Bose-Einstein Condensation of Pions in Energetic Heavy-Ion Collisions?

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Experimentally observable consequences of a possible Bose-Einstein condensation of pions in energetic heavy-ion collisions are demonstrated. An excess of low-energy pions in recent data may indicate a Bose-Einstein–condensation–like phenomenon.

In a recent paper[1] the possibility of the formation of Bose condensed pions was pointed out. This condensation is similar to the superfluid phase of liquid ^{4}He and is completely different from the Migdal-Sawyer–type condensation.[2] (The Migdal-Sawyer–type pion condensation appears as a result of the p-wave pion-nucleon-nucleon coupling, while the Bose-Einstein condensation shows up in a pure Bose system, too, as a consequence of the cooling of the system below the critical temperature.) The aim of the present work is to show how the consequences of such a "condensation" would show up in the experimental data in a heavy-ion reaction. (Because of the smallness of the volume that we are dealing with in this case, naturally no real phase transition can occur. At low enough temperatures, however, specific features of Bose statistics do show up, which in a large enough volume would correspond to a phase transition.) The present calculation does not aim to fit experimental data but rather to point out qualitative features to be expected due to Bose statistics in heavy-ion reactions.

We assume that in a central or near-central collision of heavy ions a piece of hot and dense hadronic matter is created, which we henceforth call the firecloud. The formation of hadronic matter with these properties has been suggested by several authors.[3,4] (The experimental data of Ref. 3 are reported[5] to be in error. In the spirit of our qualitative approach, mentioned above, we apply a simple model to the reaction. The reported change in the experimental data influences quantitative details of the model. This will be discussed later in this paper.)

The main interest of the present work, as well as that of Ref. 1, lies in the evolution of the firecloud. In both cases a simple model has been used to describe its formation. Head-on collision of symmetric systems was considered. The Fermi motion was negelected. All numerical results in the present paper are given in the center-of-mass frame. The volume V of the firecloud, its temperature T, and the number of pions, N, contained in it were calculated as in Ref. 1. In the case of relativistic Bose statistics, the above quantities are related by[6]

$$N=\frac{d}{\exp\{(mc^2-\mu)/kT\}-1}+\frac{Vd}{(2\pi\hbar)^3}\int\frac{d^3p}{\exp\{[(m^2c^4+p^2c^2)^{1/2}-\mu]/kT\}-1}, \qquad (1)$$

which is an implicit equation for the chemical potential μ. In Eq. (1), kT is the firecloud temperature in MeV, mc^2 is the pion rest energy, and $d=(2S+1)(2I+1)=3$ is the spin-isospin degeneracy of pions. As is well known in Bose statistics, μ never exceeds mc^2. Since in heavy-ion collisions we encounter a relatively small volume V, the contribution of zero-energy particles has been included at all temperatures T. This is uncommon in the context of usual statistical physics, where the volume has macroscopic dimensions. Once the chemical potential μ has been calculated the energy distribution of the pions is given in the rest frame of the firecloud as

$$dn(E) = \begin{cases} \dfrac{d}{\exp\{(mc^2 - \mu)/kT) - 1}, & E = mc^2 \\ \dfrac{4\pi V d}{(2\pi\hbar c)^3} \dfrac{E(E^2 - m^2c^4)^{1/2}}{\exp\{(E-\mu)/kT\} - 1} dE, & E > mc^2, \end{cases} \quad (2)$$

where $E = (m^2c^4 + p^2c^2)^{1/2}$ is the total energy of the particle. The above distribution is plotted in Figs. 1 and 2 as a function of pion kinetic energy $E_{\text{kin}} = E - mc^2$ for the reaction U + U at the laboratory bombarding energy 1400 MeV/A. Different sections of the figures refer to different temperatures corresponding to different stages of the expansion. In Fig. 1 the zero-kinetic-energy term

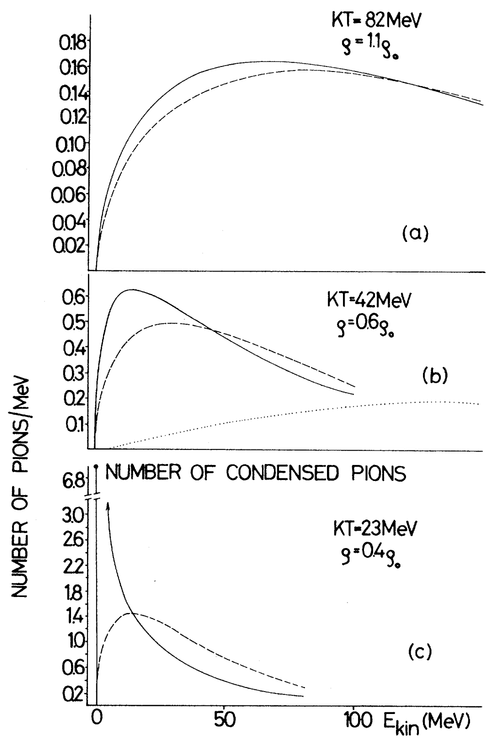

FIG. 1. The number of pions/MeV in the c.m. system is plotted as a function of pion kinetic energy in central U + U collisions at 1400 MeV/nucleon bombarding energy. The solid curve refers to the Bose distribution, the dashed line to the Boltzmann one. The Bose distribution is calculated according to macroscopic statistical physics practice (see text). Contribution from δ decay after breakup is displayed by the dotted curves. (a), (b), (c) refer to different stages of expansion characterized by the temperature kT in MeV, and the baryon density in terms of the standard nuclear matter density $\rho_0 = 0.47\ \text{fm}^{-3}$. (a) an early stage of the expansion of the firecloud; (b) an expansion stage around the breakup; (c) (probably unrealistic) a clearer demonstration of the effect of "Bose condensation".

was excluded above the critical temperature. Below the critical temperature this term ("condensate") was included and the continuous part was calculated with $\mu = mc^2$. In the case of macroscopic volumes this is the usual procedure—giving rise to a real (second-order) phase transition. (The order parameter for this case can be given as the square root of the zero-energy pion density.) In Fig. 2 the contribution of the zero-kinetic-energy term was included at all temperatures. It is clear from Fig. 2 that this term is not negligible in the expansion stage for volumes V of nuclear dimensions. In this case no real phase transition occurs. However, the contribution of the zero-kinetic-energy pions increases strongly with decreasing temperature. In order not to be in contradiction with the uncertainty principle, the zero-energy pions were uniformly smeared in the 0–5-MeV interval in Fig. 2. The corresponding Boltzmann distributions are also shown for comparison. As can be seen from the figures, at high temperatures the Bose and Boltzmann distributions are similar to each other. As the temperature decreases, however, a marked difference begins to develop. This difference has two characteristic features: (i) an excess of pions at the lowest energies, specifically, in the zero-energy state of the Bose distribution and (ii) a larger slope of the Bose distribution immediately after the maximum of the spectrum.

The experimentally observed pion spectrum will reflect the properties of the distribution at the breakup of the firecloud. This is expected to happen[7] in that stage of the reaction where the distributions are around the ones displayed in sections (b) of the figures. At this expansion stage there are approximately as many δ particles as pions. The contribution of the pions originating from the δ decay after breakup is shown by the dotted curve in the figures. These pions will make the difference between the Bose and Boltzmann distribution less observable at higher energies in the spectra.

In a recent experiment[8] an excess of very low-energy pions and a larger slope of the pion energy spectra than that of the nucleon distributions

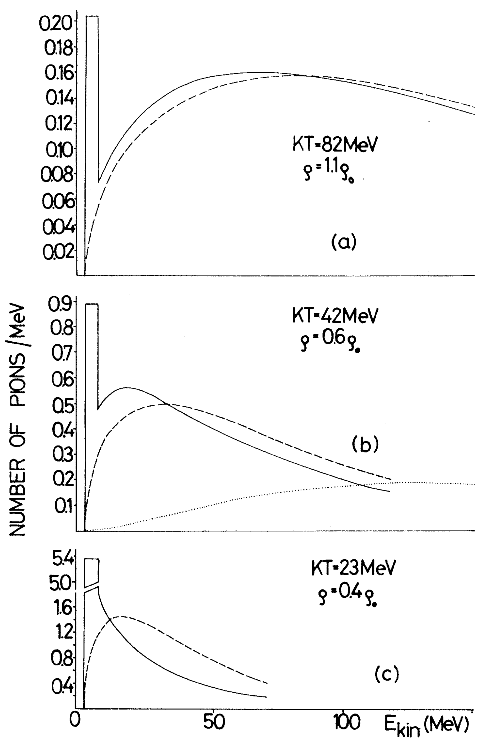

FIG. 2. As for Fig. 1, except that the contribution of zero–kinetic-energy term is included at all temperatures smeared over a 5-MeV interval. The smallness of nuclear volume V prevents the setin of a real phase transition but the contribution of the zero–kinetic-energy pions increases strongly and continuously with decreasing temperature.

was reported. These phenomena are qualitatively similar to the ones predicted above.

In the present model the possible compression of nuclear matter was not taken into account. This would undoubtedly effect the initial stages of the reaction and, specifically, the maximal temperature of the firecloud. In the expansion phase, where $\rho \lesssim \rho_0$, the initial compression will result mainly in a lower temperature and in a deviation from spherical symmetry. The lower temperature would, however, increase the number of low-energy pions [see Eq. (2)] and thereby make the effect pointed out in the present work more pronounced.

Finally we emphasize again that the predictions of the present work should be understood as qualitative ones. In order to obtain more realistic results—numerically comparable to experimental data—further investigations are needed. (For example, the omission of Fermi motion within the nuclei leads to an underestimation of the number of produced pions in the present model. Perhaps one should also consider the fact that low-lying energy states of pions are populated, therefore an induced pion emission may show up.) On the other hand, further experiments along the lines of Nagamiya[8] and Wolf *et al.*[9] in which low-energy pions can be identified in near-central collisions are called for in order to clarify the situation in connection with the importance of Bose statistics. Also event-by-event analyses in 4π-steradian detectors such as streamer chambers or nuclear emulsions could be useful to get hints of such effects.

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