Coulomb Distortion of Pion Spectra from Heavy-Ion Collisions

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(Received 11 September 1979)

The effects of final-state π-nucleus electromagnetic interactions in heavy-ion collisions are investigated in a covariant classical formulation. Experimentally observed mid-rapidity peaks in π^+ spectra are reproduced by a simple model and are shown to be sensitive to the gross features of the time-dependent nuclear charge distribution.

Experimental studies of pion production in relativistic heavy-ion collisions have recently included measurements[1-3] of inclusive pion spectra at very low pion center-of-mass (c.m.) momenta, $p \lesssim m_\pi c$. These data reveal interesting structure in the low-p region not seen in spectra at higher momenta.[4] A very distinctive feature observed in collisions of nearly equal target and beam nuclei is an enhancement of the π^+ yield in the mid-rapidity region, which appears as a peak in the invariant cross section at $\theta_{\text{c.m.}} = 90°$. Such a peak was observed at $p_\perp \approx 0.4\, m_\pi c$ by Wolf *et al.*[1] in collisions of ^{40}Ar + ^{40}Ca at a beam energy per nucleon $E/A = 1.05$ GeV, and at a slightly higher $p_\perp$ by Chiba *et al.*[2] in ^{20}Ne + NaF at $E/A = 0.8$ GeV. Speculations have attributed these peaks to pion shadowing[2] or possible hydrodynamic flow effects.[1] Additional structure in the low-p region was found at $\theta_{\text{c.m.}} = 0°$ by Benenson *et al.*[3] in the Ne + NaF system at $E/A = 0.4$ GeV. Here a peak in the invariant π^- cross section was seen at the projectile velocity, accompanied by a dip in the π^+ cross section at the same velocity. Moreover, near $p = 0$, the π^- yield was observed to be ~3 times the π^+ yield. These data are in sharp contrast to the high-p region, where π^+ and π^- are produced in equal numbers,[4] as is naively expected at all momenta for $N \approx Z$ systems.

In this Letter we discuss distortion of low-momentum charged-pion spectra by the final-state π-nucleus electromagnetic interaction, giving a simple, unified explanation for both the mid-rapidity peaks seen in π^+ spectra and the observed π^+/π^- differences. As a first approximation to this complex time-dependent quantal problem, we adopt purely classical methods, the applicability of which is supported by the momentum-space localization of the observed structure.[5] Our basic picture is that during the collision of two heavy ions a pion is emitted from an initial space-time point $s_i = (\vec{r}_i, t_i)$ with c.m. momentum $\vec{p}_i$ and subsequently follows a trajectory determined by its electromagnetic interaction with the rest of the system, attaining an asymptotic momentum $\vec{p}_f$. The nuclear charge distribution, dispersed predominantly along the beam direction, will then act to focus positive pions into the mid-rapidity region, as is seen for long-range α particles produced in fission.[6] The Lorentz-invariant inclusive pion production cross section is given by

$$E_f \frac{d^3\sigma}{d^3p_f} = \int d^2b\, d^4s_i \sum_{p_i} F(\vec{p}_i, s_i, b) \left| \frac{d^3p_i/E_i}{d^3p_f/E_f} \right|, \quad (1)$$

where b is the ion-ion impact parameter, F is a source function describing pion emission in the absence of the final-state electromagnetic interaction (number of pions emitted per unit invariant phase space per unit invariant volume), and the sum is over all $\vec{p}_i$ leading to $\vec{p}_f$ via trajectories originating at s_i. For $N = Z$ systems, F is expected to be the same for both positive and negative pions.

The two major uncertainties in the evaluation of Eq. (1) are the pion source function and the time-dependent nuclear charge distribution which determines the pion trajectories. Although cascade or hydrodynamical calculations could be used to generate detailed predictions for both these functions, we have chosen the simpler alternative of fixing a physically reasonable form for F and then investigating the effects of different charge distributions on the observed spectra. We assume that F is separable into momentum- and geometry-dependent parts, $F(\vec{p}_i, s_i, b) = f(\vec{p}_i) G(s_i, b)$, and for $f(\vec{p}_i)$ we extend to the low-p region an isotropic, exponential energy dependence, $f \sim \exp(-E_i/T)$, which approximately describes the experimental spectra at higher momenta.[4] For the Ar + Ca system at $E/A = 1.05$ GeV studied below, a slope parameter $T = 78$ MeV is taken from the empirical systematics of Nagamiya.[4] As a simple model of the geometrical dependence of F, we assume that each ion in its rest frame has a Gaussian mass distribution [$\rho \sim \exp(-r^2/r_0^2)$, with r_0 chosen so that the rms radius is equal to that of a uniform spherical distribution with radius $1.2A^{1/3}$ fm], and that the two ions are transparent both to pions and to each other. G is then taken to be proportional to the overlap of the two ions, $\rho_1\rho_2$, giving for equal-mass systems

$$G(s_i,b) \sim \exp[-2(\gamma^2 x_i^2 + y_i^2 + z_i^2)/r_0^2] \exp(-2\gamma^2 v^2 t_i^2/r_0^2) \exp(-b^2/2r_0^2), \quad (2)$$

where v is the c.m. ion velocity, $\gamma = (1 - v^2/c^2)^{-1/2}$, s_i is measured from the point of maximum density overlap, and the beam is taken to be along the x direction. Since the electromagnetic effects we are interested in manifest themselves over times much greater than the ion-ion collision time, the final spectra are fairly insensitive to the exact form of G. For the model charge distributions discussed below, Monte Carlo methods are used to evaluate Eq. (1). With the initial values $\vec{p}_i$, s_i, and b chosen from the source function F, the covariant equations of motion describing each pion trajectory are integrated numerically to obtain the asymptotic momentum $\vec{p}_f$. By integration of $\sim 2\times 10^5$ trajectories an entire spectrum with adequate statistics can be generated.

As a first approximation to the charge distribution, we take that described above for our model of G; i.e., transparent ions with Gaussian rest-frame charge distributions. Figure 1 shows the results of calculations using this "transparent" model in the Ar + Ca system for π^+ and π^-, along with the undistorted spectrum, which is taken to be the same for both pion species. It is evident that Coulomb distortion is a substantial effect which cannot be ignored in interpreting low-p spectra. The calculations show strong π^- peaking and π^+ suppression at the target and projectile velocities, in qualitative agreement with the data of Ref. 3; we expect, however, that these effects are exaggerated somewhat by our use of undisturbed nuclear charge distributions (no target or projectile fragmentation) and classical mechanics. Although the π^+ spectrum is enhanced at $y = 0$, a peak at $\theta_{\text{c.m.}} = 90°$ has not been produced by this model, a result which is insensitive to reasonable changes in $G(s_i,b)$ or in the slope parameter T.

A more realistic model which places some charge nearly at rest in the center of mass after the collision, in the spirit of the fireball[7] or firestreak[8] geometries, does result in a midrapidity peak. To demonstrate this in the above equal-mass system, we retain F as given above and assume the same two-Gaussian charge distribution until >99% of the pions have been emitted (and the ions are ~8 fm apart). At this time we change the charge configuration to two receding Gaussian spectators together with a uniform line charge along the x axis with c.m. length $2vt$. We take the total charge in this line to be a fraction, α, of the participant charge calculated with the fireball geometry of Gosset *et al.*[7]; the remaining charge is left in the spectator fragments.

Figure 2 shows the experimental π^+ data from the Ar + Ca reaction, along with π^+ spectra calculated using the above line-charge model with $\alpha = 1$ and $\alpha = \frac{1}{4}$. While $\alpha = 1$ results in a peak at too high a momentum, $\alpha = \frac{1}{4}$ gives a good reproduction of the experimental spectrum. The π^-

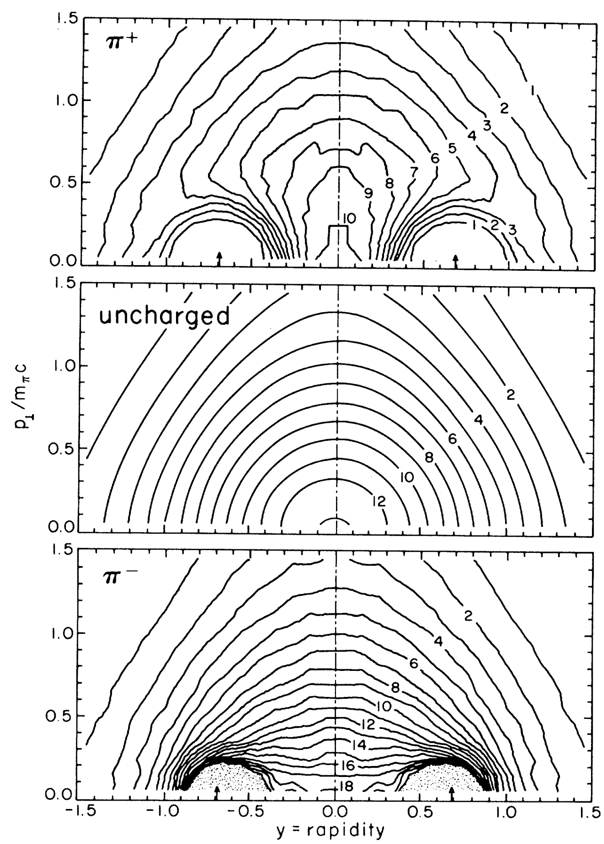

FIG. 1. Contours of constant Lorentz-invariant inclusive pion cross section, $Ed^3\sigma/d^3p$, in units of b/(sr GeV^2 c^{-3}), for the system ^{40}Ar + ^{40}Ca at E/A = 1.05 GeV. Calculations of π^+ and π^- spectra using the "transparent" model are shown, along with the undistorted (uncharged) spectrum. Normalization is as in Fig. 2. Statistical uncertainties can be inferred from the jagged computer-drawn contours, and arrows indicate beam and target rapidities. For clarity, the two large π^- peaks are not shown.

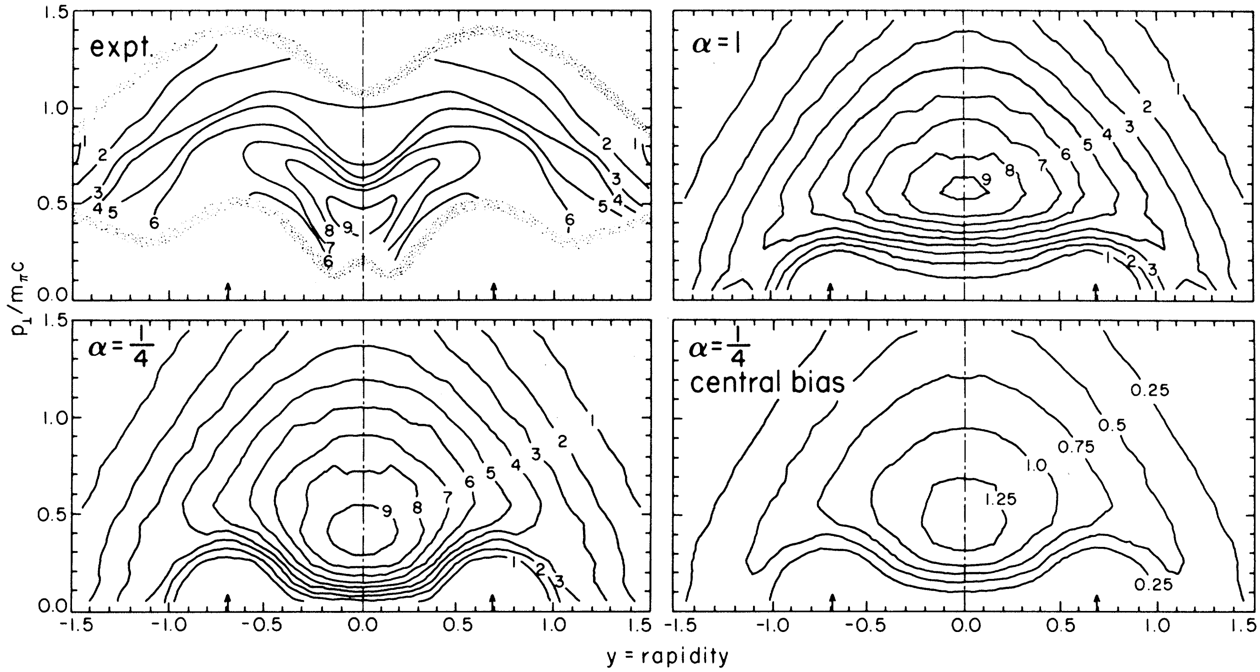

FIG. 2. Positive-pion spectra from Ar + Ca collisions at $E/A = 1.05$ GeV as in Fig. 1. Shown are the experimental data from Ref. 1, along with three spectra calculated using the "line-charge" model described in the text. The shaded edges indicate the limits of the experimental measurements. The $\alpha = \frac{1}{4}$ spectrum was normalized to agree with experiment at the peak, and the other plots were normalized to it by requiring equality at high momenta. In the lower right-hand spectrum, the $\alpha = \frac{1}{4}$ model was weighted in impact parameter to bias toward more central collisions.

spectrum for $\alpha = \frac{1}{4}$ is similar to that in Fig. 1. The small value of α needed to describe the data is probably a result of the singular nature of the assumed line charge, which also results in an unphysical vanishing π^+ yield at $p_\perp = 0$. We have found that more complex models including the dispersion of the participant charge on a pion dynamical time scale ($\sim 10^{-22}$ sec) reduce the Coulomb effects sufficiently to allow $\alpha = 1$. The experimental multiplicity independence[1] of the mid-rapidity peak is also reproduced, as can be seen from the lower right-hand spectrum in Fig. 2. Here we have biased toward more central collisions by adding a Gaussian impact-parameter weighting, $\exp(-b^2/b_0^2)$, to Eq. (2), with $b_0 = 1.5$ fm chosen so that only 15% of the total initial source strength is retained.

Despite this encouraging agreement, certain aspects of the data seem to indicate the need for a source function which is more structured in $\vec{p}_i$ than the smooth isotropic one we have assumed. For instance, note that all of the above models only slightly distort the initial spectrum for $p_\perp \gtrsim 0.5\ m_\pi c$, leaving the high-$p$ side of the calculated mid-rapidity peak rounded. Thus the peculiar "V-shaped" form of the experimental π^+ peak in the Ar + Ca system has not been reproduced. In addition, the above model with $\alpha = \frac{1}{4}$ applied to the Ne + NaF system at $E/A = 0.8$ GeV predicts a peak at a smaller value of $p_\perp$ than is observed; here the presence of a direct nucleon-nucleon component[2] in the observed pion spectrum further suggests the need for an improved source function. Measurements of π^0 spectra might be useful in this connection to better define F.

In summary, our results strongly support the hypothesis of a simple, dynamical Coulomb origin for much of the structure in low-momentum charged-pion spectra from relativistic heavy-ion collisions. This could be unambiguously verified by comparison of π^+ and π^- spectra in the entire low-p region, since for $N \approx Z$ systems the π^+/π^- differences would be predominantly a result of the electromagnetic interaction. Moreover, within the context of a plausible model for the pion source function, we have demonstrated that the observable inclusive cross sections are sensitive to the space-time evolution of the nuclear charge distribution, and that some charge must remain nearly at rest in the center of mass after the collision, probably dispersing on a time scale $\sim 10^{-22}$ sec. These Coulomb effects are thus a potentially powerful tool for investigating the dynamics of the nuclear charge distribution and the mechanisms of pion production in relativistic heavy-ion collisions.

This work was supported in part by the National Science Foundation (Contracts No. PHY-77-21602 and No. PHY76-83685), and the California Institute of Technology Summer Undergraduate Research Fellowship Program. One of us (S.E.K.) also gratefully acknowledges support by an Alfred P. Sloan Foundation Research Fellowship.

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[5]P. J. Siemens and J. O. Rasmussen, Phys. Rev. Lett. 42, 880 (1979), discuss the $\theta_{c.m.} = 90°$ π^+ spectrum in a static, quantal model for the Coulomb effects in S waves induced by a *single* charge equal to that of the whole system. This picture is unsatisfactory in that it (i) predicts a peak at all angles, contrary to experiment; (ii) assumes an implausible collision geometry, even for head-on collisions; (iii) ignores the time dependence of the nuclear charge distribution. Furthermore, the small angular size of the observed peak in momentum space indicates that partial waves up to at least $l \approx 10$ are important. Our calssical calculations emphasize the dynamical aspects of the problem, but will be invalid for regions of phase space where quantal penetration is important, i.e., near $p_\perp = 0$, where the classical cross section vanishes.

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H(2*s*) Formation and the Lyman-α Polarization in 1-7-keV H^+-H Collisions

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(Received 12 July 1979)

A multistate molecular approach to the proton–hydrogen-atom collision is formulated. Spurious long-range couplings are avoided and Galilean invariance enforced through variationally determined momentum translation factors. Well-defined radial and rotational couplings are employed in the 1–7-keV energy range in a ten-state calculation. Good agreement with Bayfield is obtained for the 2*s* charge-exchange probability. Concerning polarization of Lyman-α radiation, comparison with Kauppila *et al.* is made.

The proton–hydrogen-atom collision has been well studied over a wide range of energy, angle, and final-state products. In this Letter we shall be concerned only with energies below 25 keV where a molecular or hybrid-molecular description would appear to be necessary.

Following the pioneering work of Mott,[1] Massey and Smith,[2] and Bates, Massey, and Stewart,[3] Bates and McCarroll[4] formulated the first satisfactory impact-parameter, perturbed–stationary-states (PSS) treatment, in that the inclusion of momentum translation factors avoided spurious long-range divergent couplings and enforced Galilean invariance. This treatment was first applied in the two-state approximation by Ferguson[5] and later in the three-state approximation and in the low-velocity limit by Bates and Williams[6] and McCarroll and Piacentini.[7] Later impact-parameter multistate PSS treatments[8-10] were unfortunately not Galilean invariant and did involve spurious long-range couplings.

Crothers and Hughes,[11] to be referred to as I, set out to improve the Bates-McCarroll theory by determining the momentum translation factors

