integers.

We come now to the end of our proof. Observe that the combinatoric factors in the expansion (6), since $|\Gamma_1|+\ldots+|\Gamma_q|$ is held fixed, do not exceed $4^{|\Gamma|}K^{|\Gamma_1|+\ldots+|\Gamma_q|}$, where K is a constant (see, for instance, Ref. 11). This estimate together with (17) and the bounds on $D^{(1)}$ and $D^{(2)}(\lambda)$ proves the existence of an asymptotic expansion for $\langle F\rangle$ in the region $\lambda \gg 1$ to the first order. It is easily seen that the same proof applies to any order.

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Measurement of $\pi^{\pm}p$ Backward Elastic Scattering between 30 and 90 GeV/*c*

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Measurements of $\pi^{\pm}p$ backward elastic scattering have been made for incident pion momenta between 30 and 90 GeV/c and for $0<-u<0.5$ (GeV/c)^2. The momentum dependence of the cross sections is of a form expected from a simple Regge model, and the u dependence of the cross sections is similar to that observed at lower momenta.

There have been many experiments on $\pi^{\pm}p$ backward elastic scattering above the resonance region since the initial studies of this process over a decade ago.[1-8] Up to incident laboratory momenta (p) of ~15 GeV/c, the data show several characteristic features: The momentum dependence of $d\sigma/du$ at $u=0$ is of the form p^{-n}, where $n^-\approx 2$ for π^-p and $n^+\approx 2.7$ for π^+p scattering; for fixed incident momentum, the dependence of $d\sigma/du$ on u near $u=0$ is of the form e^{Bu}; and for π^+p scattering a sharp dip in $d\sigma/du$ is observed near $-u=0.15$ (GeV/c)^2.

These results were in general agreement with the predictions of an early model based upon Regge baryon exchanges using linear trajectories,[9] which gives

$$d\sigma/du = f(u)p^{2\alpha(u)-2},$$

where $\alpha(u)=\alpha_0+\alpha' u$ is the exchanged trajectory and $f(u)$ is some function of u only. It was assumed that the N_α and Δ_δ trajectories dominate π^+p and π^-p scattering, respectively.

Subsequent results such as those from polarized-target experiments and the reaction $\pi^-p\to n\pi^0$ could not be explained in terms of this simple Regge model, with the result that additional trajectories, absorption, and cuts were included in order to fit the data.[10-12] However, for $\pi^{\pm}p$ backward elastic scattering, the simple model is expected to remain approximately valid. Both nucleon and Δ exchange can contribute to π^+p, and only Δ exchange contributes to π^-p scattering. Since the π^-p cross section falls more slowly with increasing momentum than the π^+p cross section, it is expected that Δ exchange will dominate both reactions above ~50 GeV/c. This gives the prediction that for $u=0$ and above 50 GeV/c incident momentum, $d\sigma/du$ for π^-p should become larger than that for π^+p (contrary to the situation at lower momenta) and that the π^+p u distribution should become similar to that for π^-p.[13]

When results became available from Babaev and co-workers at 25 and 40 GeV/c,[14,15] they showed that $d\sigma/du$ at $u=0$ for $\pi^-n \to n\pi^-$ (isotopically equivalent to $\pi^+p \to p\pi^+$) was about a factor of 3 larger than expected from a $p^{-2.7}$ dependence extrapolated from the results at lower momenta. Since this was not easily explained by simple Regge models, at least using linear trajectories, models using considerably different ideas were proposed in order to fit the data.[16-18] In addition, while the u distributions near $u=0$ had the form e^{Bu} as at lower momenta, the values of B obtained at 25 and 40 GeV/c for π^- were substantially larger than those found below 15 GeV/c, also in disagreement with simple Regge predictions.

We have carried out an experiment in the $M6$ beam at Fermilab on $\pi^\pm p$ backward elastic scattering from 30 to 90 GeV/c, covering $0<-u<0.5$ (GeV/c)^2, in order to test the theoretical predictions at the highest possible momentum, and also to repeat measurements in the momentum region studied by Babaev and co-workers.

The experimental layout is shown in Fig. 1. The beam intensity was $\sim 2\times 10^6$ particles per 1-sec spill. Incident pions were identified by a threshold gas Cherenkov counter and their direction onto the 1-m hydrogen target recorded by proportional wire chambers (PWC's). Both outgoing particles were detected in magnetic spectrometers with use of PWC's. Three scintillation-counter hodoscopes (S, F^α, F^β) were used in the trigger, with electronic coincidence logic strongly favoring backward elastic events. In addition, there were several veto scintillators in the target region and a 9-m-long threshold gas Cherenkov counter in the forward arm to veto forward pions. To maintain resolution at higher momenta, the forward spectrometer for -50, ± 70, and -90 GeV/c was stretched by about a factor of 2 compared to that shown in Fig. 1, and a second BM109 magnet was added.

In the analysis, tracks were first reconstructed from the PWC data in the backward arm with the requirement that they extrapolate both to the target and to the S hodoscope element involved in the trigger. By use of elastic kinematics and the measured backward scattering angle, predictions were made for the momentum of the backward pion as well as the scattering angle and momentum of the corresponding forward proton. The measured quantities were compared to the predictions, and cuts were made on the differences. In addition, cuts were made on the number of PWC and hodoscope hits and on the time of flight of the backward pion. The inelastic background which remained after all such cuts was subtracted for each u bin, such subtractions always being under 10%.

Corrections have been made to the data for absorption and decay of particles, Coulomb scattering, muon and electron contamination of the pion beam, and PWC inefficiencies. Studies showed

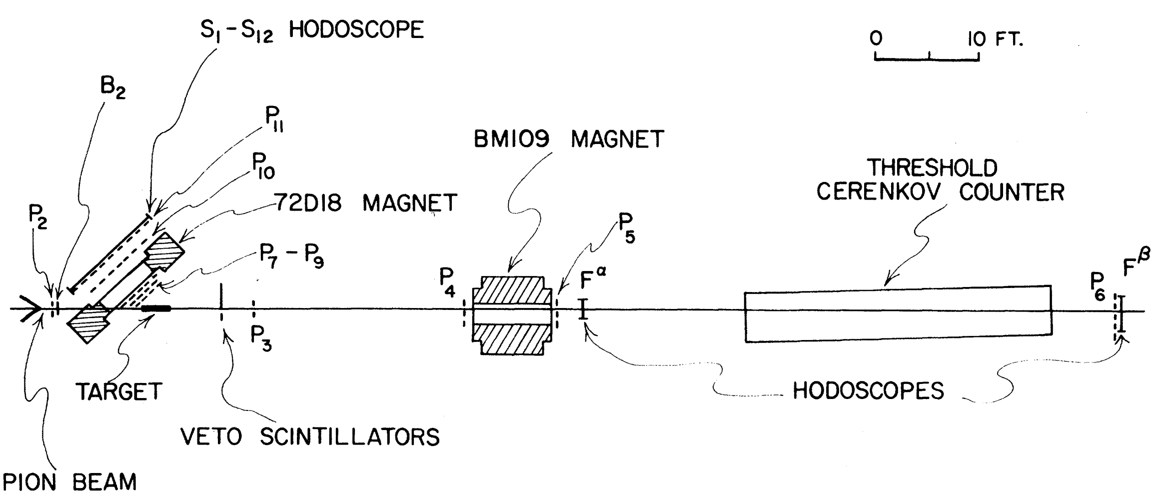

FIG. 1. Layout of the experiment for 30 and 50 GeV/c. P_2–P_{11} represent groups of PWC's; S_1–S_{12}, F^α, and F^β are scintillation-counter hodoscopes. B_2 is a scintillation counter. Not shown are proportional chambers, scintillation counters, and a threshold Cherenkov counter in the incident pion beam upstream from the region shown.

that the combined efficiency of the PWC's and analysis programs for observing elastic events was ~ 80%. Geometrical acceptance was determined by means of a Monte Carlo calculation. Empty-target runs showed that events not originating in the hydrogen constituted a negligible background. We have applied radiative corrections to the data.[19-22] These change the overall normalization of our data by factors between 1.0 for 30-GeV/c π^+ and 1.20 for 90-GeV/c π^-, but have negligible effect on the u distributions. Because of uncertainties in the corrections made to the data noted above, the overall uncertainty in the absolute magnitude of our cross sections is estimated to be ± 15%.

The results are shown in Fig. 2 together with some earlier data. Several features should be noted.

(i) The u distributions for both π^+ and π^- are very similar to those obtained below ~ 15 GeV/c. In particular, for π^+p, the steep backward peak and the dip at $-u = 0.15$ (GeV/c)^2 are present at least up to 50 GeV/c. The 70-GeV/c data are also consistent with these features.

(ii) The $d\sigma/du$ distributions near $u = 0$ show exponential dependence on u. Fitting the data for $-u \lesssim 0.1$ for π^+ and $-u \lesssim 0.5$ for π^- with the form e^{Bu}, we obtain values of B shown in Fig. 3; for both π^+ and π^- they are consistent with a slow increase between 5 and 90 GeV/c.

(iii) $d\sigma/du$ at $u = 0$ for both π^+ and π^- falls with increasing momentum; all data above ~ 5 GeV/c including our own can be fitted with the form p^{-n} as shown in Fig. 4. Using the data shown (except for Ref. 14) and quadratically adding the quoted systematic and statistical errors for each experiment, we obtain the values $n^+ = 2.33 \pm 0.07$ and $n^- = 2.08 \pm 0.06$. Data at other values of u can also be fitted with the form p^{-n}, where n is a function of u.

(iv) Our data do not confirm the large value of $d\sigma/du$ at $u = 0$ for π^+, or the large value of B for

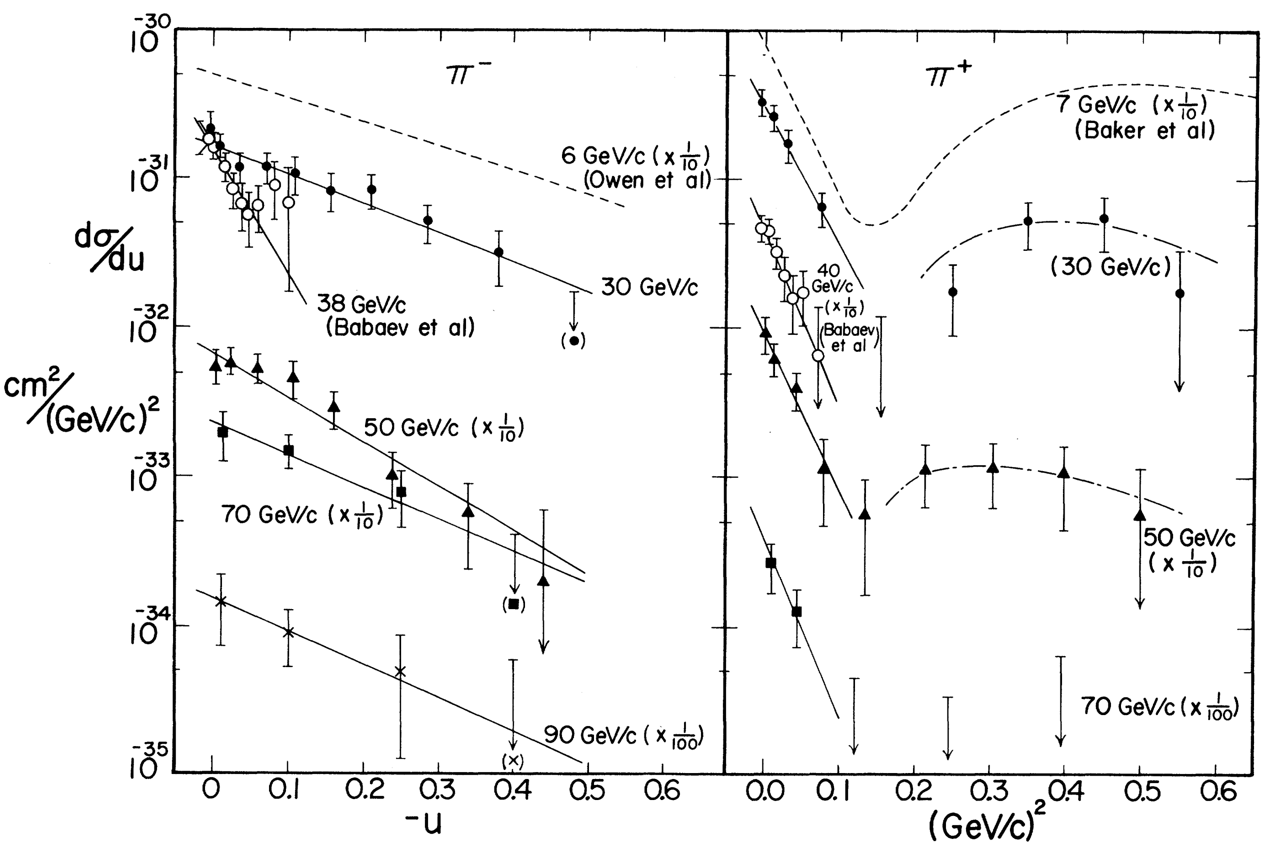

FIG. 2. Results of this experiment for the u dependence of $d\sigma/du$ for π^-p and π^+p backward elastic scattering; upper error limits are 90% confidence. Solid lines are fits of the form e^{Bu} to the present data and those of Babaev and co-workers (Refs. 14 and 15). Results of Owen *et al*. (Ref. 3) and Baker *et al*. (Ref. 4) are represented by dashed curves. Remaining dashed lines are to guide the eye.

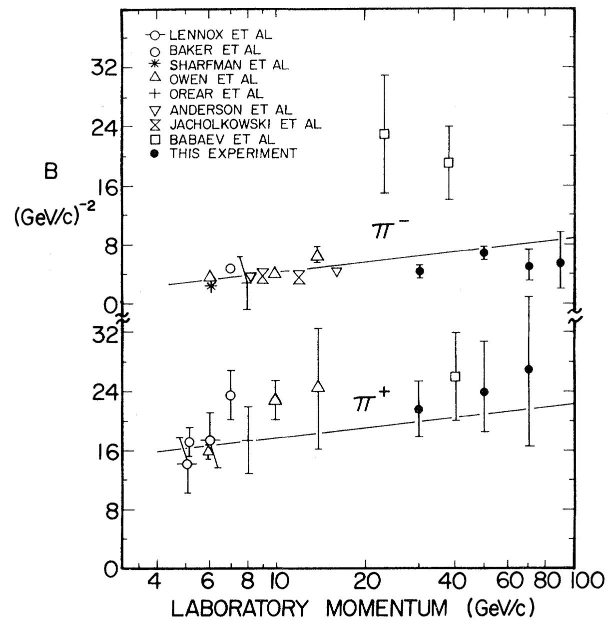

FIG. 3. Results for B. Data are shown from this experiment and Refs. 1–6, 8, 14, and 15. The two lines shown are fits of the form $B = 2\ln p$ + const to the data for each pion sign.

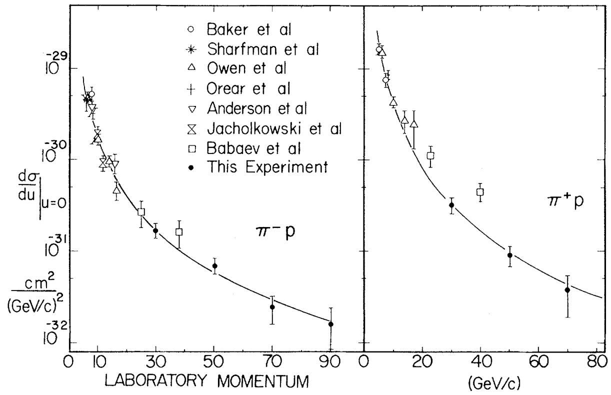

FIG. 4. Results for $d\sigma/du$ at $u = 0$. Data are shown from this experiment and Refs. 1–4, 6, 8, 14, and 15. The curves are the fits described in the text.

π^-, observed in the data of Babaev and co-workers.[14,15] In the latter case, it should be noted that the data used in their fit cover a much narrower u range than ours.

All data on $\pi^{\pm}p$ backward elastic scattering above the resonance region can be fitted with the parametrization given by simple Regge models. The values of B shown in Fig. 3 increase with increasing momentum for π^+ and π^- consistent with the relation $B = 2\ln(p) + \text{const}$, as predicted by such models. These models also predict the p^{-n} dependence of $d\sigma/du$ for any value of u. While the value for n^- at $u = 0$ is very close to that predicted, the result for n^+ (2.33) is somewhat lower than the expected 2.7; the prediction for the π^- cross section to become larger than π^+ is consequently raised to ~400 GeV/c. This discrepancy suggests that the form assumed for the exchanged trajectory near $u = 0$ may not be strictly correct; an explanation awaits a more detailed theoretical analysis.

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