2)$ where g is a gauge coupling. We will refer to (1) as the weak $|\Delta I| = \frac{1}{2}$ rule.

The basic idea of the dynamical Higgs mechanism is to dispense with elementary scalar fields in the fundamental Lagrangian. Instead of elementary fields the Higgs scalars responsible for symmetry breaking are Lorentz-invariant spinless bound states of the fundamental fermions. Models of this type have been constructed by Migal and Polyakov,[1] Jackiw and Johnson,[2] and Cornwall and Norton.[2] It is anticipated that such dynamical mechanisms will avoid the proliferation of parameters characteristic of weak interaction theories with elementary scalars.[3]

Recently Susskind[4] and Weinberg[5] have pointed out that the weak $\Delta I = \frac{1}{2}$ rule follows from the dynamical Higgs mechanism provided the fermions are isospin degenerate. In the hypercolor model[4] the bound-state Higgs particles are built out of the isodoublet (U,D) of heavy hypercolor quarks. These hyperpion fields transform like $\overline{U}_R\gamma_5 D_L$, $\overline{D}_R\gamma_5 U_L$, $\overline{U}_R\gamma_5 U_L - \overline{D}_R\gamma_5 D_L$, i.e., like $(\frac{1}{2}, 1)$ representations of the weak group.

The condition of isospin degeneracy is essential. Zakharov, Voloshin, and Susskind[6] have emphasized that if one attempts to incorporate isospin breaking, as evidenced by the light-quark mass ratio $m_d/m_u \simeq 2.7$, in models of extended hypercolor one loses the weak $\Delta I = \frac{1}{2}$ rule. In such models $m_u/m_d \simeq \langle \overline{U}U\rangle_0/\langle \overline{D}D\rangle_0$. This implies large weak isospin breaking in the vacuum. The hyperpions that are absorbed by the weak gauge fields are then strongly mixed with representations other than $(\frac{1}{2}, 1)$. The weak $\Delta I = \frac{1}{2}$ rule is completely lost. One could avoid this conclusion by making the origin of the light-quark mass difference distinct from the origin of the W and Z boson masses, but this entails an unattractive proliferation of groups and fermions.

It is interesting to note how the standard Weinberg-Salam model avoids this problem on account of the presence of elementary Higgs scalars. Isospin breaking in the fermion sector, such as manifested by an e-ν or u-d mass difference, can only affect the weak $\Delta I = \frac{1}{2}$ rule via the Higgs sector—through loop effects. For this reason the rule holds to $O(g^2)$. In the dynamical mechanism—since the Higgs bosons are built from the fermions—isospin breaking affects the W and Z

masses directly. How can this be avoided?

Let us suppose that we have a gauge theory with gauge fields and fermions (no scalars) and that gauge symmetry precludes bare fermion masses. The effective potential $V(\Sigma)$ is a functional of the fermion self-energy. In general there exists an extremum $\Sigma = 0$ corresponding to an unbroken symmetry. In addition there may be other nontrivial extrema, one of which may be the true minimum. These can correspond to the regular and irregular solutions to the Bethe-Salpeter equation for $\Sigma(p)$ or linear combinations thereof.[7] The asymptotic behavior of the regular and irregular solutions, which for simplicity we give in a model without vector boson self-energy insertions, is specified by (for $p^2 \to \infty$)

$$\Sigma^{(+)}(p) \sim (-p^2/\mu^2)^{-\gamma} \text{ (irregular)},$$
$$\Sigma^{(-)}(p) \sim (1/p^2)(-p^2/\mu^2)^{+\gamma} \text{ (regular)}, \quad (2)$$

where γ is an anomalous dimension of order g^2. The terms regular and irregular correspond to the behavior at the origin of the pseudoscalar bound-state wave function which is directly related to Σ by Ward identities.

It is difficult to determine which solution corresponds to a true minimum of the vacuum energy. In dynamically broken γ_5 invariance in quantum chromodynamics it is evidently the regular solution, corresponding to an ordinary bound-state pion with falling electromagnetic form factor, that is physically relevant.[7] By contrast, the irregular solution would correspond to a pointlike structure for the bound-state pion. The major assumption we make is that for the dynamically broken $SU_L(2)\otimes U(1)$ model it is the irregular solution that minimizes vacuum energy. If this is so then the weak $\Delta I = \frac{1}{2}$ rule follows up to corrections of $O(g^2)$. Physically, bound-state Higgs states corresponding to the irregular solutions mimic the elementary Higgs fields of the standard model. The integral equations for the dynamically generated gauge-boson and fermion masses, in the weak-coupling limit without vector-boson self-energy insertions, are completely dominated by the high-energy components in the loop integrals. At high energy the isospin symmetry is restored and the bound-state Higgs particle is in the $(\frac{1}{2}, 1)$ representation.

To see this explicitly we apply the integral equations of Refs. 2 to an $SU_L(2)\otimes U(1)$ model with a doublet consisting of a heavy lepton E^- and its accompanying massless neutrino ν_E. The $SU_L(2)$ and $U(1)$ gauge couplings are g and g', respectively, and $\tan\theta = g'/g$. Then in the weak-coupling limit the dynamically generated W and Z masses are

$$M_W^2 = \frac{-ig^2}{2(2\pi)^4}\int \frac{d^4p\,\Sigma_E^2(p)}{(p^2-\Sigma_E^2)p^2}, \quad (3a)$$

$$M_Z^2 = \frac{-ig^2\sec^2\theta}{2(2\pi)^4}\int \frac{d^4p\,\Sigma_E^2(p)}{(p^2-\Sigma_E^2)^2}. \quad (3b)$$

For the irregular solutions the integrals (3) are dominated by the high-frequency domain $p^2 \to -\infty$, $\Sigma_E(p) = m_E(-p^2/m_E^2)^{-\gamma}$, $\gamma = (3g^2/32\pi^2)\tan^2\theta + O(g^4)$. In the weak-coupling limit, $g^2 \to 0$, the difference in the denominators of the equations (3a) and (3b) can be ignored. Consequently we obtain from (3)

$$M_W^2/M_Z^2 = \cos^2\theta + O(g^2), \quad (4)$$

the weak $\Delta I = \frac{1}{2}$ rule. Had we used the regular solution, the difference of the denominators in (3a) and (3b), a consequence of the large isospin breaking, could not be ignored and we would not obtain (4). For example, hyperpions are regular solutions, and in that scheme M_W/M_Z is sensitive to isospin breaking.

There is a relation between the dynamically generated gauge-boson mass and the fermion mass. In this simple model it is given by

$$m_E^2 = 6\tan^2\theta M_W^2, \quad (5)$$

up to $O(g^2)$. This relation would imply the existence of a heavy lepton of mass $\cong$ 100 GeV.

The above model can be generalized. The pseudoscalar bound states could be built out of any isodoublets including quarks with ultraviolet mass differences. The anomalous dimension γ of the fermion self-energy, besides receiving contributions from the $SU(2)_L\otimes U(1)$ gauge bosons, will, in unified models of weak and strong interactions, receive contributions from other gauge bosons as well. (Consistency requires $\gamma > 0$. Vector gauge fields contribute positively to γ while axial vectors contribute negatively, and so we assume that the vector contributions dominate.) There results a sum rule between the weak boson mass M and the ultraviolet fermion masses m_i of the form

$$M^2 = \sum_{\substack{\text{doublet}\\ \text{fermions}}} c_i m_i^2, \quad (6)$$

where $c_i > 0$ are numbers of order unity which are nonvanishing in the weak-coupling limit. In principle these numbers can be computed in specific models. Thus if these ideas are correct the W

and Z bosons will have to be accompanied by comparably massive fermions. Nonetheless, the familiar corrections of perturbation theory to the tree graph amplitudes, proportional to g^2m^2/M^2, remain small.

Besides the bound-state Higgs particles that get absorbed, there will be at least one Higgs particle which does not disappear from the spectrum. This bound state will be essentially indistinguishable from an elementary Higgs field. Conceivably it could be almost exclusively a bound state of either heavy quarks or heavy leptons. Then the Higgs particle would decay primarily to hadrons or quarks, respectively.

In the standard $SU(2)\otimes U(1)$ model with elementary Higgs scalars, the mass of this physical Higgs field is given by

$$M_H^2/M_W^2 = 4\lambda/g^2,$$

where λ is the quartic Higgs coupling. In our dynamical model, this coupling can be calculated as a loop effect, and to lowest order is given by

$$\lambda = 3g^2\tan^2\theta,$$

so that

$$M_H^2 = 12\tan^2\theta\, M_W^2.$$

This completes the calculation of all the mass ratios in the standard $SU(2)\otimes U(1)$ model in terms of the gauge couplings.

It seems difficult to show that the irregular solution is actually the one that minimizes the vacuum energy in weak-interaction models. This is tantamount to proving that dynamical symmetry breaking actually occurs. However, it is clear that there already exist weak isodoublets of quarks and leptons with large ultraviolet masses. Whether these ultraviolet masses are described by the irregular solution or not, they will contribute to the W and Z masses from the ultraviolet regions of the integrals (3a) and (3b) [although heavier fermions are probably required to saturate the sum rule (6)]. The existence of these massive fermion doublets is sufficient to ensure the $\Delta I = \frac{1}{2}$ rule.

We would like to thank the Aspen Center for Physics for hospitality. One of us (H.P.) thanks V. I. Zakharov and M. B. Voloshin for discussions about the weak $\Delta I = \frac{1}{2}$ rule.

This work was supported in part by the U. S. Department of Energy under Contract Grant No. EY-76-C-02-2232B.*000.

Note added.—After completion of this work, we became aware of work by S. Englert and R. Brout, Phys. Lett 49B, 77 (1973), containing results similar to ours but with a different emphasis.

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Partons and Their Transverse Momenta in Quantum Chromodynamics

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It is argued that the parton distribution function $\mathcal{P}_{a/A}(x,\vec{k}_T;\zeta)$ is well defined and useful in quantum chromodynamics. Here $\zeta=(2P_A\cdot n)^2/(-n^2)$ arises as a variable because of the use of the axial gauge $n\cdot A=0$. An approximation for $\mathcal{P}_{a/A}(x,k_T;\zeta)$ is given which is obtained from an approximate solution of the Bethe-Salpeter equation in the axial-gauge ladder model.

There has recently been great progress[1,2] in deriving an improved version of the parton model (including scaling violations) from quantum chromodynamics (QCD). One can now derive from QCD the improved version of those results in which, in the original parton model, only the parton distributions $\mathcal{P}_{a/A}(x)=\int d^3k_T\, \mathcal{P}_{a/A}(x,\vec{k}_T)$ integrated over transverse momenta appear. In this paper, I adapt the line of argument of Ref. 2 to include the Drell-Yan[3] process at measured Q_T, a process that is sensitive to the full parton distribution $\mathcal{P}_{a/A}(x,\vec{k}_T)$. This argument is quite

