Operator Analysis of Nucleon Decay

Frank Wilczek
Department of Physics, Princeton University, Princeton, New Jersey 08544

and

A. Zee
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 14 September 1979)

The operator structure of the effective Hamiltonian mediating nucleon decay is analyzed. Selection rules diagnostic of the underlying superunified theories are derived. A kinship hypothesis important in analyzing nucleon decay is formulated and tests of it are proposed.

We now have gauge theories of the strong and of the electroweak interactions, based on SU(3) and SU(2) ⊗ U(1), respectively, for which the evidence is becoming persuasive. It has become of great interest to see if these mathematically similar theories can be united in a larger superunified gauge theory.[1] Some evidence in favor of this step is provided by the successful calculation of $\sin^2\theta_W$.[2]

The accessible direct consequences of superunification [i.e., which cannot be described in the framework of SU(3) ⊗ SU(2) ⊗ U(1)] are probably quite limited. In fact, only nucleon instability seems to fall into this category.[3,4] It is therefore important to analyze this process fully for tests of the superunification idea in general and for means of distinguishing different patterns of superunification.

Classification of operators.—Nucleon decay in superunified theories arises from exchange of very heavy particles. There is a standard two-step method for analyzing effects of heavy particles at low energies.[5] In the first step the graphs involving heavy particles are evaluated at large external momenta and an effective Hamiltonian involving only light particles is formed. In the second step the operators appearing in the effective Hamiltonian are renormalized down to small external momenta by exchange of light particles. In both these steps, until the external momenta become comparable to the light-particle masses, the light particles may be regarded as massless and the full symmetry of the massless theory is valid to a good approximation. Subsequent renormalization (from $m_w \approx 10^2$ GeV to 1 GeV or so) is well understood.

The most important operators in the effective Hamiltonian for any process are in general those of minimal dimension with the appropriate quantum numbers. Higher-dimensional operators will be suppressed by successive powers of (light mass)/(heavy mass).

Operators with nonzero baryon number must have at least three quark fields to be color singlets, and then at least four fermion fields to form a Lorentz scalar. So their dimension is at least six. In analyzing nucleon decay we are therefore led to the problem of classifying operators of dimension 6 with $\Delta B \neq 0$, forming SU(3) ⊗ SU(2) ⊗ U(1) singlets. A straightforward analysis leads to the following exhaustive list of possibilities: Vector type,

$$O_1 = \bar{e}\gamma_\mu au\bar{u}\gamma_\mu ad + \bar{\nu}\gamma_\mu au\bar{d}\gamma_\mu ad, \quad (1)$$

$$O_2 = \bar{u}\gamma_\mu ae\bar{d}\gamma_\mu au; \quad (2)$$

scalar type,

$$O_3 = \bar{e}Cu\bar{u}Cd + \bar{\nu}Cd\bar{d}Cu, \quad (3)$$

$$O_4 = \overline{Cau}ae\bar{u}Cd, \quad (4)$$

$$O_5 = \bar{e}Cu\overline{Cau}ad + \bar{\nu}Cd\overline{Cau}ad, \quad (5)$$

$$O_6 = \overline{Cau}ae\overline{Cau}ad; \quad (6)$$

and tensor type, similar to scalar type. In writing these operators we have for simplicity written only fermions of the first family, ν, e, u, d, suppressed color indices, and understand both f and af to be left-handed. For any fermion field f,

$$af \equiv Cf_R = \tfrac{1}{2}i\gamma_2(1+\gamma_5)f^*$$

destroys the left-handed anti-f; C, of course, represents charge conjugation.

It is important to notice that O_1 is *antisymmetric* in the exchange $au \leftrightarrow ad$—there are minus signs for Fermi statistics, Fierz transformation, and color. Similarly O_5 and O_6 are antisymmetric under $au \leftrightarrow ad$ and O_2, O_3, O_4 are antisymmetric under an exchange $u \leftrightarrow d$. The tensor operators are *symmetric* under the corresponding exchanges.

Physical consequences.—All these operators conserve $B-L$, so in any superunified theory we expect $B-L$ to be conserved[6] up to powers of (light mass)/(heavy mass) $\sim 10^{-13}$ in nucleon decay. We also have the rule $\Delta S \geq 0$ up to weak interaction corrections.

In the most likely case that nucleon decay is mediated by vector-boson exchange we get only operators O_1 and O_2 in the effective Hamiltonian for $\Delta S=0$ decay. This structure is quite restrictive, and allows some definite predictions free of uncertainties in hadronic matrix elements. This is because the hadronic pieces of the two parts of O_1 form a strong isodoublet, and the hadronic piece of O_2 is just the parity transform of the corresponding piece of O_1. Note that O_1 creates a right-handed, O_2 a left-handed, positron. Suppose that the coefficients of operators O_1, O_2 in the effective Hamiltonian are in the ratio $1{:}r$. Then we find, for example,

$$\tfrac{1}{2}(1+r^2)\Gamma(p\to\pi^+\bar\nu)=\Gamma(p\to\pi^0e^+)=\tfrac{1}{2}\Gamma(n\to\pi^-e^+)=(1+r^2)\Gamma(n\to\pi^0\bar\nu), \quad (7)$$

$$\Gamma(pe^+ + \text{any})=(1+r^2)\Gamma(n\to\bar\nu+\text{any}), \quad (8)$$

$$(1+r^2)\Gamma(p\to\bar\nu+\text{any})=\Gamma(n\to e^+ +\text{any}), \quad (9)$$

$$\Gamma(O^{16}\to e^+ +\text{any})=(1+r^2)\Gamma(O^{16}\to\bar\nu+\text{any}), \quad (10)$$

$$\Gamma(p\to e^+ +\text{any})\geq\tfrac{1}{2}(1+r^2)\Gamma(p\to\bar\nu+\text{any}), \quad (11)$$

and so forth.

The value of r is characteristic of different superunified theories and symmetry breaking patterns. For example, we have the following:

(i) In SU(5) (Ref. 7) at the tree-graph level (i.e., step one in the language above) we find $r=2$. However, the coefficient is subject to renormalization effects from the SU(2) ⊗ U(1) bosons. The calculation of these effects is presented below.

(ii) SU(5) has been extended to SO(10),[8] which gives a satisfactory theory. Symmetry breaking with a Higgs field in the adjoint representation, as used in breaking SU(5) → SU(3) ⊗ SU(2) ⊗ U(1), is readily generalized to SO(10) and breaks SO(10) → SU(3) ⊗ U(1) ⊗ U(1). The extra U(1) is coupled to $B-L-\frac{2}{5}Y$, and results of neutrino experiments indicate that it must be broken. Still we can ask if this breaking might be weaker than the breaking due to the adjoint Higgs field.[9] If we assume that just the adjoint breaking is relevant in this way, then we find that nucleon decay is mediated mainly by some bosons which are in SO(10) but not in SU(5), which couple the $\underline{5}$ to the $\underline{10}^*$ fermions in SU(5). These bosons are 5 times lighter than the others mediating nucleon decay; so their contribution is cominant by a factor of $5^2=25$ in amplitude. For these leading contributions we have $r=0$.

(iii) It has often been conjectured[10] that $SU(2)_L \otimes SU(2)_R \otimes U(1)$ might become the effective electroweak interaction theory at momenta well below 10^{15} GeV (though presumably $\gtrsim 10^3$ GeV). In this case we find $r=1$ to a first approximation.

Kinship hypothesis.—These predictions for $\Delta S=0$ nucleon decay should be supplemented with a discussion of $\Delta S=1$ decays, and of decays involving a μ^+. Jarlskog[11] has noted that nucleon decay in principle brings in new angles, analogous to the Cabibbo angle, governing, e.g., the ratio of $d\to e^+$ to $s\to e^+$ couplings. It is attractive to suppose, and has been implicitly assumed in some analyses, that "cross-family" couplings like $s\to e^+$ are small. In view of the importance of this assumption we shall give it a name and formulate it precisely, as the *kinship hypothesis* (KH): The charge-$(-\frac{1}{3})$ quarks and -(-1) leptons of definite mass are rotated by the same angles measurable at low energies in weak interactions (e.g., the Cabibbo angle), relative to the charge-$\frac{2}{3}$ quarks. In this pairing of quarks and leptons, the lightest go together, then the next lightest, etc.

The kinship hypothesis is a consequence of SU(5) if there is only one vector Higgs field, and is motivated by the partially successful relation between masses of charge-$(-\frac{1}{3})$ quarks and charge-(-1) leptons derived on this assumption.[11] It is very much open to doubt, however.

Fortunately, KH is also open to experimental test. Its consequences depend in detail on the particular superunifying theory, and we will not go into details here. In the SO(10) theory broken as described above, for example, KH leads to

$$\Gamma(n\to\mu^+\pi^-)=\tan^2\theta_C\Gamma(n\to e^+\pi^-) \quad (12)$$

One result of potential importance is that KH allows us to predict the polarization of the μ^+ in nucleon decay. In SU(5) at the tree-graph level the μ^+ is essentially unpolarized (in contrast to the e^+, as discussed above) but this will be modified by renormalization effects as calculated below. In SO(10) broken as above the μ^+ is essentially always right handed (independent of KH).

An immediate consequence of KH is that $\Delta S=1$ nucleon decays are usually associated with a μ^+, $\Delta S=0$ almost always with e^+.

Renormalization.—We have calculated the effects of renormalization due to SU(2) ⊗ U(1) boson exchange[12] from the superunification mass down to ~10 GeV on operators O_1 and O_2 in SU(5). Let the number of fermion families by f and assume that there is only one light Higgs multiplet. Then

$$O_1(\mu)=\left[\frac{g'^2(M)}{g'^2(\mu)}\right]^{-33/(80f+6)}\left[\frac{g^2(M)}{g^2(\mu)}\right]^{-27/16f-86)}O_1(M), \tag{13}$$

$$O_2(\mu)=\left[\frac{g'^2(M)}{g'^2(\mu)}\right]^{-69/(80f+6)}\left[\frac{g^2(M)}{g^2(\mu)}\right]^{-27/(16f-86)}O_2(M). \tag{14}$$

Here $O_i(M)$ and $O_i(\mu)$ are the operators normalized at external momenta M and μ, respectively, and g' and g are U(1) and SU(2) effective couplings, respectively. If $f=3$ both operators are suppressed equally by a factor ~1.4 from the SU(2) renormalizations and the U(1) renormalizations are small, ~10%. The coefficients of the operators in the effective Hamiltonian have to be adjusted accordingly, and thus the effect is to increase both r and the overall decay rate, and to make the μ^+ predominantly left handed.

One of us (F.W.) is grateful to S. Treiman for several helpful discussions. One of us (A.Z.) thanks the Aspen Center for Physics for hospitality. This research was partially supported by the Department of Energy under Contract No. EY-76-C02-3072 and No. AT(E11-1)3071.

Note added.—After this work was completed, we learned that Steven Weinberg [preceding Letter, Phys. Rev. Lett. 43, 1566 (1979)] has performed a similar analysis.

[1]J. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[2]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

[3]Some superunified theories also lead to nonzero neutrino masses; however, the predictions so far are not very definite.

[4]Nucleon decay in superunified theories has been studied recently by M. Machacek, Harvard University Report No. HUTP/79/A021, 1979 (to be published), and reviewed by P. Langacker (to be published).

[5]For a congenial exposition see J. Collins, F. Wilczek, and A. Zee, Phys. Rev. D 18, 242 (1978). The basic framework is due to K. Wilson, refined by T. Appelquist and J. Carazzone, Phys. Rev. D 11, 2856 (1975); E. Witten, Nucl. Phys. B104, 445 (1976); H. Georgi and H. D. Politzer, Phys. Rev. D 14, 1829 (1976).

[6]This implication of SU(3)⊗SU(2)⊗U(1) invariance was brought out in an analysis of one-boson exchange contributions to nucleon decay by S. Weinberg, Phys. Rev. Lett. 42, 850 (1979). Our result is slightly stronger, not restricted to one-boson exchange.

[7]Georgi and Glashow, Ref. 1; A. Buras, J. Ellis, M. Gaillard, and D. Nanopoulos, Nucl. Phys. B135, 66 (1978).

[8]H. Georgi and D. Nanopoulos, Phys. Lett. 82B, 392 (1979), and references therein.

[9]The vacuum expectation value of the adjoint Higgs may have SU(5) 24 and 1 parts. We assume that only the 24 contributes, for concreteness.

[10]For instance, R. N. Mohapatra and D. P. Sidhu, Phys. Rev. Lett. 38, 667 (1977).

[11]C. Jarlskog, Phys. Lett. 82B, 401 (1979). We disagree with her assertion that KH does not follow from the simplest symmetry breaking.

[12]For the SU(3) renormalizations see Buras *et al.*, Ref. 7.