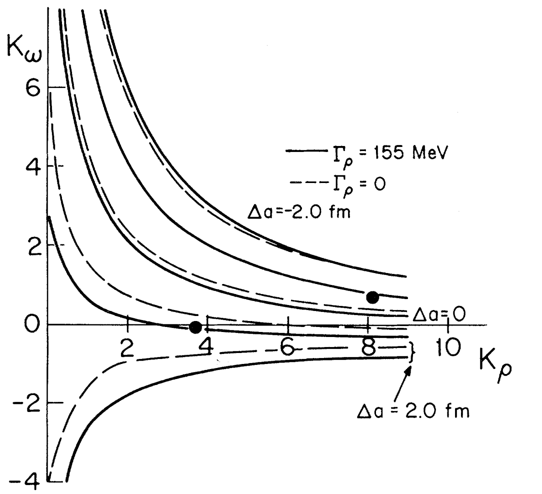


FIG. 2. Dependence of scattering length on magnetic couplings and $\rho$ width. The magnetic couplings from Refs. 4 and 6 are indicated by the points.

Because of the uncertainties due to coupling constants, it seems inappropriate to add the pseudoscalar and vector contributions. Given the importance of those effects, they must be included and better magnetic couplings determined before detailed comparison with experiment can be made.

We have presented calculation of charge-symmetry breaking in the *N*-*N* system arising from intrinsic SU(2)-symmetry breaking in the strong interaction arising from quark mass differences. There are large effects on the scattering length, of order 1 fm, even after considerable apparently accidental cancellations in both the pseudoscalar- and vector-meson contributions. These cancellations imply that the effects of heavier mesons cannot be ignored *a priori* simply because of their shorter range. Further, isospin seems likely to be a better symmetry for the nucleon-nucleon system than even the small quark-mass ratios would suggest. Finally, we have explicitly exhibited these cancellations, as well as the importance of treating the $\rho$ width correctly.

The authors are particularly grateful to Carl Dover for many helpful discussions. This work was supported in part by the U. S. Department of Energy under Contract No. EY-76-C-02-3071, and in part by a grant from the National Science Foundation.


[1]P. Langacker and H. Pagels, Phys. Rev. D 19, 2070 (1979); S. Weinberg, in *Festschrift for I. I. Rabi*, edited by Lloyd Motz (New York Academy of Sciences, New York, 1977).

[2]P. Langacker, Phys. Rev. D (to be published).

[3]E. M. Henley and G. A. Miller, in ***Mesons in Nuclei***, edited by M. Rho and D. Wilkinson (North-Holland, Amsterdam, 1979), Vol. 1, p. 405.

[4]M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D 12, 744 (1975).

[5]R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

[6]P. C. McNamee, M. D. Scadron, and S. A. Coon, Nucl. Phys. A249, 483 (1975).

[7]R. D. Brandenburg, S. A. Coon, and P. U. Sauer, Nucl. Phys. A294, 305 (1978).

[8]B. F. Gibson and G. J. Stephenson, Phys. Rev. C 8, 1222 (1973).

[9]S. A. Coon, M. D. Scadron, and P. C. McNamee, Nucl. Phys. A287, 381 (1977).

[10]J. L. Friar and B. F. Gibson, Phys. Rev. D 17, 1752 (1978).


## Effects of Heavy Particles through Factorization and Renormalization Group

Yoichi Kazama[(a)] and York-Peng Yao
*Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48109*
(Received 30 July 1979)

For renormalizable theories without spontaneous symmetry breaking, a formalism is developed to systematically extract and evaluate the effects of heavy particles in low-energy physics through factorized local operators and the renormalization group. As an application, quantum electrodynamics with electrons and muons is considered and the effects of muons on the electron anomalous magnetic moment are assessed.

It is an interesting view which is shared by many of us that the dimensionless coupling constants in various interactions are almost universal in strength. The breakup into various hierarchies is really due to the mass scales involved. In other words, the apparent strengths are intimately tied up with the relative lightness and heaviness of species of particles and the experimental conditions under consideration.

It has been shown[1] that in the absence of the Higgs mechanism,[2] when dealing with low-energy light-particle physics, the heavy particles can be neglected to the zeroth-order approximation. While this is a true statement, it can hardly be the settling point of any physical theory. On the one hand, one can increase the incident energy in an experiment so as to approach the threshold of the heavier particles. In such a situation, it should be useful to have a formalism to interpolate between the "far away" and the "near" regions, relative to these thresholds.

More important yet, even when we are far away from directly producing the heavy objects, their effects can still be felt because of virtual exchanges and vacuum fluctuations. These give rise to phenomena quite distinct from those due to the lighter sector in isolation. A case in point is, of course, the weak interaction.

These are what have motivated us to engage in a systematic study of the following questions: (1) To a certain level of accuracy to be specified later, can we catalog the effects of heavy particles when all other scales are small? If so, can we make these results quantitative? We have affirmative answers to both, and this is a short report of our findings. Details are being prepared and to be published elsewhere.[3]

Specifically, we consider quantum electrodynamics (QED) which contains light photons and electrons and heavy muons. The charges are assumed to have a universal value ($e$). We may for the moment think it to be quite strong, so that this model is a prelude to quantum chromodynamics, which is also being pursued. Let $m$ be the mass of an electron and $M$ that of a muon. Let $\Gamma^{B,F}$ be the proper amputated Green's functions with $B$ external photons and $F$ external electrons. We have been able to summarize effects of the muons to all orders in $e$ by the following formula:

$$\Gamma_{\text{full}}{}^{B,F} = \Gamma_{\text{light}}{}^{B,F} + (1/M^2)\sum_{N,b} C_{Nb}\,\Gamma_{\text{light}}{}^{B,F}(\mathcal{O}_{Nb}) + O(1/M^4), \tag{1}$$

where "light" theory is QED with photons and electrons, i.e., with all muon loops of the "full" theory deleted. The $C_{Nb}$'s are a set of universal coefficient functions, which have all the muon effects in the form of powers of $\ln(M/m)$. The $\mathcal{O}_{Nb}$'s are a set of gauge-invariant local operators made up of the light fields only. $N \leq 6$ specifies the naive dimension of the density and $b$ is a label for operators having the same dimension. In QED, there are two $N=5$ and eight $N=6$ relevant operators, although it is convenient in the intermediate stage to introduce two $N=4$ operators, which correspond to photon and electron kinetic insertions, and one $N=3$ operator which effects electron mass insertions.

In the remainder of this article, we shall indicate how Eq. (1) is derived and then write down a set of Callan-Symanzik-like[4] equations which allow us to do "improved perturbative" calculation for $C_{Nb}$. The explicit solutions to these equations, which include anomalous dimensions evaluated to one-loop order, are given. As an application, we comment on the effects of muons on the electron anomalous magnetic moment $(g-2)/2$.

Equation (1) is an application of Zimmermann's algebraic identity,[5] which is basically an algorithm to rearrange renormalized integrands. It allows us to take the large-$M$ limit at some stage after the relevant subintegration has been made sufficiently finite by oversubtraction. This is how $C_{Nb}/M^2$ are isolated. An example should make the procedure transparent. Consider the second-order electron self-energy in which the second-order vacuum polarization due to a muon loop is inserted (altogether fourth order). It is rendered finite by first subtracting the photon polarization $\overline{\pi}(l^2)$ and then by an overall renormalization, i.e.,

$$\Sigma(p) = -\frac{ie^2}{(2\pi)^4}\int d^4l\,(1-t_1{}^p)I(1-t_0{}^l)\overline{\pi}(l^2), \tag{2}$$

where in the Landau gauge, which will be used throughout,

$$I = (-g_{\mu\nu} + l_\mu l_\nu/l^2)l^{-2}\gamma^\mu(\not{p}-\not{l}-m)^{-1}\gamma^\nu, \tag{3}$$

the $t$'s are the Taylor operators, viz.

$$(1-t_0{}^l)\overline{\pi}(l^2) = \overline{\pi}(l^2) - \overline{\pi}(0), \tag{4}$$

and the generalization is obvious. We can rearrange Eq. (2) by adding and subtracting higher Taylor operators. This gives

$$\Sigma(p) = -\frac{ie^2}{(2\pi)^4}\int d^4l\,[(1-t_3{}^p)I(1-t_2{}^l)\overline{\pi} + (1-t_3{}^p)I(t_2{}^l - t_0{}^l)\overline{\pi} + (t_3{}^p - t_1{}^p)I(1-t_0{}^l)\overline{\pi}]. \tag{5}$$

The first term is readily shown to be $O(1/M^4)$ by simple power counting. The second term converges sufficiently rapidly in $l$ because of the oversubtraction $1-t_3{}^p$, and the limit $M^2 \gg l^2$ can be taken to replace $(t_2{}^l - t_0{}^l)\bar{\pi}$ by $(\alpha/15\pi)l^2/M^2$. We may consider it to be the operator-inserted Green's function $\Gamma^{0,2}(\frac{1}{4}i\int d^4x\, F^{\mu\nu}\partial^2 F_{\mu\nu})$, multiplied by a coefficient $C_{65}=\alpha/15\pi$. The third term is also of order $1/M^2$. However, the limit $M \gg p$ should be taken after the $l$ integration. We can regard it as inducing operators $O_i \sim \int d^4x\, \bar{\psi}(\not\partial^3, \not\partial^2)\psi$, which are to be inserted into the electron two-point function.

The general case is treated by extending this procedure. We partition a diagram $\Delta$ with muon loop(s) into an upper part and a lower part. The former is connected, one-particle irreducible, and contains all the muon loops. The rest is the lower part. A degree $\delta$ of divergence is assigned to this upper part according to $\delta = d+2 = 6-B-3F/2$, where $d$ is the naive degree of divergence and $B$ and $F$ are, respectively, the number of entering photon and electron lines. There are, in general, many partitions we can make for each diagram. We disregard all those with $\delta<0$ and call those with $\delta>0$ heavy parts. Then, we rearrange the renormalized integrand corresponding to $\Delta$ into

$$R_\Delta = \sum_{\tau\in T}\sum_{U_1\in\mathfrak{M}^\tau}\prod_{\gamma\in U_1}(-t_{d+2}{}^\gamma)I_{\Delta/\tau}(t_{d+2}{}^\tau - t_d{}^\tau)\bar{I}_\tau + O(1/M^4),$$
$$\bar{I}_\tau = \sum_{U_2\in N^d(\tau)}\prod_{\gamma\in U_2}(-t_d{}^\gamma)I_\tau. \qquad (6)$$

In the above formula, $\tau$ is a heavy part and all the subdivergences in the integrand $\bar{I}_\tau$ are removed conventionally. This is done by applying the fundamental forest formula,[5] where $U_2$ is a forest of $\tau$ which contains nonoverlapping renormalization parts. The set of all forests not having $\tau$ as an element is $N^d(\tau)$. The operation $t_{d+2}{}^\tau - t_d{}^\tau$, besides making the final integration in $\tau$ finite, induces local operator vertices. Such operators have dimensions higher than 4 and therefore the subsequent subtractions are done with $d+2$ counting if the renormalization parts under consideration contain $\tau$. Otherwise, they are counted by $d$. The Taylor operator $t_{d+2}{}^\gamma$ must be understood in this sense. The program of renormalization on the reduced integrand $I_{\Delta/\tau}$ is again carried out via the fundamental forest formula in which the divergent parts are grouped into forest $U_1$. $\mathfrak{M}^\tau$ is the set of all these forests. $T$ is the set of all heavy parts.

If we just want a factorization formula, then the mission is almost accomplished. However, we want to have a method to calculate the coefficient functions. Note that what we are after are $1/M^2$ effects. We need to account for mass insertions, just as in short-distance expansions when nonleading singularities are extracted. The bookkeeping of their effects[6,7] can be efficiently done by generalizing the subtraction procedure. Thus we add a small parameter $\lambda$ uniformly to the free-electron inverse propagator and renormalize electron self-energy according to $\Sigma(\not p,\lambda)=\bar{\Sigma}(\not p,\lambda)-\bar{\Sigma}(0,0)-\not p(\partial/\partial\not p)\bar{\Sigma}(0,0)-\lambda(\partial/\partial\lambda)\bar{\Sigma}(0,0)$, where $\bar{\Sigma}$ means that the subdivergences have been recursively removed. The renormalizations of the photon polarization and the vertex are similarly generalized. For counting purposes, one may treat $\lambda$ like an extra momentum. At the end, physical quantities are recovered by taking the limit $\lambda\to 0$. Since the forest formula is an algebraic statement, it can be extended to include $\lambda$ in the Taylor operators. Equation (6) will now be so understood.

We use $p$ generically to denote momenta external to $\tau$. Then Eq. (6) becomes

$$R_\Delta = \sum_{\tau\in T}\sum_{U_1\in\mathfrak{M}^\tau}\sum_{\gamma\in U_1}\sum_{a=1}^{2}\sum_{l=0}^{d(\tau)+a}(-t_{d+2}{}^\gamma)(\lambda^l p^{d(\tau)+a-l}I_{\Delta/\tau})C_{\tau,l}{}^{(a)}, \qquad (7)$$

where the coefficient functions are

$$C_{\tau,l}{}^{(a)} = \{l![d(\tau)+a-l]!\}^{-1}(\partial/\partial\lambda)^l(\partial/\partial p)^{d(\tau)+a-l}\bar{I}_\tau|_{p=\lambda=0}. \qquad (8)$$

Because the factor $\lambda^l p^{d(\tau)+a-l}$ depend only on what joins $\tau$ and $\Delta/\tau$, but not on the details of either of them, they can be converted into a set of local operators. Thereupon Eq. (1) is obtained. We now make two remarks: (i) Our subtraction procedure respects Ward identities and therefore the operators $\mathcal{O}_{Nb}$ are gauge invariant. (ii) In Eq. (8), the factors $\lambda^l p^{d(\tau)+a-l}$ for $a=1,2$ differ in dimension by one unit. However, the same Taylor operator $-t_{d+2}{}^\gamma$ subsequently acts on them. This means that those operators with $a=1$ are oversubtracted. We reexpress them by the set of normally subtracted ones with $\lambda$-dependent coefficients. After some analysis, it is found that the coefficient functions have the structure

$$C_{Nb} = m\lambda^{5-N}\xi_{Nb}{}^{(5-N)} + \lambda^{6-N}\xi_{Nb}{}^{(6-N)}, \qquad (9)$$

where $\xi$'s are dimensionless and $\lambda$ independent. We should remind ourselves that only $\xi_{5a}^{(0)}$ and $\xi_{6a}^{(0)}$ are physical.

The derivation of the scaling equations follow the usual argument. The starting point is to make use of the multiplicative nature of the renormalization. In the light theory without the muons, this is

$$[\Gamma^{B,F}(\mathcal{O}_{Ma})]_{\rm bare}=Z_2^{-F/2}Z_3^{-B/2}Z_{Ma,Nb}^{-1}\Gamma^{B,F}(\mathcal{O}_{Nb}),$$

where $Z_{2,3}$ are the wave-function renormalization constants and $Z$ is the operator renormalization matrix. We vary with respect to $m$ and hold $e_0$, $m_0$, and $\Lambda$ fixed. This yields (repeated indices summed)

$$\{[m(\partial/\partial m)+\beta_e(\partial/\partial e)-m(1-\beta_\lambda)(\partial/\partial\lambda)-B\gamma_A-F\gamma_e]\,\delta_{Ma,Nb}+\gamma_{Ma,Nb}\}\,\Gamma^{B,F}(\mathcal{O}_{Nb})=0, \tag{10}$$

where the $\beta$'s and $\gamma$'s are related to mass variation of $e$ and $Z$. The unfamiliar term $m\,d\lambda/dm\equiv-m(1-\beta_\lambda)$ arises because the renormalization procedure requires the electron bare mass to have the form $m_0=a_0+\lambda b_0$ with $a_0$ and $b_0$ independent of $\lambda$. A variation in $m$ induces a change in $\lambda$. Furthermore it is seen that $\beta_\lambda=\beta_\lambda^{(0)}+\lambda\beta^{(1)}/m$, where $\beta^{(0)}$ and $\beta^{(1)}$ are $\lambda$ independent. The mixing matrix elements $\gamma_{Ma,Nb}$ have the properties that (a) they vanish for $N>M$ and (b) they can be written as $\gamma_{Ma,Nb}=\lambda^{M-N}\gamma_{Ma,Nb}^{(M-N)}$, where $\gamma_{Ma,Nb}^{(M-N)}$ is $\lambda$ independent. The rest of the $\beta$'s and $\gamma$'s are also $\lambda$ independent. The equation for $\Gamma^{B,F}$ itself has the form of Eq. (10) in which the replacement $\delta_{Ma,Nb}\to1$, $\gamma_{Ma,Nb}\to0$ should be made.

Let us turn to the full theory, where only Green's functions are needed. A parallel argument leads to an equation similar to that for $\Gamma_{\rm light}^{B,F}$, except that all the $\beta$'s and $\gamma$'s now have rather complicated dependence on $\lambda$. Let us star them (e.g., $\beta_e^*$, etc.) to differentiate from those in the light theory. There is an extra term due to an induced change of $M$. This can be shown to give $1/M^4$ effects and can be dropped. After we substitute Eq. (1) into the scaling equation of the full theory and make use of Eq. (10), we find

$$\Gamma^{B,F}(\mathcal{O}_{Ma})\Big(\{m(\partial/\partial m)+\beta_e(\partial/\partial e)+[-m(1-\beta_\lambda^{(0)})+\lambda\beta_\lambda^{(1)}](\partial/\partial\lambda)\}\,\delta_{Ma,Nb}-\gamma_{Ma,Nb}^{T}\Big)C_{Nb}$$
$$+m^2[\Delta\gamma_A e(\partial/\partial e)-m\Delta\beta_\lambda(\partial/\partial\lambda)\ -B\Delta\gamma_A-F\Delta\gamma_e]\,\Gamma^{B,F}=0, \tag{11}$$

where we have defined $\gamma_A^*-\gamma_A=(m^2/M^2)\Delta\gamma_A$, etc., and superscript $T$ denotes transposition. In this expression, the Green's functions and the operator-inseted ones are all of the light theory. The $\ln(M/m)$-dependent quantities are $C_{Nb}$, $\Delta\beta_\lambda$, $\Delta\gamma_A$, and $\Delta\gamma_e$. We decouple Eq. (11) by looking into the two-, three-, and four-point functions and expanding them in powers of $\lambda$. After using some identities which account for the effects of $\mathcal{O}_{3a}$ and $\mathcal{O}_{4a}$ on general Feynman diagrams, we obtain

$$\{[m(\partial/\partial m)+\beta_e(\partial/\partial e)+1]\,\delta_{5a,5b}-\gamma_{5a,5b}^{t(0)}\}\xi_{5b}^{(0)}-(1-\beta_\lambda^{(0)})\xi_{5a}^{(1)}=0, \tag{12}$$

$$[m(\partial/\partial m)+\beta_e(\partial/\partial e)+\beta_\lambda^{(1)}]\xi_{5a}^{(1)}-\gamma_{5a,5b}^{t(0)}\xi_{5b}^{(1)}-\gamma_{5a,6b}^{(1)}\xi_{6b}^{(0)}=0, \tag{13}$$

$$[(m(\partial/\partial m)+\beta_e(\partial/\partial e)\delta_{6a,6b}-\gamma_{6a,6b}^{t(0)}]\,\xi_{6b}^{(0)}=0. \tag{14}$$

It is surprising that all we need in solving these equations are $\beta$'s and $\gamma$'s of the light theory and the values of $\xi_{5a}^{(0,1)}$ and $\xi_{6a}^{(0)}$ at a point in $M/m$. Because the $\beta$'s and $\gamma$'s do not have large $\ln(M/m)$'s, we can calculate them perturbatively. After we feed in the one-loop information, we obtain the leading-logarithm result[8]

$$\xi_{61}^{(0)}=\xi_{64}^{(0)}=-(\tfrac{3}{16})(\alpha/15\pi)(y-y^{-1}),\quad \xi_{63}^{(0)}=-(\tfrac14)(\alpha/15\pi)(y-y^{-1}),\quad \xi_{65}^{(0)}=(\tfrac12)(\alpha/15\pi)(y+y^{-1}),$$
$$\xi_{51}^{(0)}=\xi_{51}^{(1)}=-\xi_{52}^{(0)}=-\xi_{52}^{(1)}=(\alpha/15\pi)[(\tfrac38)y-(\tfrac{27}{8})y^{-1}+3y^{-5/4}]; \tag{15}$$

all other $\xi$'s $=0$ (i.e., nonleading);

where $y=(\bar e/e)^2=1/[1-(\frac83)(\alpha/4\pi)\ln(M/m)]$ is the factor for the running coupling constant $\bar e$. We have explicitly verified these $\xi$'s to the two-loop order.

As an application we have used these results to assess the effects of muons on the electron anomalous magnetic moment. Taking the on-shell corrections into account, we found that to all orders in $\alpha$ the leading logarithms cancel.[9] The effects are thus of order $(m/M)^2\alpha^2[\alpha\ln(M/m)^2]^n$. The details of this calculation will be reported elsewhere.

We are presently extending this work to quantum chromodynamics. In particular we hope to have an interpolation formula for $e^+e^-\to$ anything before and after various quark thresholds are passed.

One of us (Y.K.) would like to thank Professor Trueman and the members of the theory group at Brookhaven National Laboratory for hospitality, where a part of this work was done. This work has been partially supported by the U. S. Department of Energy.

[a] Present address: Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, Ill. 60510.

[1] T. Appelquist and J. Carazzone, Phys. Rev. D 11, 2856 (1975).

[2] E. G., J. Collins, F. Wilczek, and A. Zee, Phys. Rev. D 18, 242 (1978); D. Toussaint, Phys. Rev. D 18, 1626 (1978).

[3] Yoichi Kazama and York-Peng Yao, to be published.

[4] C. Callan, Phys. Rev. D 2, 1541 (1970); K. Symanzik, Commun. Math. Phys. 18, 227 (1970).

[5] W. Zimmermann, in *Lectures on Elementary Particles and Quantum Field Theory*, edited by S. Deser *et al.* (MIT Press, Cambridge, Mass., 1971), Vol. I, p. 397.

[6] S. L. Adler and W. A. Bardeen, Phys. Rev. D 4, 3045 (1971).

[7] C. K. Lee, Ph.D. thesis, Stanford University, 1975 (unpublished).

[8] The corresponding operators are

$$\mathcal{O}_{51-52} = i\int d^4x\,\bar{\psi}[(iD)^2, (e/2)\sigma_{\mu\nu}F^{\mu\nu}]\psi,$$

$$\mathcal{O}_{61-65} = i\int d^4x\,\bar{\psi}[(iD)^2 i\not{D}, eF_{\mu\nu}\gamma^\nu D^\mu, e\gamma^\mu(\partial^\nu F_{\mu\nu}), -ieF_{\mu\nu}\sigma^{\mu\nu}\not{D}/2, F_{\mu\nu}\partial^2 F^{\mu\nu}/4]\psi;$$

$\mathcal{O}_{66-68}$ are four-fermion operators.

[9] It may be noted that the lowest-order contribution to electron $(g-2)/2$ is $(\alpha/\pi)^2(m/M)^2\times 1/45$ [B. E. Lautrup and E. de Rafael, Phys. Rev. 174, 1835 (1968)]. The question arises whether the next-order correction may be of order $(\alpha/\pi)^3(m/M)^2[\ln(m/M)]^2$, which is comparable to $(\alpha/\pi)^4$ and therefore should become important in view of the work in progress by T. Kinoshita. Our answer is that such effects due to muons do not exist.

# Baryon- and Lepton-Nonconserving Processes

Steven Weinberg
*Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, and Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138*
(Received 13 August 1979)

A number of properties of possible baryon- and lepton-nonconserving processes are shown to follow under very general assumptions. Attention is drawn to the importance of measuring $\mu^+$ polarizations and $\bar{\nu}_e/e^+$ ratios in nucleon decay as a means of discriminating among specific models.

Of the supposedly exact conservation laws of physics, two are especially questionable: the conservation of baryon number and lepton number. As far as we know, there is no necessity for an *a priori* principle of baryon and lepton conservation. As we shall see, even without such a principle, the fact that the weak, electromagnetic, and strong interactions of ordinary quarks and leptons conserve baryon and lepton number can be understood as simply a consequence of the $SU(2)\otimes U(1)$ and $SU(3)$ gauge symmetries. Also, in contrast with the conservation of charge, color, and energy and momentum, the conservation of baryon number and lepton number are almost certainly not unbroken local symmetries.[1] Not only is baryon conservation unnecessary as a fundamental principle, the apparent excess of baryons over antibaryons in our universe provides a positive clue that some sort of physical processes have actually violated baryon-number conservation.[2] Violations of baryon and lepton conservation are likely to occur in grand unified theories that combine the gauge theory of weak and electromagnetic interactions with that of strong interactions and have leptons and quarks in the same gauge multiplets, and such violations have been found in various of these models.[3]

The purpose of this paper is to point out those features of baryon- or lepton-nonconserving processes that are to be expected on very general grounds. Other features will be indicated that may be used to discriminate among specific models.

No grand unified model or other specific gauge model of baryon- and lepton-nonconserving processes will be adopted here. Instead, it will simply be assumed that these processes are mediated by some unspecified "superheavy" particles, with a characteristic mass $M$ above, say, $10^{14}$ GeV. Such large masses are indicated by the experimental lower bound[4] on the proton lifetime, and are also required in order that these parti-