Possibility of a Spin-Isospin Density Phase in Finite Nuclei

G. Do Dang
Laboratoire de Physique Théorique et Hautes Energies, Université de Paris-Sud, 91405 Orsay Cédex, France
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It is suggested that in a finite nucleus there may exist a condensed phase in which the spins of the protons are all aligned in one direction and the spins of the neutrons in the other and which has a lower energy than that of the normal phase above some density. It is found that this critical density is a few times larger than the normal density and that the nucleus will have a strongly oblate shape.

The possibility of a condensed state of pions in nuclear matter and in finite nuclei has been the subject of numerous studies.[1] From the start, it has been suggested[2] that such a state might exist, even at densities around normal nuclear density $\rho_0 \sim 0.5 m_\pi{}^3$, and that, due to the existence of a finite critical momentum k_c, nuclear matter might have a laminated structure. This double observation has led to more detailed studies. As far as the critical density is concerned, it seems at present that, despite intensive efforts,[3] one is still far from being capable of predicting its value with reasonable confidence. The difficulty comes mainly from the treatment of short-range correlations, even more so when one is interested in the region of very high densities. From an experimental point of view, even though a strong condensation does not seem to exist in ordinary nuclei,[4] nothing has come to confirm, nor to contradict, the existence of a weak condensation the presence of which would not perturb their known properties appreciably. It has also been suggested[5] that a condensed state might be formed in the collision of two ions during which higher densities might be reached. The question of the laminated structure seems to be better understood, at least for infinite nuclear matter, in particular, with the help of recent works.[6] It is demonstrated there that this structure allows one to get the most energy out of the tensor force, in agreement with a previous work[7] in a somewhat different context. The question is to see how one could reconcile such a structure which is perfectly valid for an infinite system, with the known shell-model structure of finite nuclei, i.e., in the case where one insists on maintaining this structure even at higher densities. This Letter suggests a possible solution. Specifically, we propose that a condensed phase which we call spin-isospin density phase (SID) might exist which would have a lower energy than the normal phase above some critical density.

For illustration, let us consider only a symmetric nuclear medium $N=Z$ and neutral pions. Addition of charged pions can but reinforce the conclusion obtained below as to the existence of the phase transition. In the field description of the nucleons and the pions, respectively, with the field operators $\psi(\vec{r})$ and $\varphi_0(\vec{r})$, the condensation is intimately related to the existence of a nonvanishing source function $\vec{S}(\vec{r})$ defined by[6]

$$\vec{S}(\vec{r}) \equiv \langle \psi^\dagger(\vec{r}) \tau_3 \vec{\sigma} \psi(\vec{r}) \rangle , \tag{1}$$

where $\langle\ \rangle$ indicates the expectation value in the condensed state. Using the mean-field approximation, it has been shown that the pion field $\varphi_0(\vec{r})$ has a nonvanishing expectation value given in terms of $\vec{S}(\vec{r})$ by

$$(\nabla^2 - m_\pi{}^2)\langle \varphi_0(\vec{r}) \rangle = \nabla \cdot \vec{S}(\vec{r}) , \tag{2}$$

and that the energy of the condensed pions is simply given as the direct part of the expectation value of the one-pion–exchange potential $V_{\rm OPE}$ in the given state.

For an infinite system, the source function $\vec{S}(\vec{r})$ is obtained by assigning, on the one hand, the nucleons to be localized in parallel layers, thus creating a modulation of the mass density, and on the other hand, the spins of the protons and neutrons to be opposite on the same layer and to change directions from one layer to the next, thus giving rise to a modulation of the spin-isospin density. It is clear that for a finite nucleus, if one maintains its description as a system of independent particles, it is not possible to create a modulation of the mass density in such an artificial way. Furthermore, if each one-particle orbital state α was occupied either by a neutron and a proton or by two neutrons (or two protons) with the two directions of spins, then the source function $\vec{S}(\vec{r})$ would vanish everywhere and there would be no condensation. The only remaining alternative is that each state α is occupied by a single proton with spin up (or spin down) and a

single neutron with spin down (or spin up). In other words, the spins of the protons are all oriented in one direction and the spins of the neutrons all in the other. It is to be compared with the normal configuration where each state α is occupied simultaneously by two protons and two neutrons with the two spin directions. As this configuration gives rise to a nonvanishing source function, it will allow one to gain energy from the tensor force. The price one has to pay for it is that one is forced, according to the Pauli principle, to place a number of nucleons, which normally would occupy lower orbitals, to higher ones, whereby increasing their kinetic energy. The energy balance depends very much on the way these extra nucleons are distributed on the higher orbitals. From the result of the infinite system, one may expect that a configuration which gives rise to a strongly oblate shape of the nucleus will allow one to get the most out of the tensor force.

To simplify the problem, let us suppose that nucleons are placed in a harmonic-oscillator potential with axial symmetry characterized by the frequencies ω_z and $\omega_x=\omega_y$ or equivalently by the oscillator lengths b_z and $b_x=b_y$ with $b_i^2=\hbar/M\omega_i$. A single-particle state α is then defined by the quantum numbers n_z and (n,m) corresponding to the wave function

$$\lambda_\alpha(\vec{r})=R_{n_z}(z)R_n(\rho)e^{im\varphi}/(2\pi)^{1/2}, \tag{3}$$

which is the product of harmonic-oscillator wave functions in one and two dimensions, where z, ρ, and φ are the usual cylindrical coordinates. To calculate the condensation energy, it is convenient to write the one-pion–exchange potential in the form

$$V_{\mathrm{OPE}}(1,2)=-(f/m_\pi)^2(\vec{\tau}_1\cdot\vec{\tau}_2)\int d^3k\,(2\pi)^{-3}(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{k})e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}\rho(k^2)/\omega_k^2, \tag{4}$$

with $\omega_k^2=m_\pi^2+k^2$ and where one has introduced a reduction factor $\rho(k^2)=\Lambda^2/(\Lambda^2+k^2)$ in order to test the sensitivity of the short-range part, with $\Lambda=\infty$ corresponding to the usual form of the OPE potential. In the mean-field approximation, the condensation energy is just the direct part of the expectation value of V_{OPE} in the SID configuration:

$$\langle V_{\mathrm{OPE}}\rangle=-(f/m_\pi)^2\textstyle\sum_{\alpha\beta}\int d^3k\,(2\pi)^{-3}k_z^2\rho(k^2)\omega_k^{-2}\langle\alpha|e^{i\vec{k}\cdot\vec{r}_1}|\alpha\rangle\langle\beta|e^{-i\vec{k}\cdot\vec{r}_2}|\beta\rangle. \tag{5}$$

The one-body matrix can be calculated immediately, with use of Eq. (3):

$$\langle\alpha|e^{i\vec{k}\cdot\vec{r}}|\alpha\rangle\equiv G_z^\alpha(k_z)G_\rho^\alpha(k_\perp)=\int_0^\infty dz\,2R_{n_z}^2(z)\cos k_z z\int_0^\infty\rho\,d\rho\,R_n^2(\rho)J_0(k_\perp\rho), \tag{6}$$

where J_0 is the usual regular Bessel function of zeroth order. The final result for the condensation energy is

$$\langle V_{\mathrm{OPE}}\rangle=-(f/\pi m_\pi)^2\textstyle\sum_{\alpha\beta}\int_0^\infty dz\,k_z^2G_z^\alpha(k_z)G_z^\beta(k_z)\int_0^\infty k_\perp dk_\perp G_\rho^\alpha(k_\perp)G_\rho^\beta(k_\perp)\rho(k^2)/\omega_k^2. \tag{7}$$

The total energy of the SID configuration relative to the bottom of the potential well is the sum of V_{OPE} and the kinetic energy of the nucleons (with $\hbar=1$):

$$E_c=\langle V_{\mathrm{OPE}}\rangle+2\sum_{\alpha=(n_z,n,m)}[(n+1)\omega_x+(n_z+\tfrac{1}{2})\omega_z], \tag{8}$$

where the sum runs over all occupied orbitals and the factor 2 coming from the summation over the proton and neutron. This energy is to be compared with the energy of the normal phase which we shall take as a spherical nucleus in a harmonic-oscillator potential with frequency ω_s. As the tensor force does not contribute in this case, one gets simply

$$E_s=4\sum_{\alpha=(N,l,m)}(N+\tfrac{3}{2})\omega_s. \tag{9}$$

Of course, the frequencies ω_s, ω_z, and ω_x are not independent: We shall impose the condition that the volumes of the two systems are the same, which gives

$$b_z b_x^2=b_s^3. \tag{10}$$

For a given value of b_s, namely for a given density, the SID configuration will be more stable if one can get a set of values b_z and b_x satisfying Eq. (10) for which the energy E_c is lower than E_s.

As a test to see whether such a situation might effectively occur, we have considered the carbon nucleus $N=Z=6$ for which the ground state corresponds to $b_s^0=1.65$ fm. Of course, the question is not to see whether, at the density corresponding to b_s^0, the SID phase is more stable than the normal phase: We know that the answer is negative and that at best one could expect a small mixture. On the other hand, it may be that at higher densities (smaller b_s), the situation is reversed:

One then has a phase transition and the SID solution becomes more favorable.

The importance of the choice of the SID configuration will be illustrated by the following two examples: Configuration I, $(n_z,n,m)=(0,0,0)$, $(0,1,1)$, $(0,1,-1)$, $(1,0,0)$, $(1,1,1)$, and $(1,1,-1)$; configuration II, $(n_z,n,m)=(0,0,0)$, $(0,1,1)$, $(0,1,-1)$, $(0,2,2)$, $(0,2,0)$, and $(0,2,-2)$. Notice that for $b_z=b_x$, the configuration I corresponds to a spherical nucleus and configuration II to an oblate one. This oblate shape can furthermore be accentuated by decreasing b_z, thus increasing b_x according to Eq. (10).

Figure 1 shows first of all that for each value of b_s, there exists a set of values b_z and b_x satisfying Eq. (10) for which E_c is minimum: This value $E_c^{\min}$ is the lowest energy of the SID phase at a given density. The curves have been obtained for configuration II with the cutoff energy $\Lambda=900$ MeV. For configuration I, the corresponding curves are very similar, except that they lie much higher. This fact is illustrated in Fig. 2 where we show the energy differences $E_c^{\min}-E_s$ in terms of b_s for the three values $\Lambda=500$, 900, and ∞ MeV. The ratio of the densities $\rho/\rho_0=(b_s^0/b_s)^3$ is also given. We see that, even at very high densities, configuration I is still less favorable $(E_c^{\min}>E_s)$ than the normal configuration. On the contrary, for configuration II, there always exists a critical density ρ_c above which the SID solution is more stable. For $\Lambda=\infty$, 900, and 500 MeV, one gets, respectively, $\rho_c=1.95\rho_0$, $2.53\rho_0$, and $4.36\rho_0$ $(b_s^c=1.32,\ 1.21$, and 1.01 fm). It should be emphasized that these values are just rough estimates of the critical density. As in previous calculations, the largest uncertainty comes from the treatment of the short-range part of the interaction which is illustrated here by the variation of Λ. Furthermore, the total energy of the system should be better calculated by directly using, instead of Eqs. (8) and (9), a Hamiltonian with a properly chosen two-body interaction (in addition to the OPE potential). The effect of the $\Delta(1232)$ and of the exchange part of the interaction,[6,8] which are both important and contribute in opposite directions, should also be

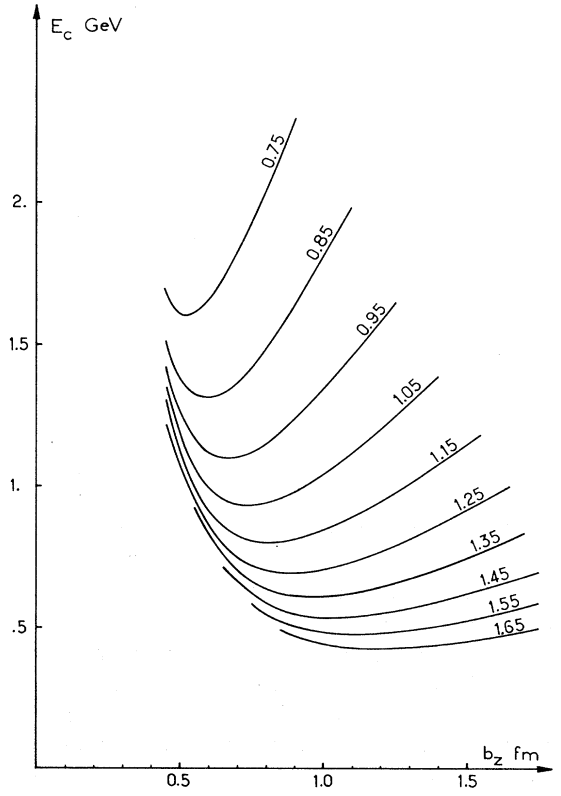

FIG. 1. Variation of the SID phase energies (configuration II) vs the shape parameter b_z for different values of the density parameter b_s. The cutoff energy is Λ=900 MeV.

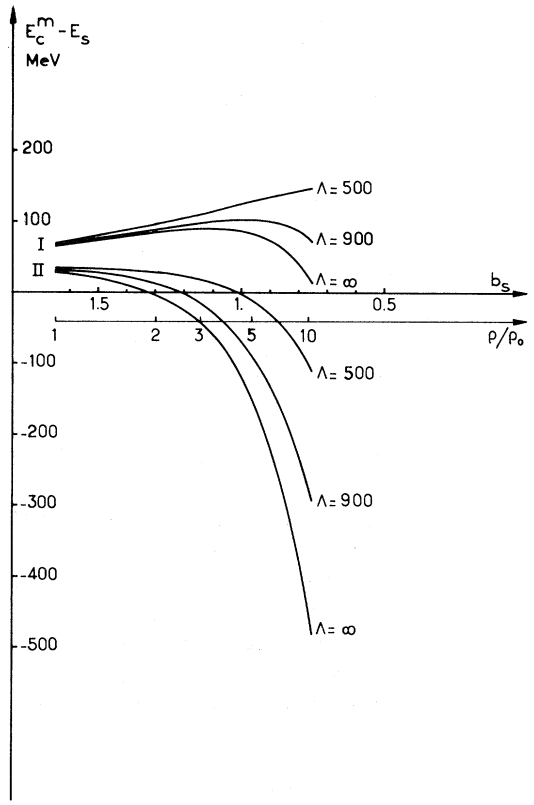

FIG. 2. Relative energies $E_c^{\min}-E_s$ vs density variations for configurations I and II and for three values of Λ.

taken into account.

Aside from the uncertainty in the value of the critical density ρ_c, the present calculation shows clearly that this value exists above which the SID phase is more favorable than the normal phase. In this phase the oblate configuration in which one puts as much as possible the nucleons in states of the (x,y) plane, is favored. The oblate shape is even strongly accentuated by the small values of $b_z < b_x$ obtained. It should be emphasized, however, that the proposed SID phase does not represent a stable, but a transient state. As can be seen from Fig. 1, even though its energy is the lowest at densities above the critical density, it is always higher than that of the normal phase at the *normal* nuclear density. At these very high densities, the energy of the SID phase may even be higher if the short-range repulsion between the nucleons is included.

It is obvious that it would be most interesting if one could observe such a phase, if it exists. For this purpose, the proposed[5] idea of observing it in a heavy-ion collision is of interest. During the short interval where two ions crash against each other, the density may be very high, which would allow the combined system to reach the condensed phase. For the case of the nucleus $N=Z=6$, the energy necessary for reaching such a phase may be estimated. Taking $\Lambda = 500$ MeV, the energy of the SID phase at the critical density is 1.056 GeV and that of the normal ground state of the carbon nucleus is 0.396 GeV. In its center of mass, the energy required is therefore $E' = 0.66$ GeV. In assuming that this system is obtained by the head-on collision of two ^{6}Li nuclei, the incident energy of the projectile in the laboratory system must be, neglecting binding energies and with use of nonrelativistic kinematics, at least $E_i = 2E' = 1.32$ GeV. Obviously, this value should not be taken as such for an experiment. In the present situation, what one should do is to really carry out experiments for all available energies E_i and to look for possible anomalies. The SID phase, if it exists, would open up a new channel and would manifest itself as an increase in the total cross section.

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Discontinuity in the Transient Magnetic Field around $Z_1 = 9$ and $Z_2 = 26$

K. Dybdal, J. S. Forster,[a] and N. Rud
Institute of Physics, University of Aarhus, DK-8000 Aarhus C, Denmark

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K-vacancy fractions have been measured for O ions moving in Fe and for F ions moving in Fe, Co, and Ni at velocities $2.1 \leq v/v_0 \leq 10.5$. Discontinuities that explain those found for the transient magnetic field are observed. The present findings indicate that (i) the transient field cannot be approximated by a linear velocity dependence for $Z_1 \leq 8$ in Fe and for $Z_1 \leq 9$ in Co and Ni, and (ii) the discontinuity in the transient field is only present at velocities below $4v_0$.

Much effort has recently been put into the investigation of the origin of the transient magnetic field (TMF) which acts on nuclei slowing down in ferromagnetic materials.[1-5] It is presently believed that the TMF is caused by polarized electrons in bound *ns* states of the moving ion

